COLOOQUIUM PACO YNDURAIN
MADRID, MARCH 6th, 2019

EMERGENT GRAVITY IN AN ENTANGLED UNIVERSE

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University of Amsterdam

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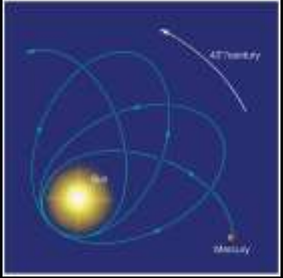
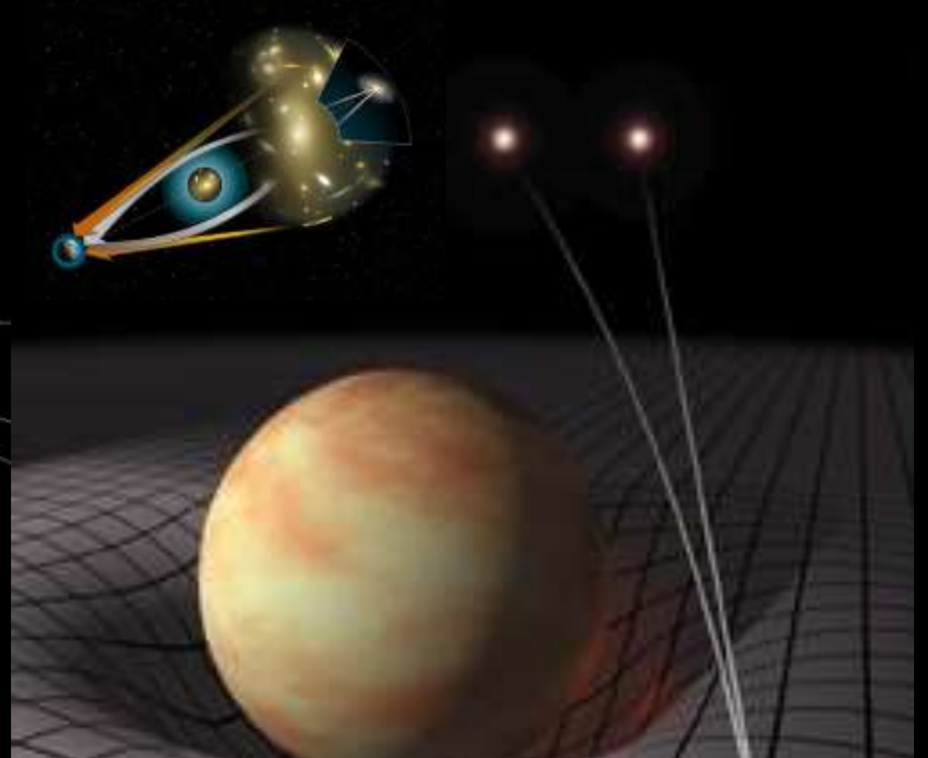
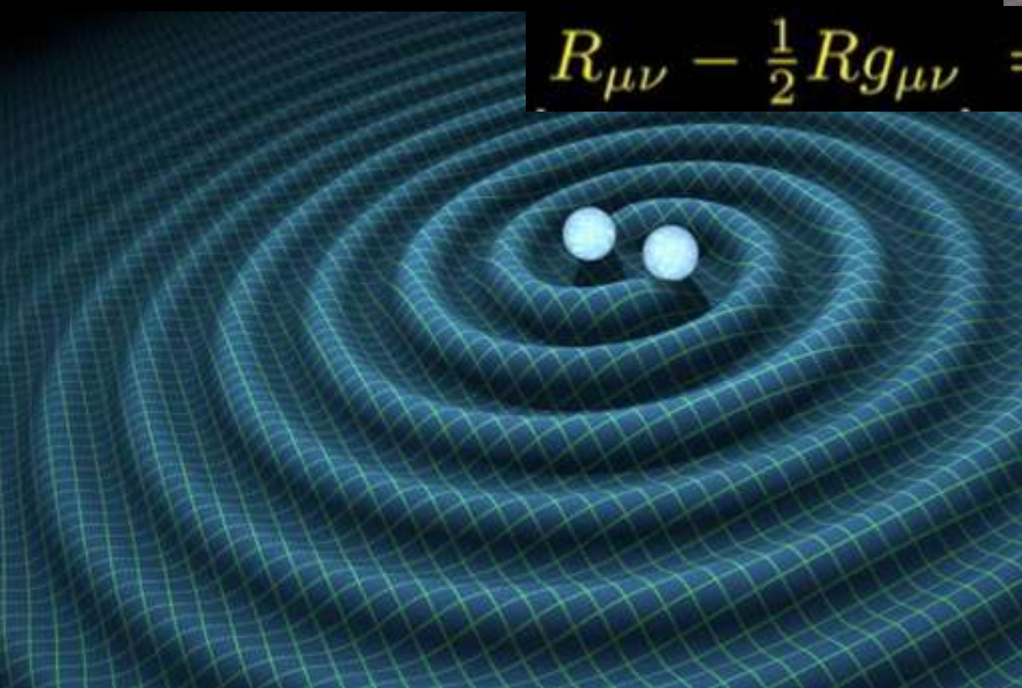
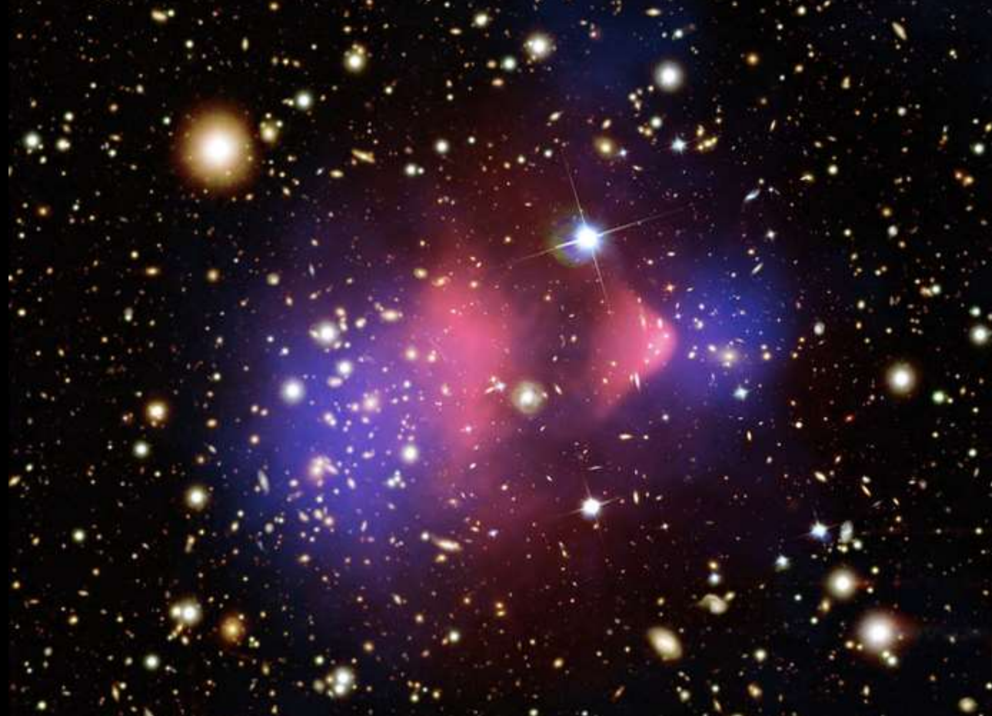
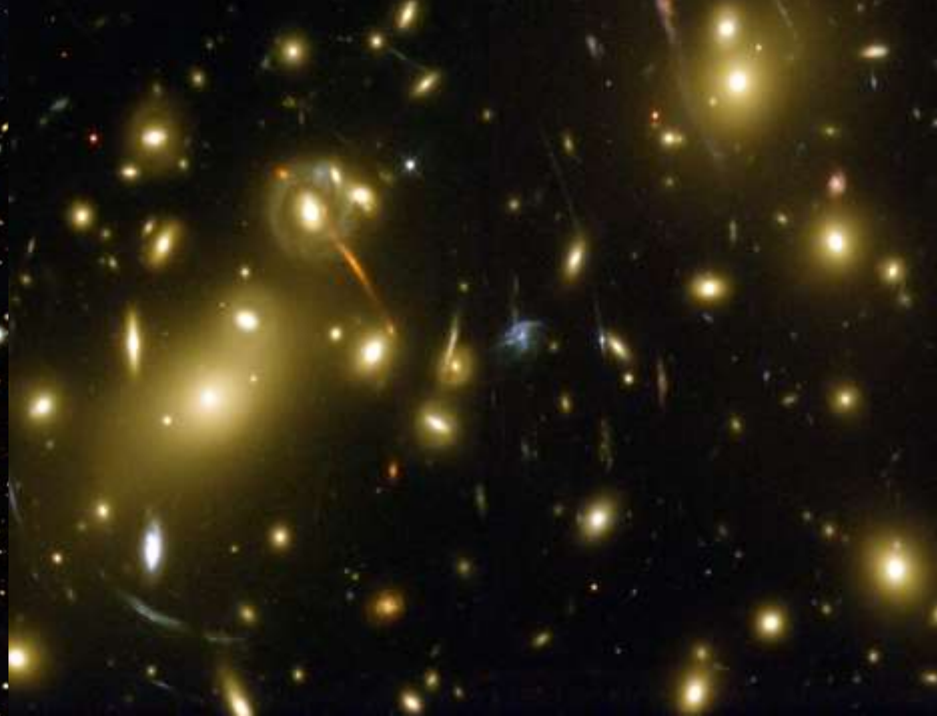


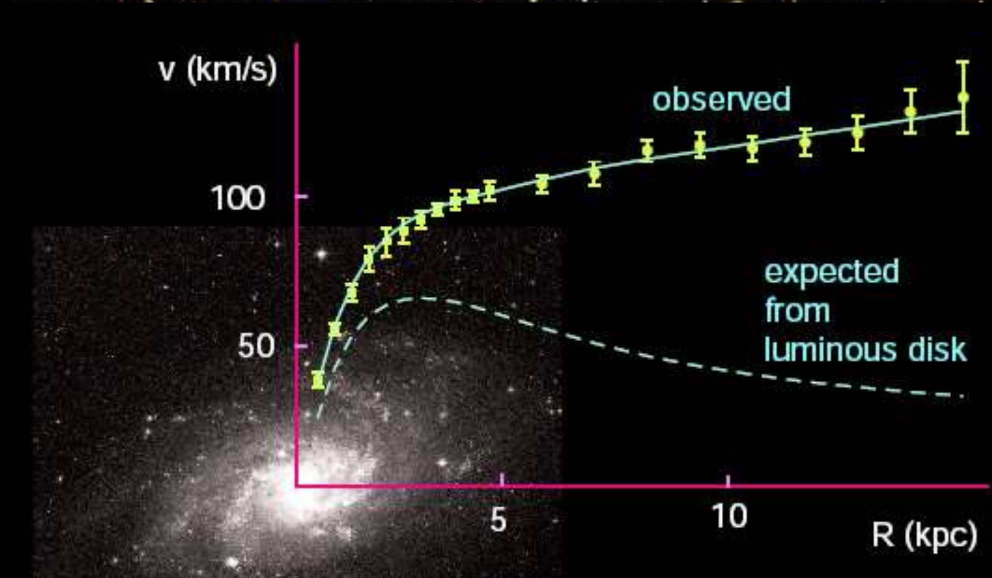


$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = (8\pi G)T_{\mu\nu}$$



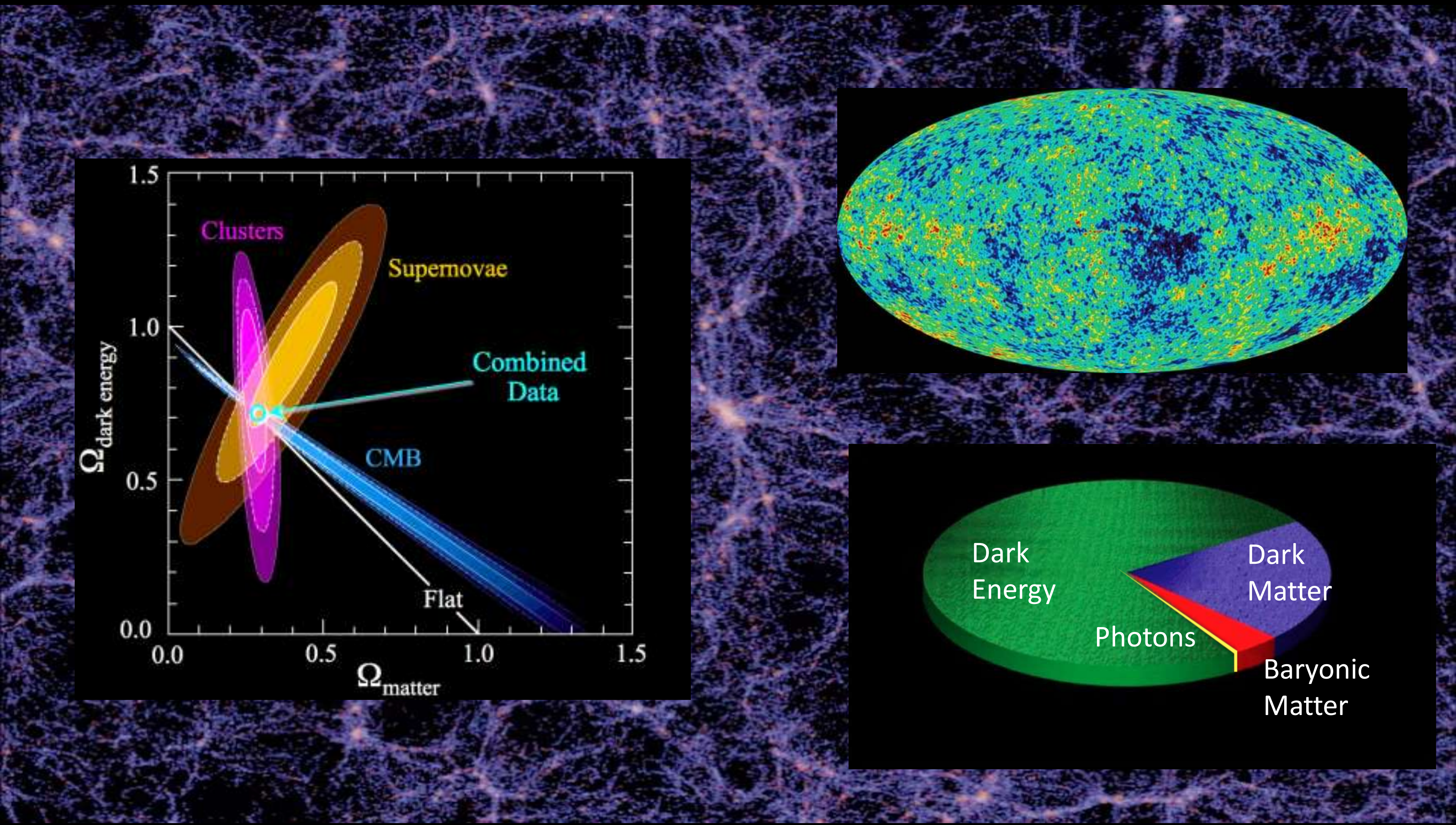






M33 rotation curve

(fig. 1)



1.5
1.0
0.5
0.0
0.0
0.5
1.0
1.5
$\Omega_{\text{dark energy}}$
Ω_{matter}
Clusters
Supernovae
Combined Data
CMB
Flat
Dark Energy
Dark Matter
Photons
Baryonic Matter

1. Gravity is accurately described by General Relavity, in particular in circumstances with `high acceleration'.

2. GR is a conceptually elegant and convincing Theory based on 1) the equivalence principle, 2) coordinate invariance.

3. Until recently there was no conceptual reason why GR would fail at `large distances' or `low acceleration'.

4. The Dark Matter Hypothesis appears well motivated.

5. `Proof ' of existence of DM is based:
    - firm believe in GR (points 2. and 3.)
    - phenomological successes.

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On the origin of gravity and the laws of Newton

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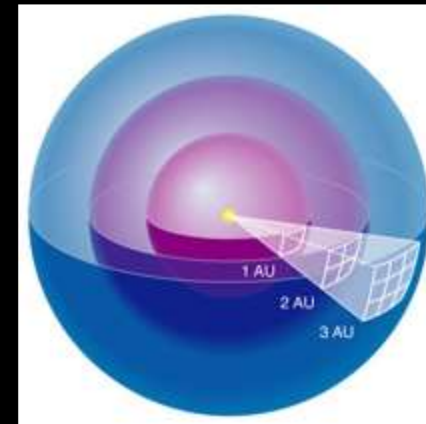
ABSTRACT: Starting from first principles and general assumptions we present a heuristic argument that shows that Newton's law of gravitation naturally arises in a theory in which space emerges through a holographic scenario. Gravity is identified with an entropic force caused by changes in the information associated with the positions of material bodies. A relativistic generalization of the presented arguments directly leads to the Einstein equations. When space is emergent even Newton's law of inertia needs to be explained. The equivalence principle suggests that it is actually the law of inertia whose origin is entropic.



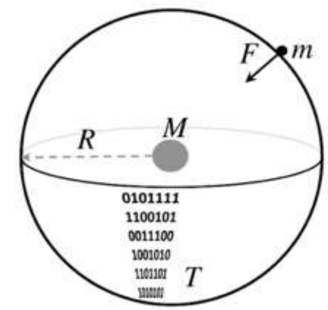


Figure 3. A particle with mass m near a spherical holographic screen. The energy is evenly distributed over the occupied bits, and is equivalent to the mass M that would emerge in the part of space surrounded by the screen.

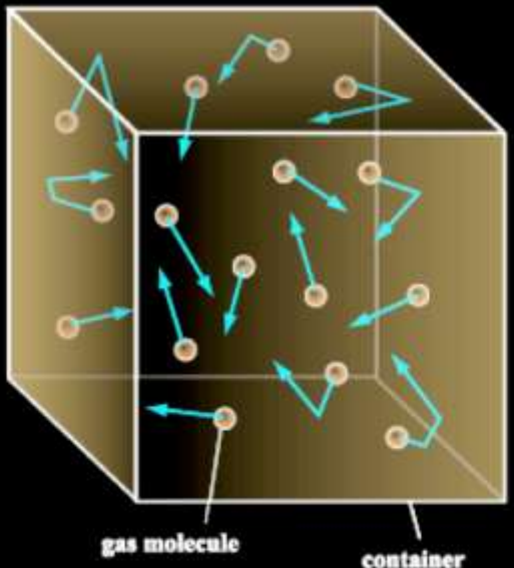
and one obtains the familiar law:

$$F = G\frac{Mm}{R^2}. \tag{3.9}$$

We have recovered Newton's law of gravitation, practically from first principles!

Emergence

'The whole is more than the sum of its parts.'



We use concepts and observe phenomena at a macroscopic scale that are derived from a microscopic scale where they have no a priori meaning.

Temperature: T

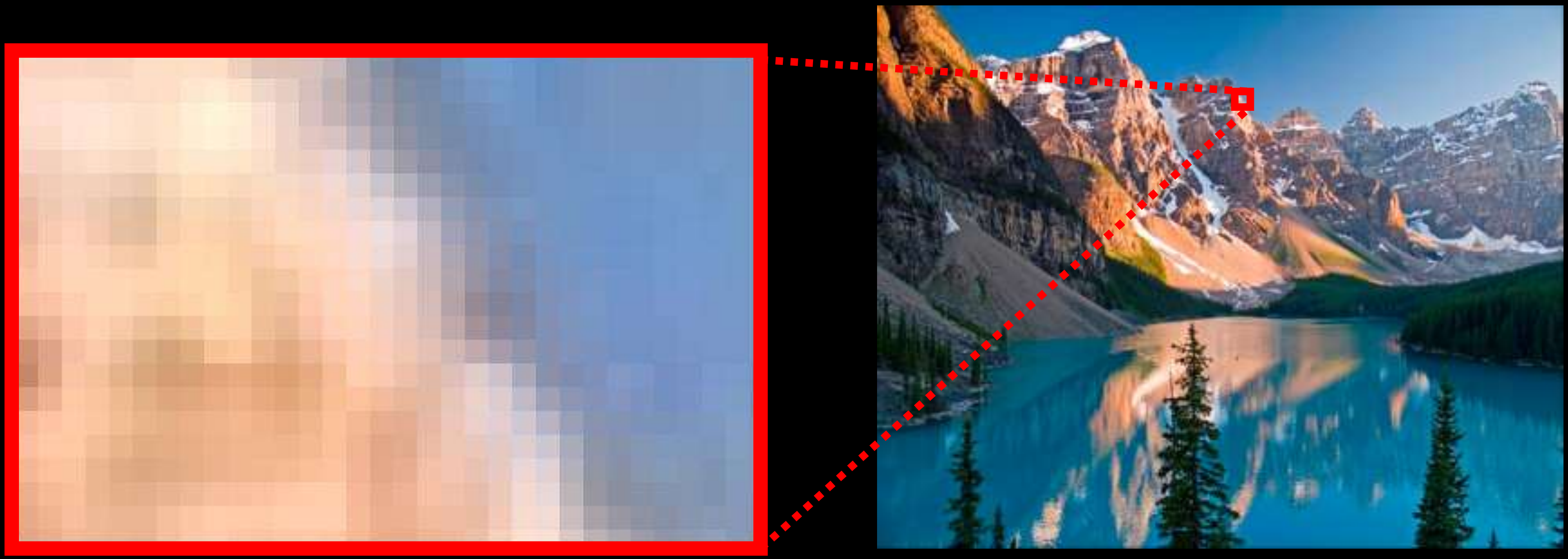
Entropy: S

1st Law: $dE = TdS$

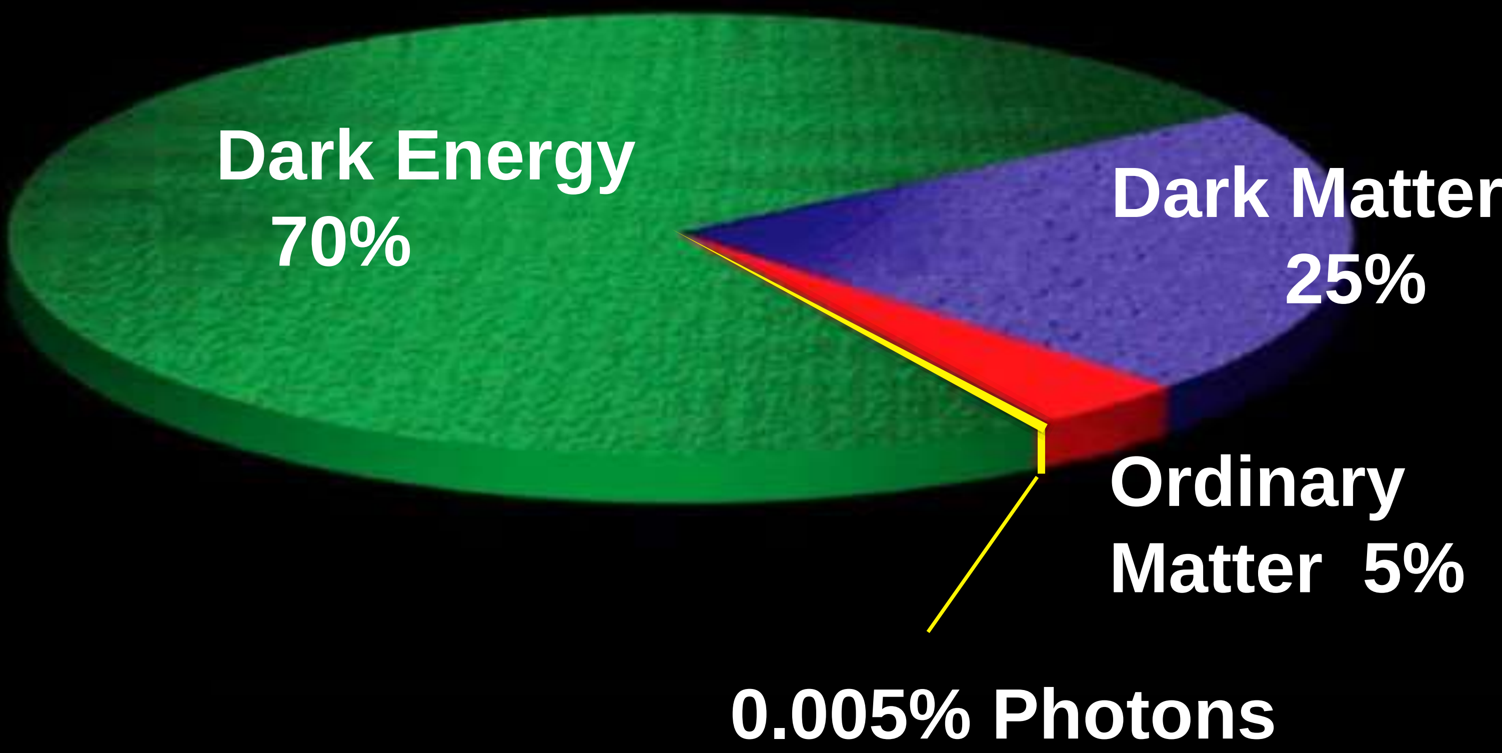
Emergence

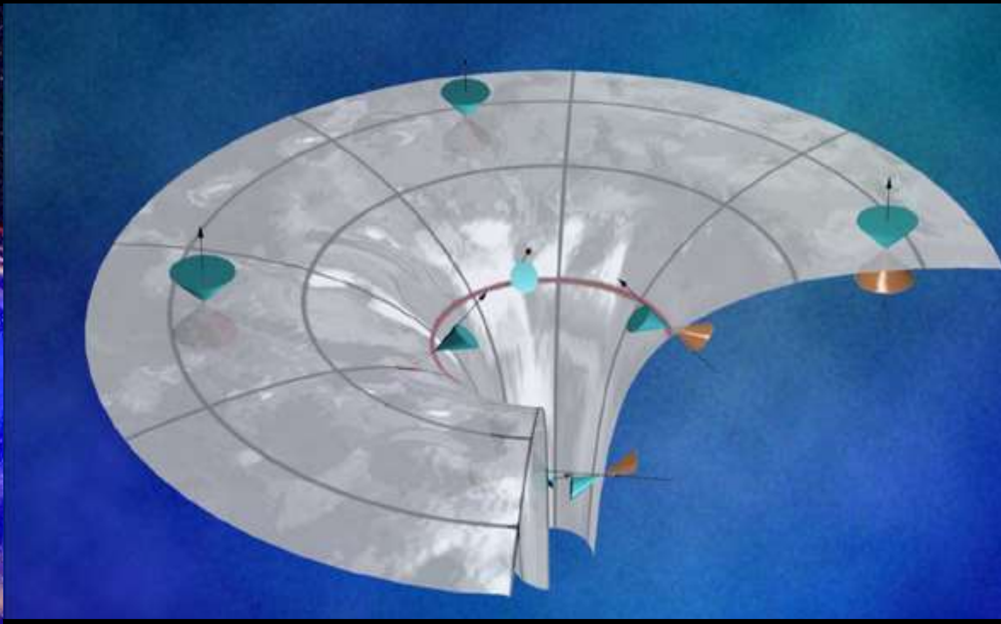
'The whole is more than the sum of its parts.'

We use concepts and observe phenomena at a macroscopic scale that are derived from a microscopic scale where they have no a priori meaning.



Cosmological energy budget

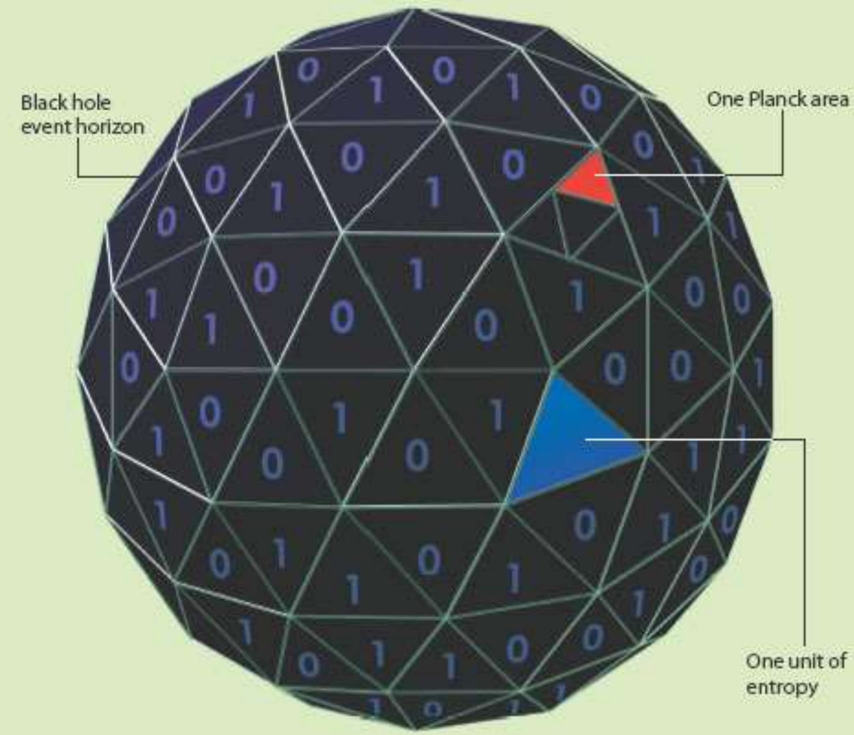




$$ds^2 = -\left(1 - 2GM/r\right) dt^2 + \frac{dr^2}{1 - 2GM/r} + r^2 d\Omega^2$$

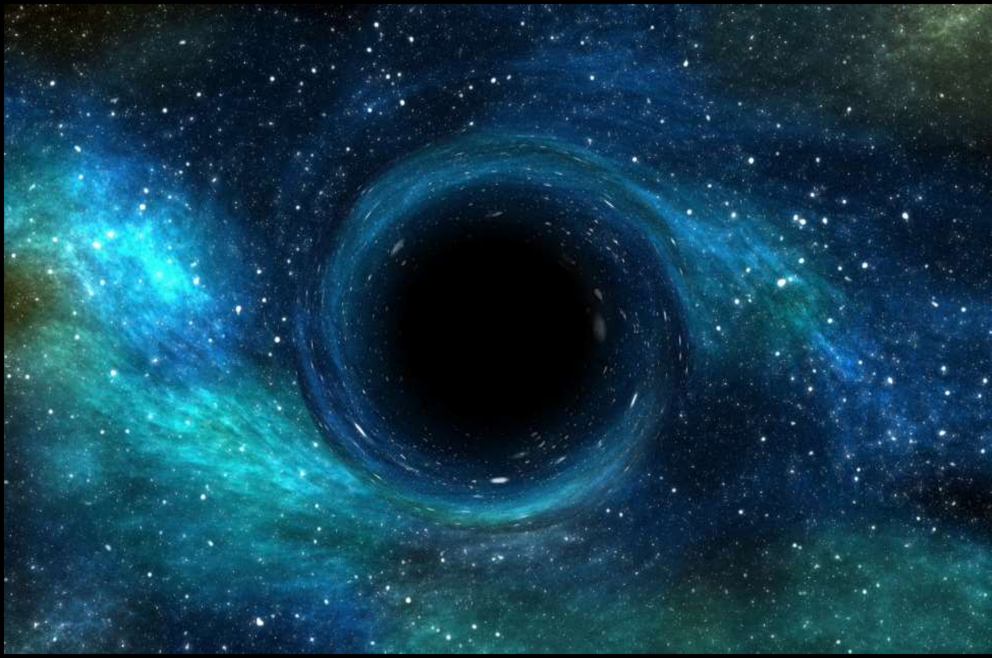
Bekenstein-Hawking Entropy Formula

$$S = k\frac{A\,c^3}{4G\hbar}$$



ENTROPY OF A BLACK HOLE is proportional to the area of its event horizon, the surface from within which even light cannot escape the gravity of the hole. Specifically, a hole with a horizon spanning A Planck areas has $A/4$ units of entropy. (The Planck area, approximately 10^{-66} square centimeter, is the fundamental quantum unit of area determined by the strength of gravity, the speed of light and the size of quanta.) Considered as information, it is as if the entropy were written on the event horizon, with each bit (each digital 1 or 0) corresponding to four Planck areas.

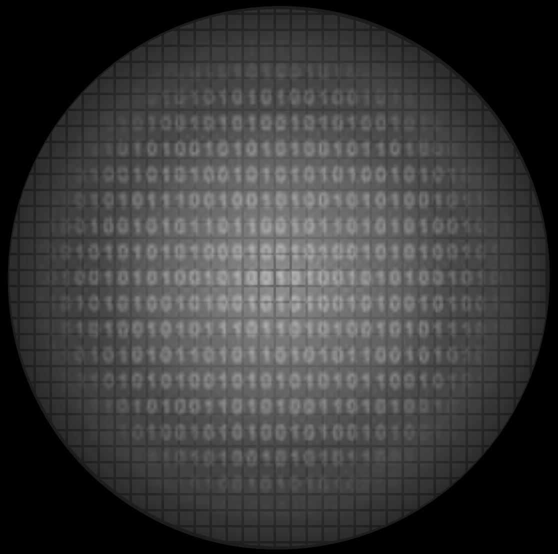
Black Holes



Bekenstein & Hawking:

Black holes carry entropy

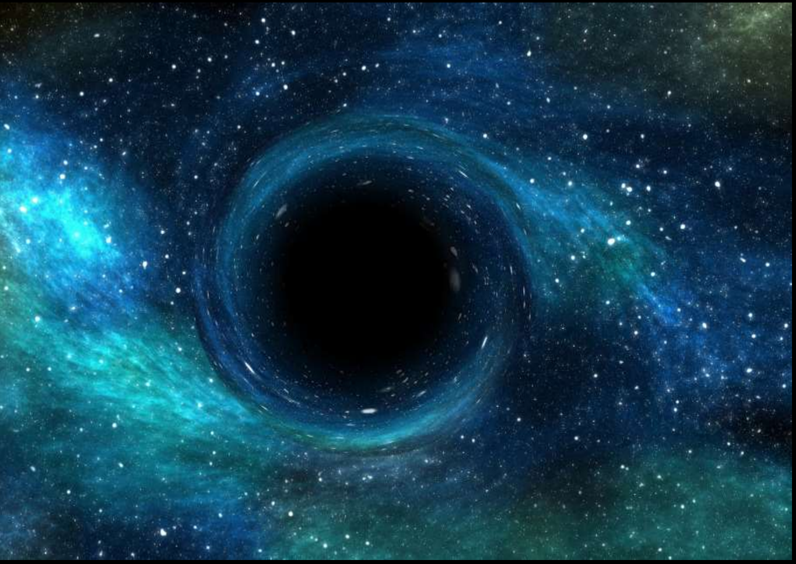
$$S = k_B \frac{Ac^3}{4\pi G\hbar}$$

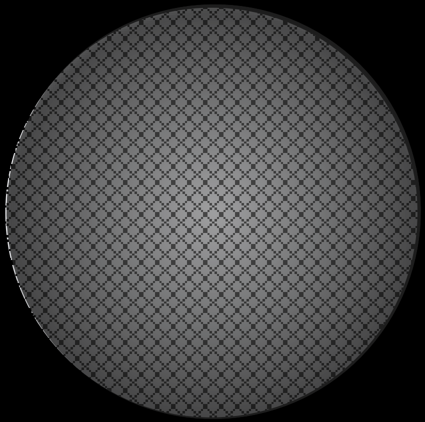


and have a temperature

$$k_B T = \frac{\hbar\kappa}{2\pi c}$$

$\kappa = \text{surface gravity}$





$$S = k_B \frac{Ac^3}{4\pi G\hbar}$$

$$k_B T = \frac{\hbar\kappa}{2\pi c}$$

Black hole hole thermodynamics relates Einstein equations
to 1st law of thermodynamics = derivable from microscopic theory

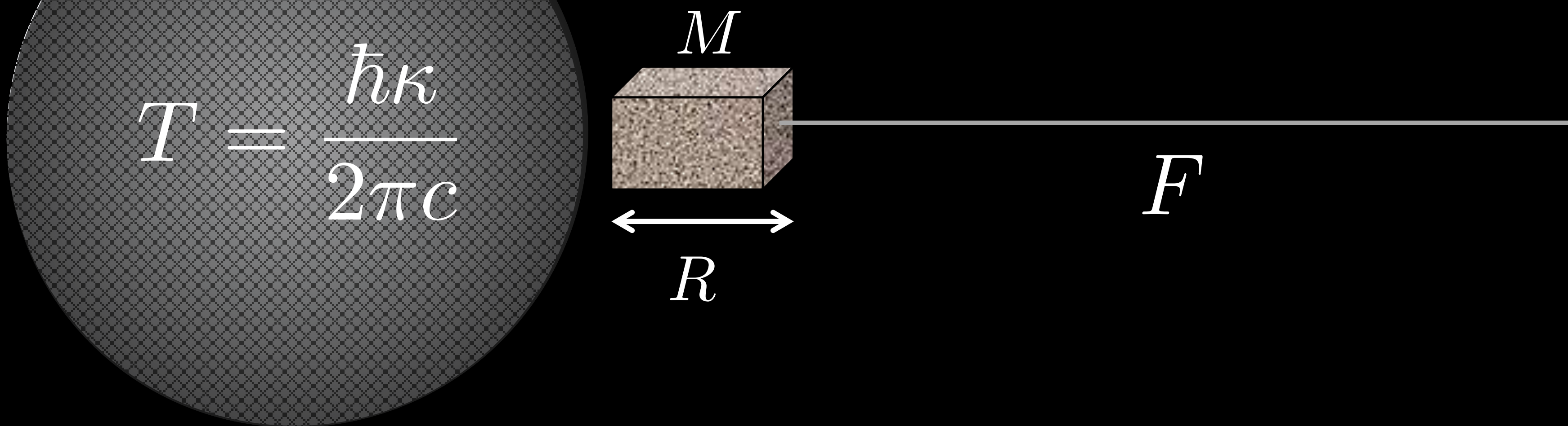
$$dM = \frac{\kappa}{2\pi}\frac{dA}{4G}$$

$$dE = TdS$$

$$\kappa = \text{surface gravity}$$

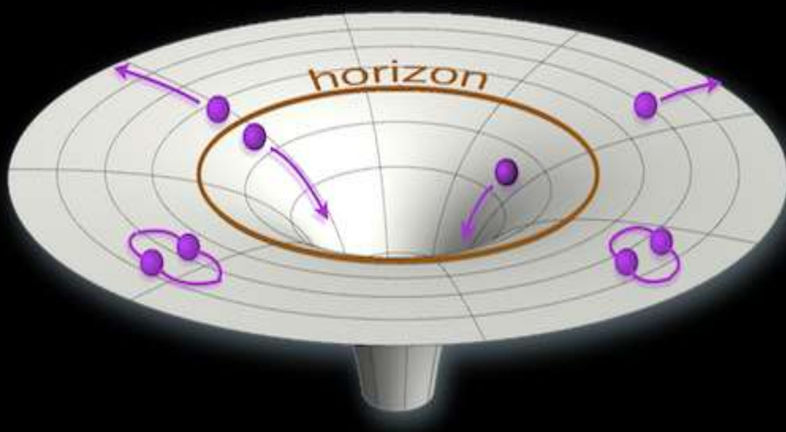
Emergent gravity = derivable
from microscopic theory!

$$\kappa = \text{surface gravity} = \frac{F}{M}$$

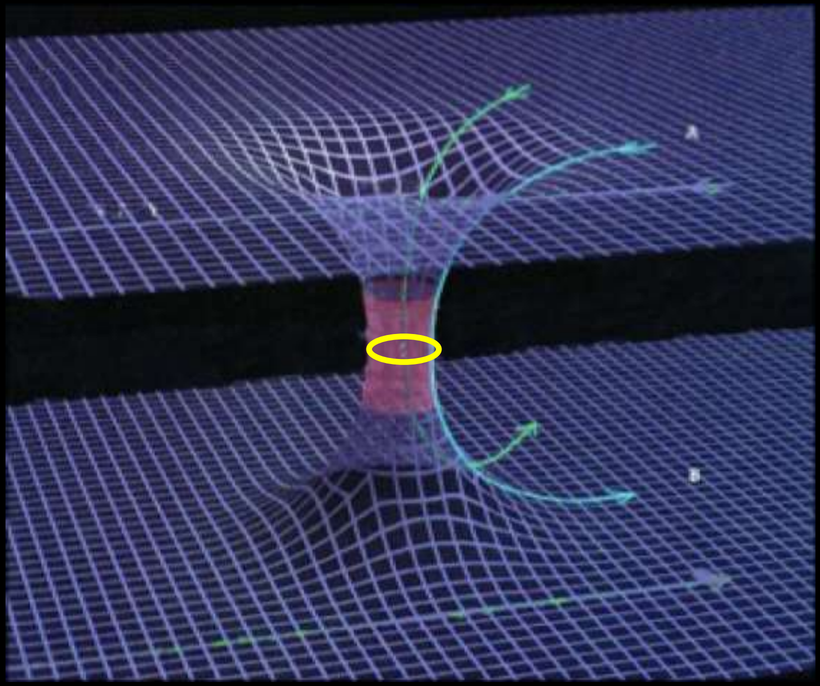


Bekenstein bound: the entropy contained inside a region of size *R* and mass *M* is bounded.

$$F \cdot R = TS \quad \Longrightarrow \quad S = 2\pi \frac{McR}{\hbar}$$



Pair-production s

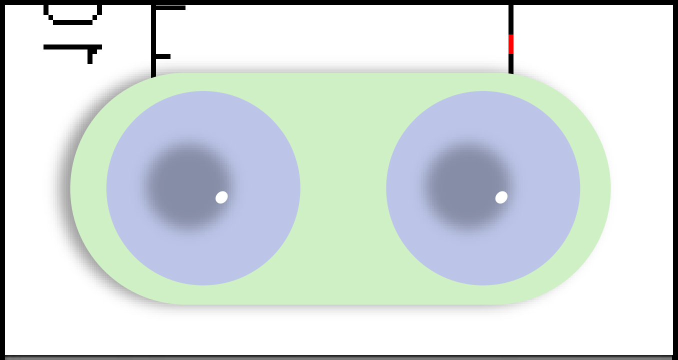


Einstein-Rosen-bridge

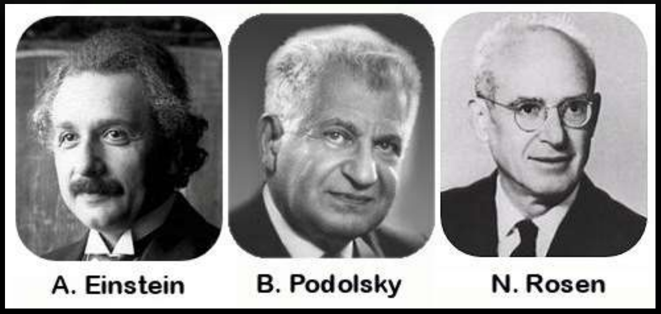
Entanglement of Quantum Information

qubit: $\alpha|0\rangle + \beta|1\rangle$

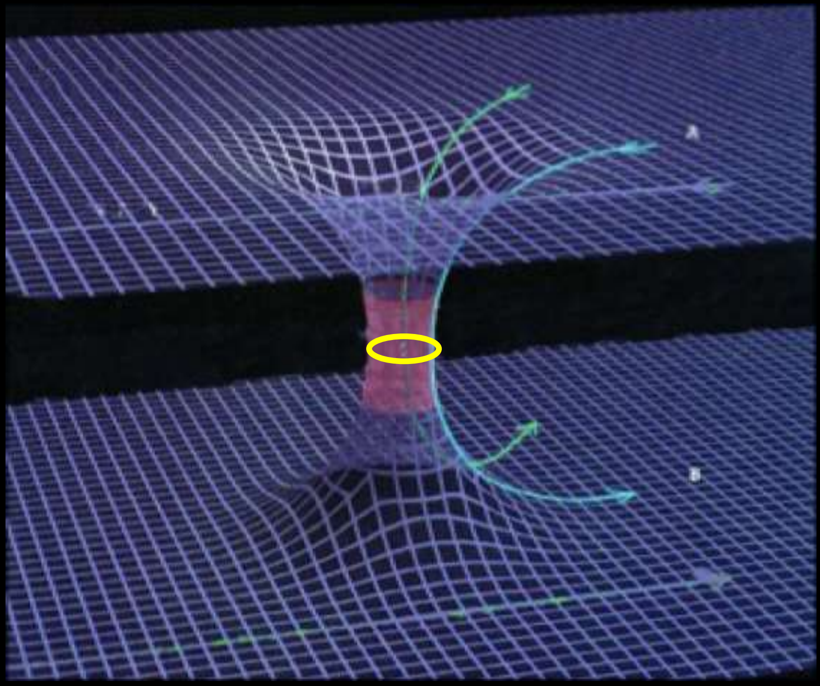
$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$



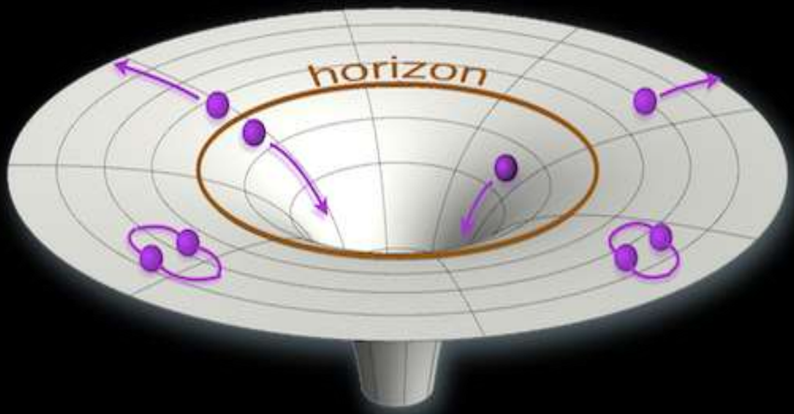
EPR pair



Worm holes and Entanglement: ER=EPR



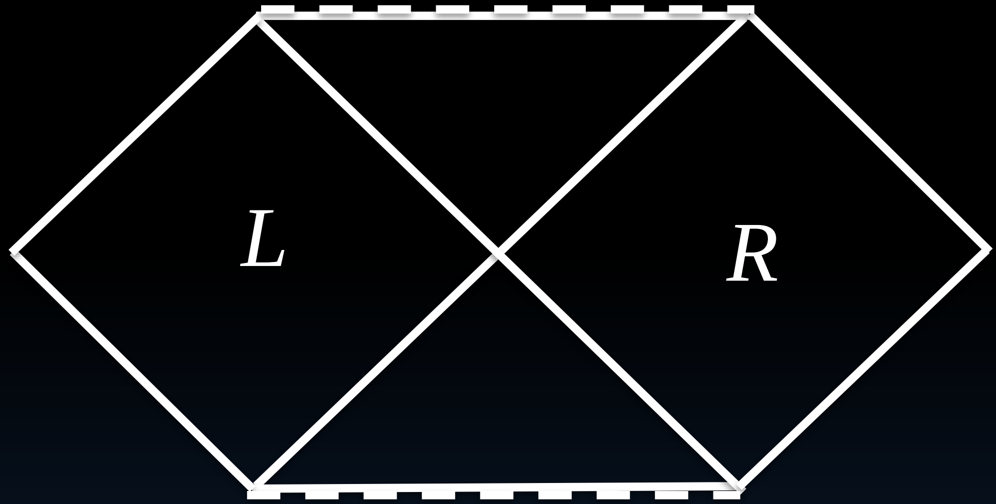
Einstein-Rosen-bridge

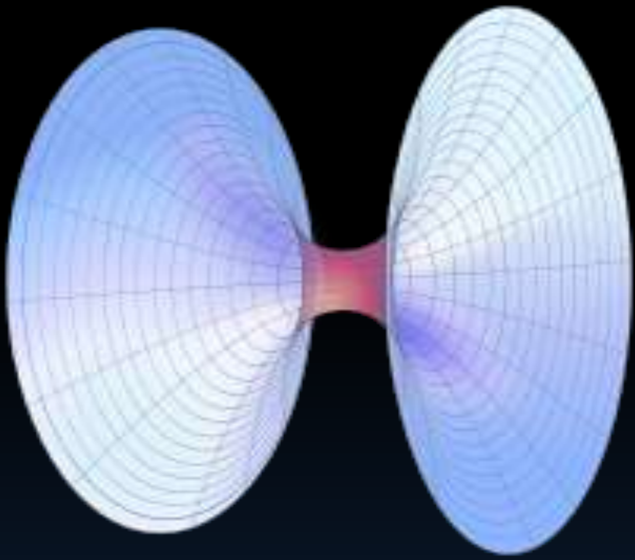


Quantum Entanglement
(Einstein-Podolsky-Rosen)

EPR = ER

Einstein-Rosen bridge





Microscopic BH-vacuum state

$$|vac\rangle_{BH} = \frac{1}{\sqrt{Z}} \sum_i |E_i\rangle_L |E_i\rangle_R e^{-\beta E_i/2}$$

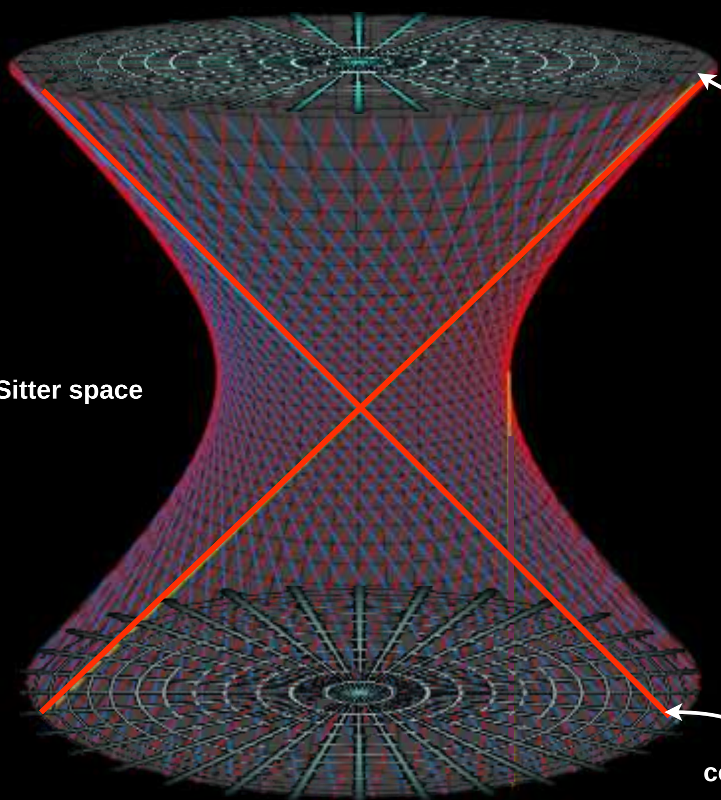
Entanglement => connectivity of spacetime

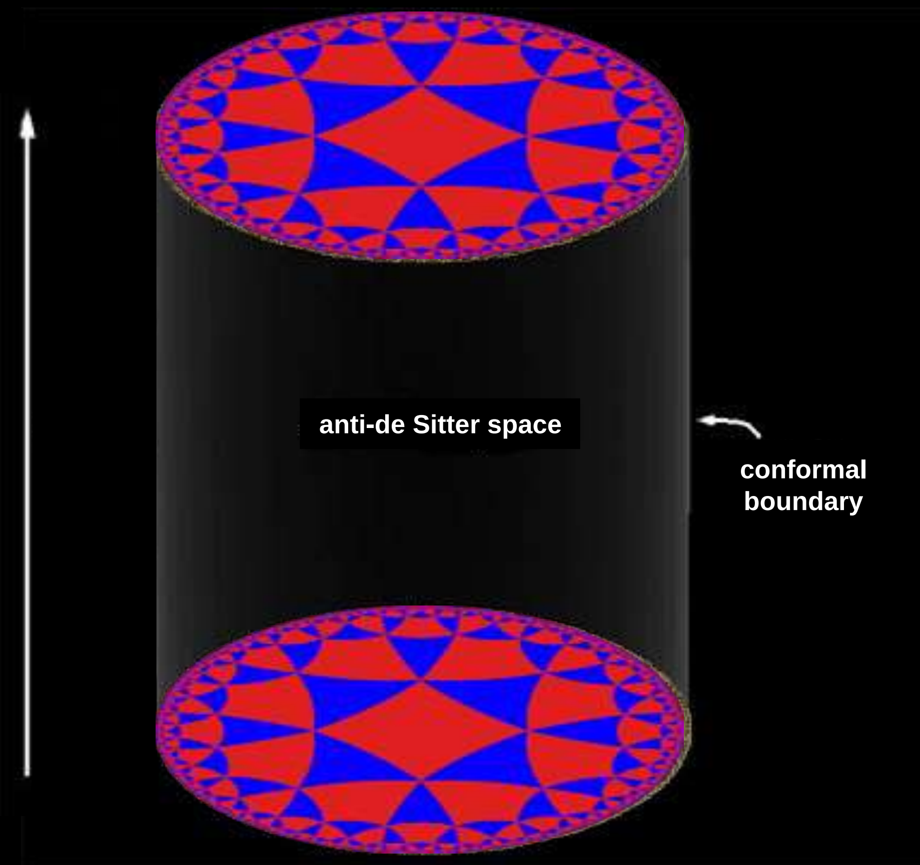
$$S_{ent} = \frac{A}{4G\hbar}$$

Van Raamsdonk
Maldacena-Susskind
E.V.- H.Verlinde

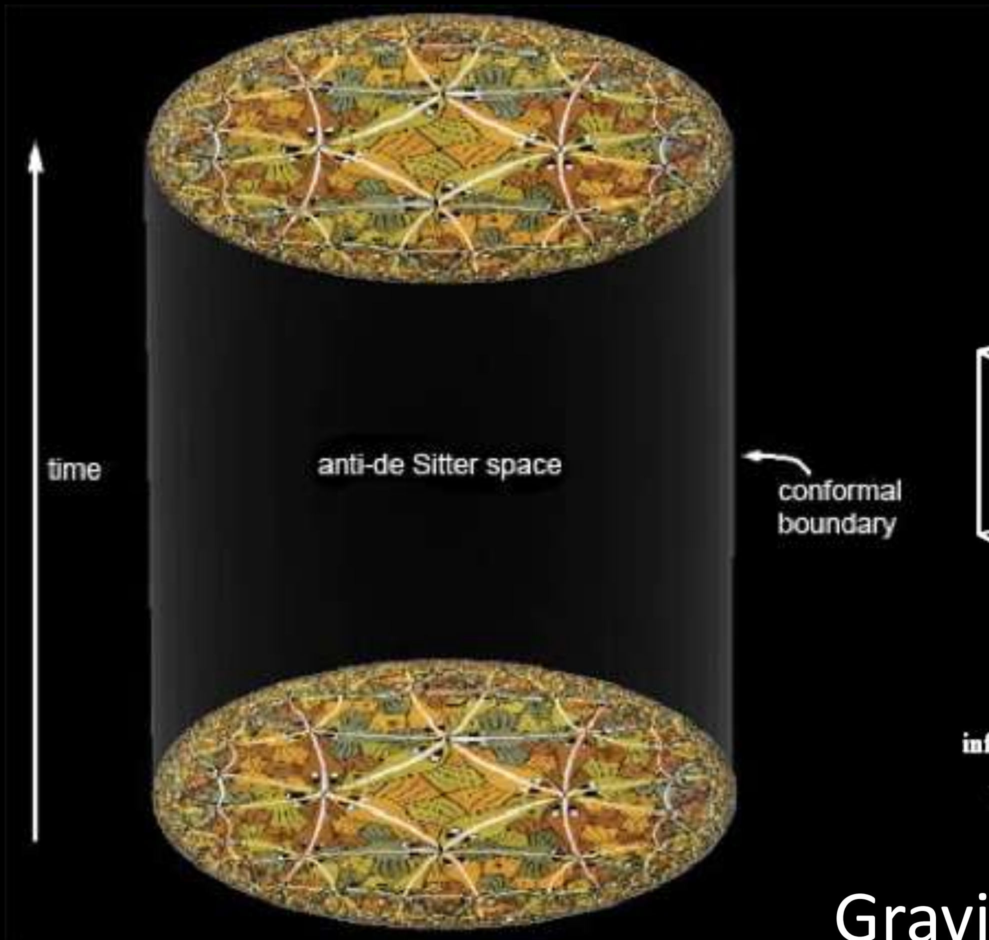
(Anti-) de Sitter space

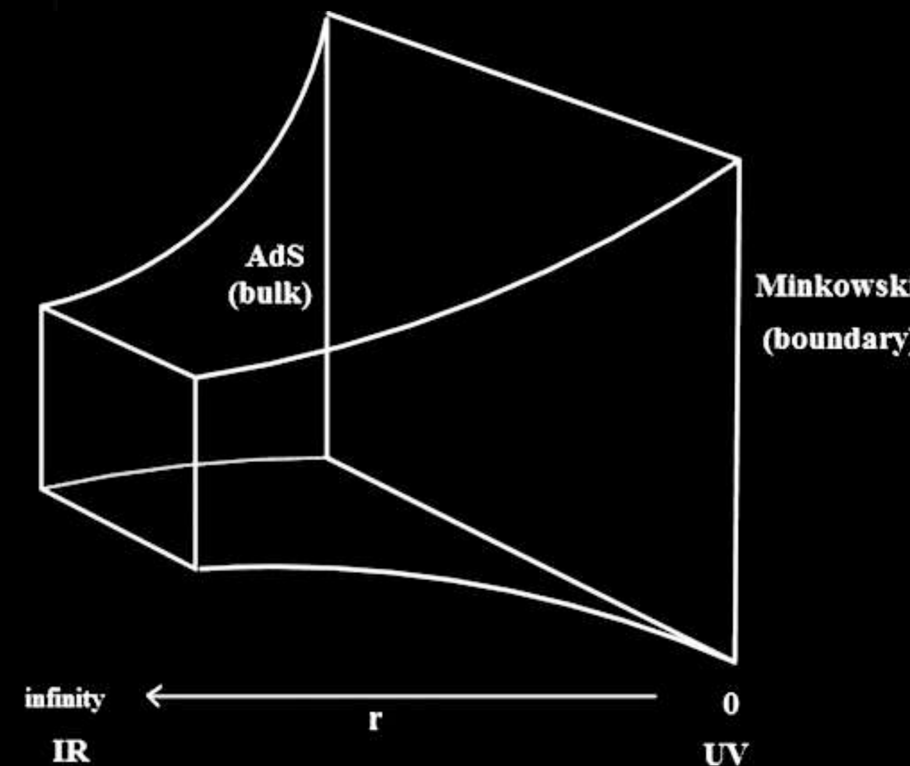
$$ds^2 = -(1 \pm R^2/L^2)dt^2 + \frac{dR^2}{1 \pm R^2/L^2} + R^2 d\Omega^2$$





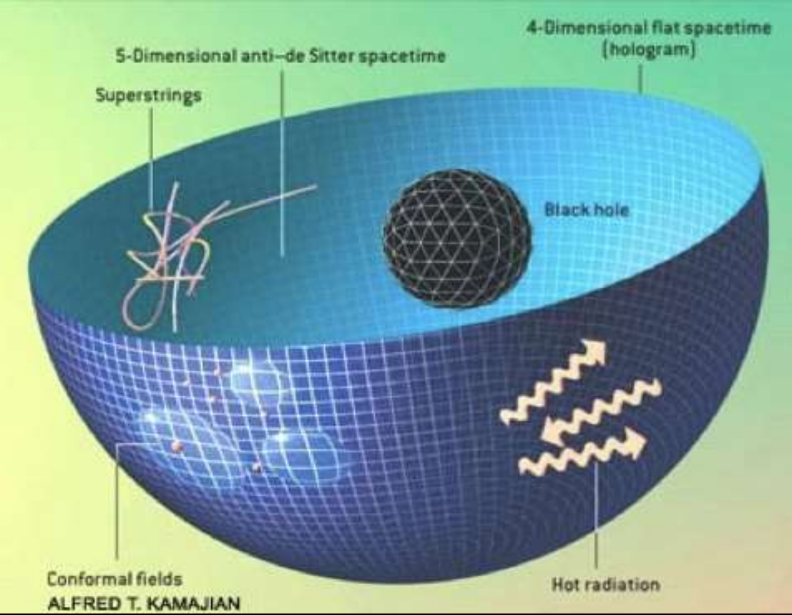
Anti-de Sitter/Conformal Field Theory Correspondence

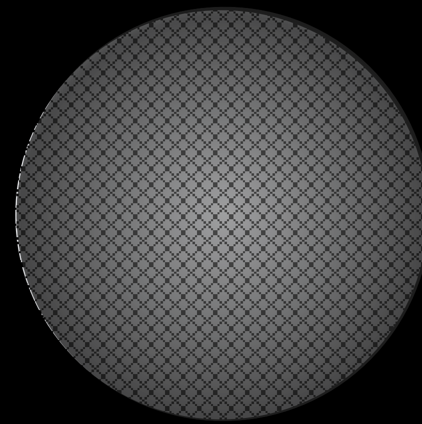




AdS spacetime = groundstate

Gravity in the bulk spacetime emerges from microscopic Theory (= CFT) on the boundary

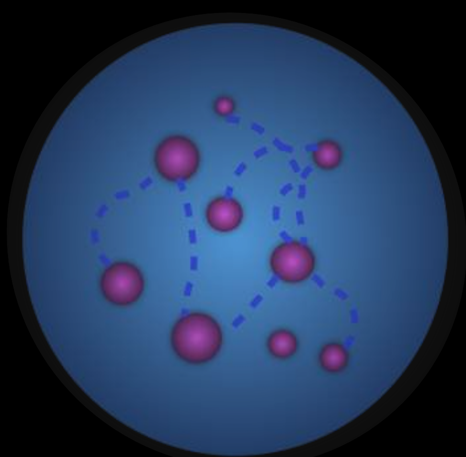




$$S = k_B \frac{Ac^3}{4\pi G\hbar}$$

$$k_B T = \frac{\hbar\kappa}{2\pi c}$$

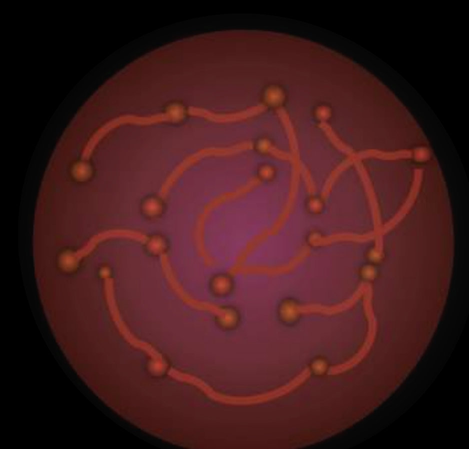
Microscopic explanation of black hole entropy: not possible in terms of ordinary phases or matter, but requires a different `entropic' phase (of string theory) with extreme high density of states and low temperature.



Coulomb branch

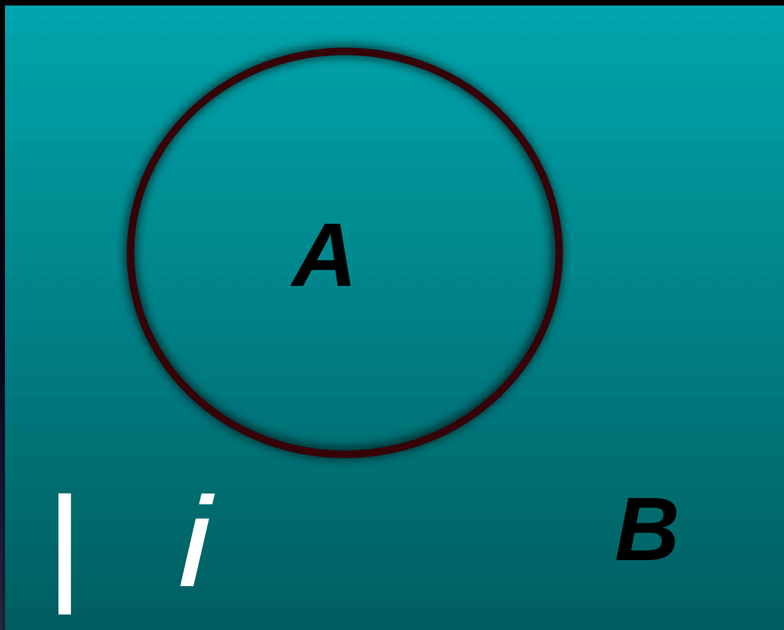
Thermalization

Deconfinement



Higgs branch

Entanglement entropy



$$\rho_A = \mathrm{tr}_{\mathcal{H}_B}(|\psi\rangle\langle\psi|)$$

$$S_A = -\mathrm{tr}_{\mathcal{H}_A}(\rho_A \log \rho_A)$$

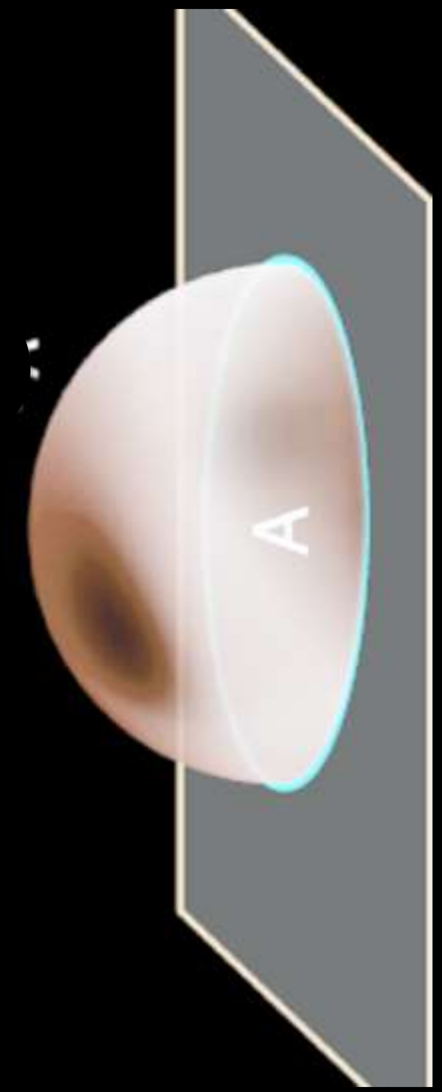
The entanglement entropy measures the number of "entangled Bell pairs" that connect the regions *A* en *B*. One has

$$S_A = S_B$$

In AdS/CFT the
Bekenstein-Hawking formula

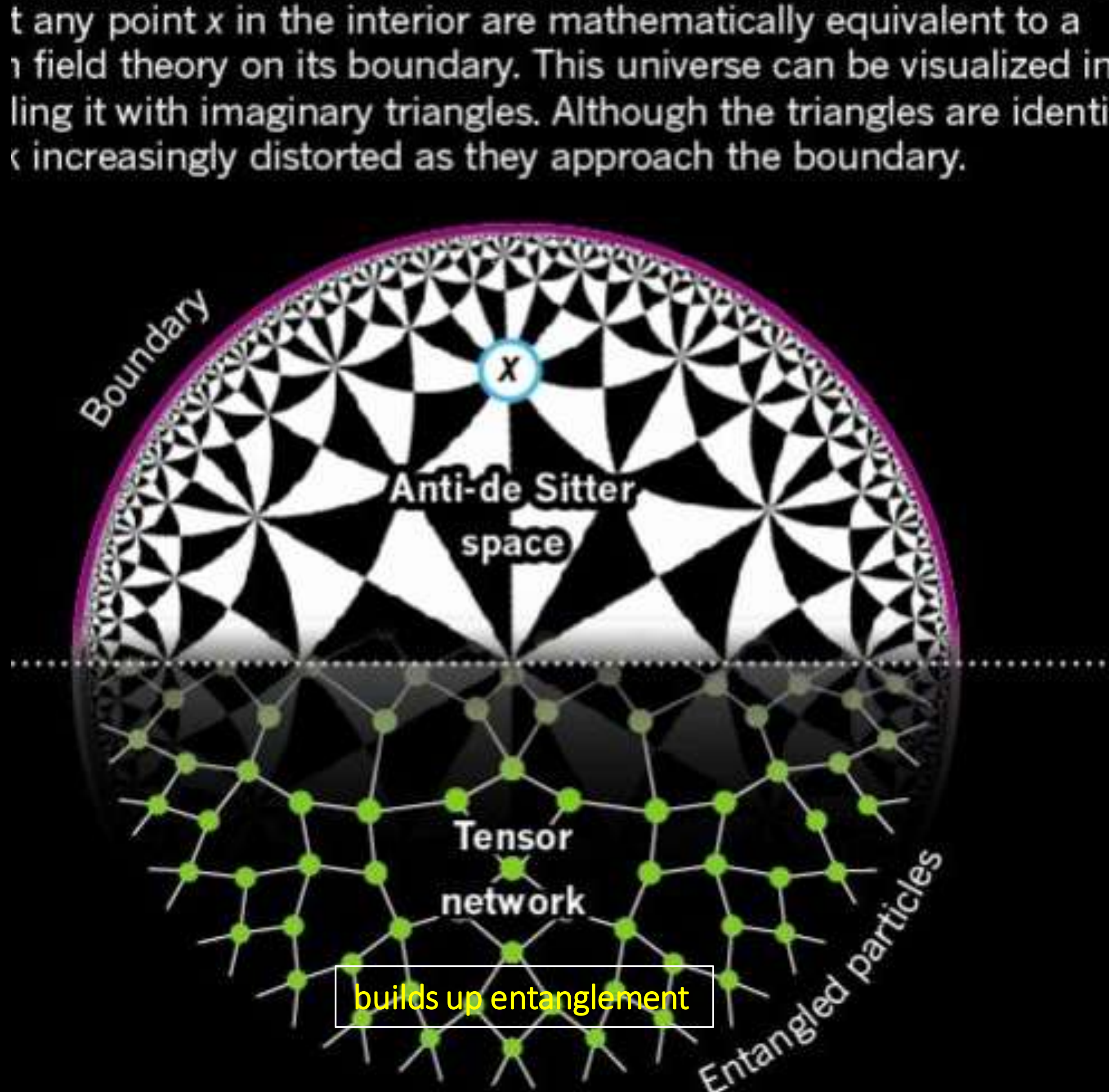
$$S = k_B \frac{c^3 Area}{4G\hbar}$$

gives the amount of **quantum entanglement** in the vacuum state.



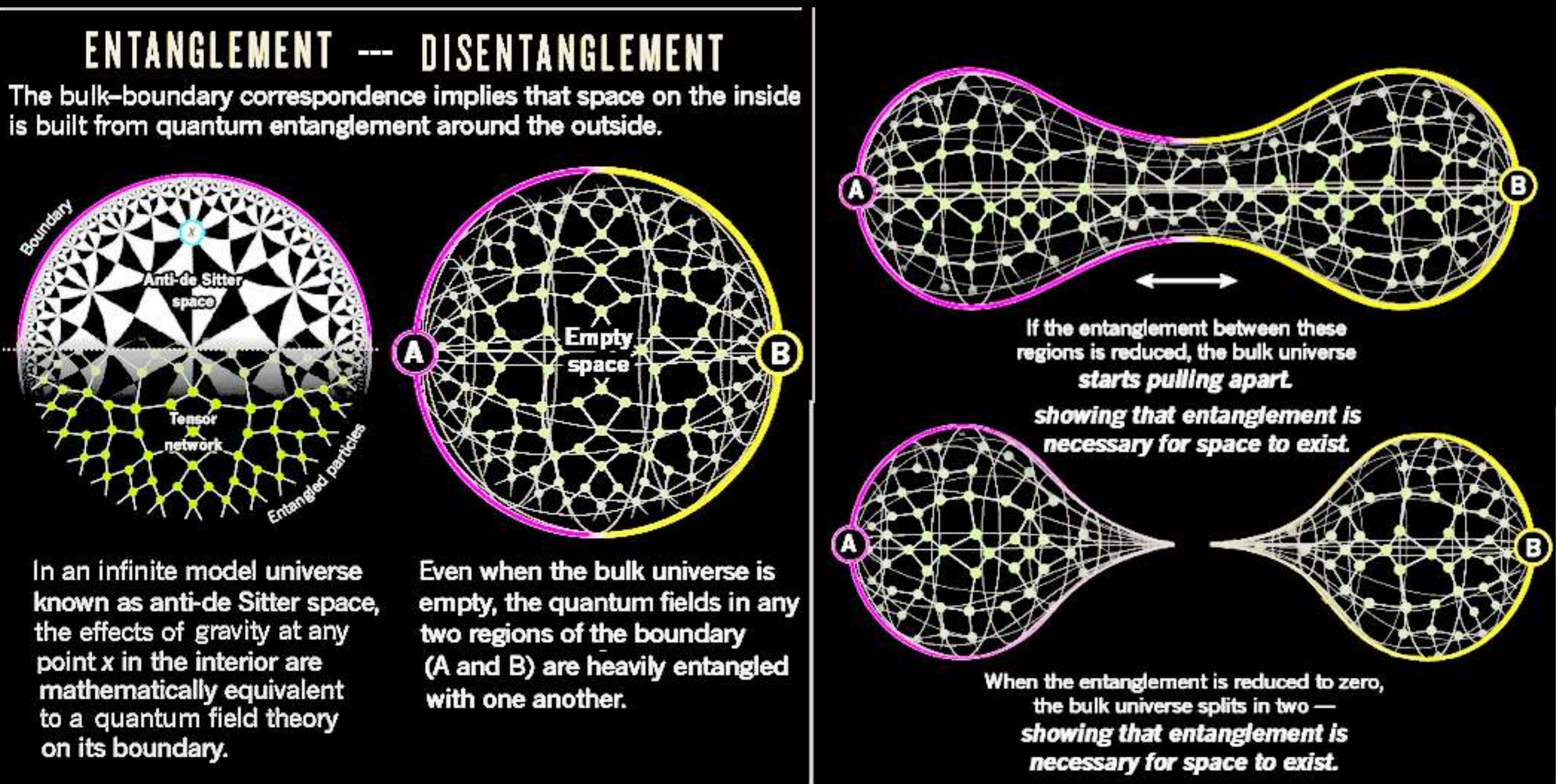
AdS/CFT:

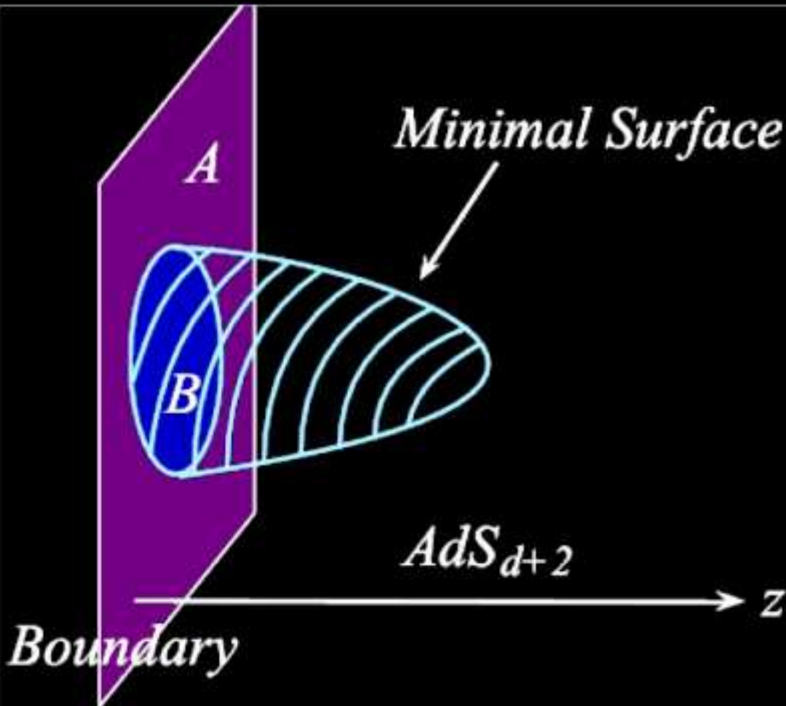
Space-time and Gravity emerge from quantum-entanglement



Caveat: Entanglement is not enough

Emergent Spacetime and Gravity from Quantum Entanglement





Emergent Gravity from Quantum Information

"It from Qubit"

$$S_{ent} = \frac{Area}{4G\hbar}$$

Ryu, Takanayagi
van Raamsdonk
Myers, Casini etal.

Gravity derived from 1st law of **entanglement entropy**

GR requires Area law => holds in Anti-de Sitte space. What about de Sitter space?

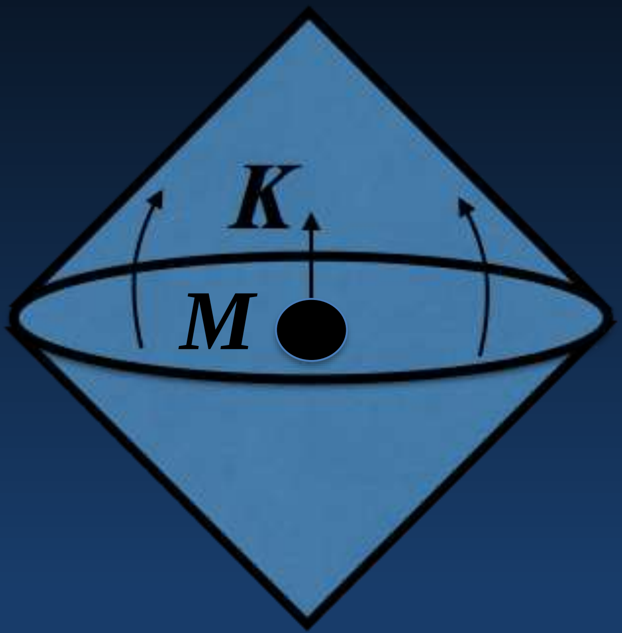
General Relativity derived from quantum entanglement

First law of entanglement entropy

$$\rho_A = \frac{1}{Z} e^{-K}$$

implies the Einstein equations

$$\delta S = \delta K$$



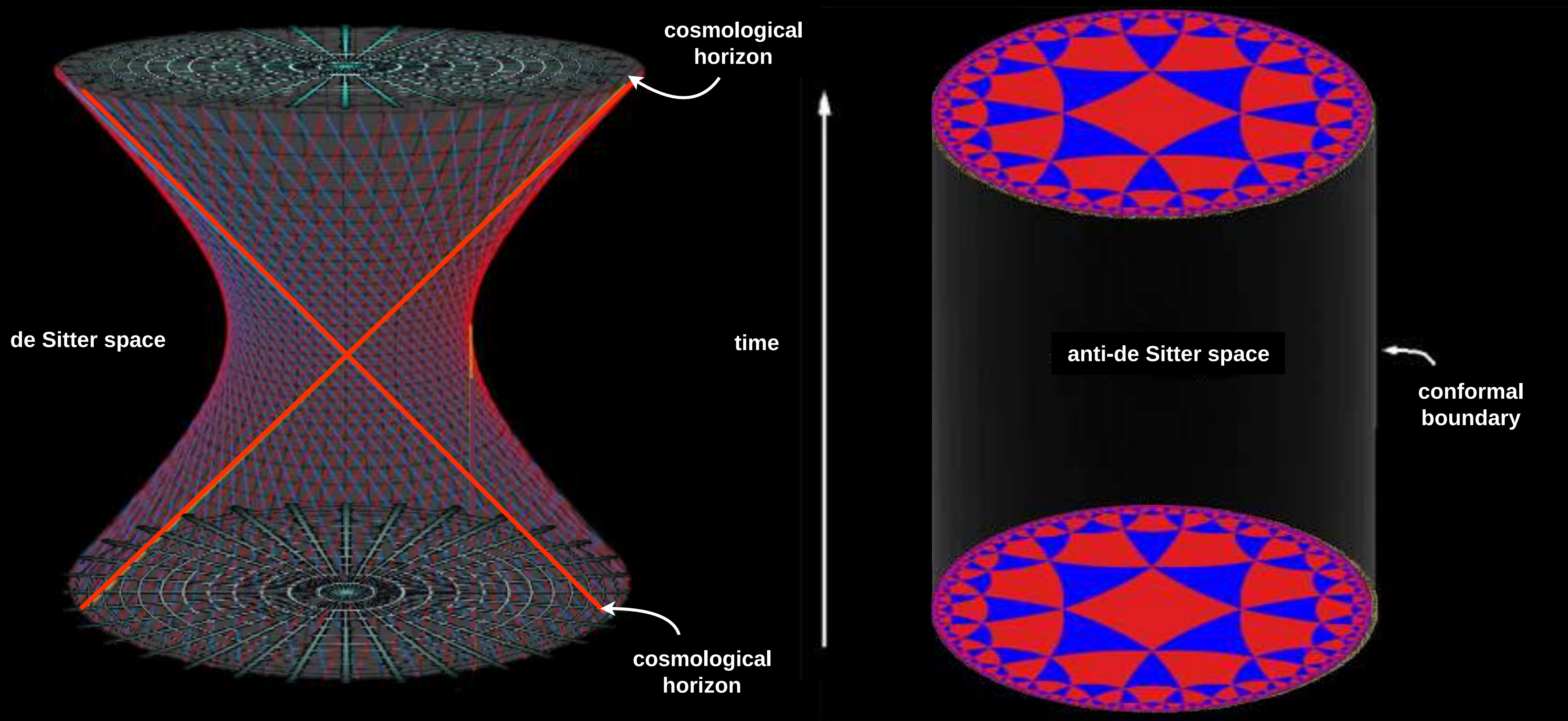
$$\delta K = -\frac{\delta A}{4G\hbar}\Big|_V$$

$$K = 2\pi \int \xi^a T_{ab} d\Sigma^a$$

Van Raamsdonk etal, Jacobson

(Anti-) de Sitter space

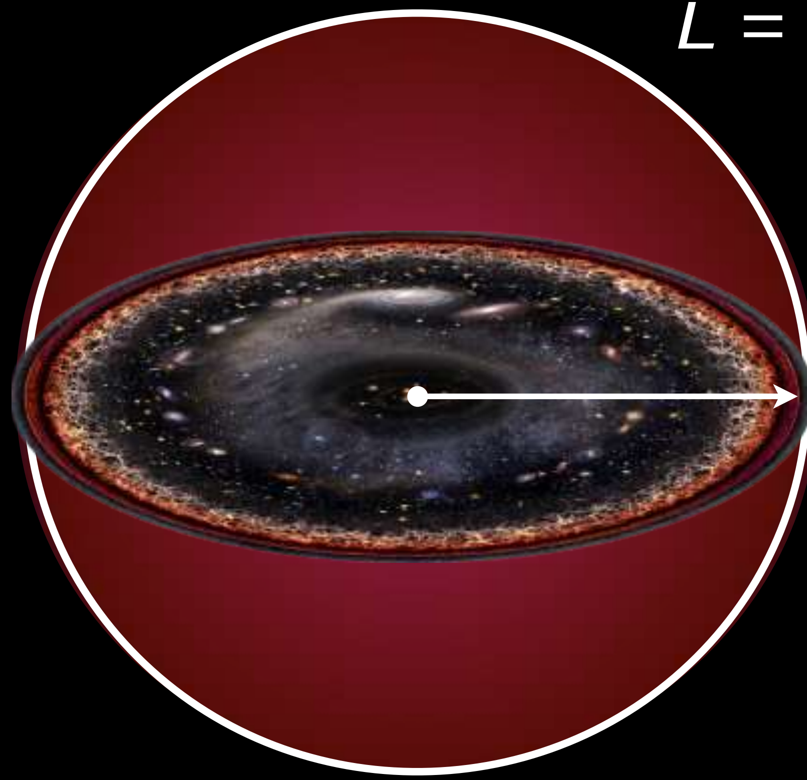
$$ds^2 = -(1 \pm R^2/L^2)dt^2 + \frac{dR^2}{1 \pm R^2/L^2} + R^2 d\Omega^2$$



de Sitter Space

cosmological horizon

$$L = \frac{c}{H_0}$$

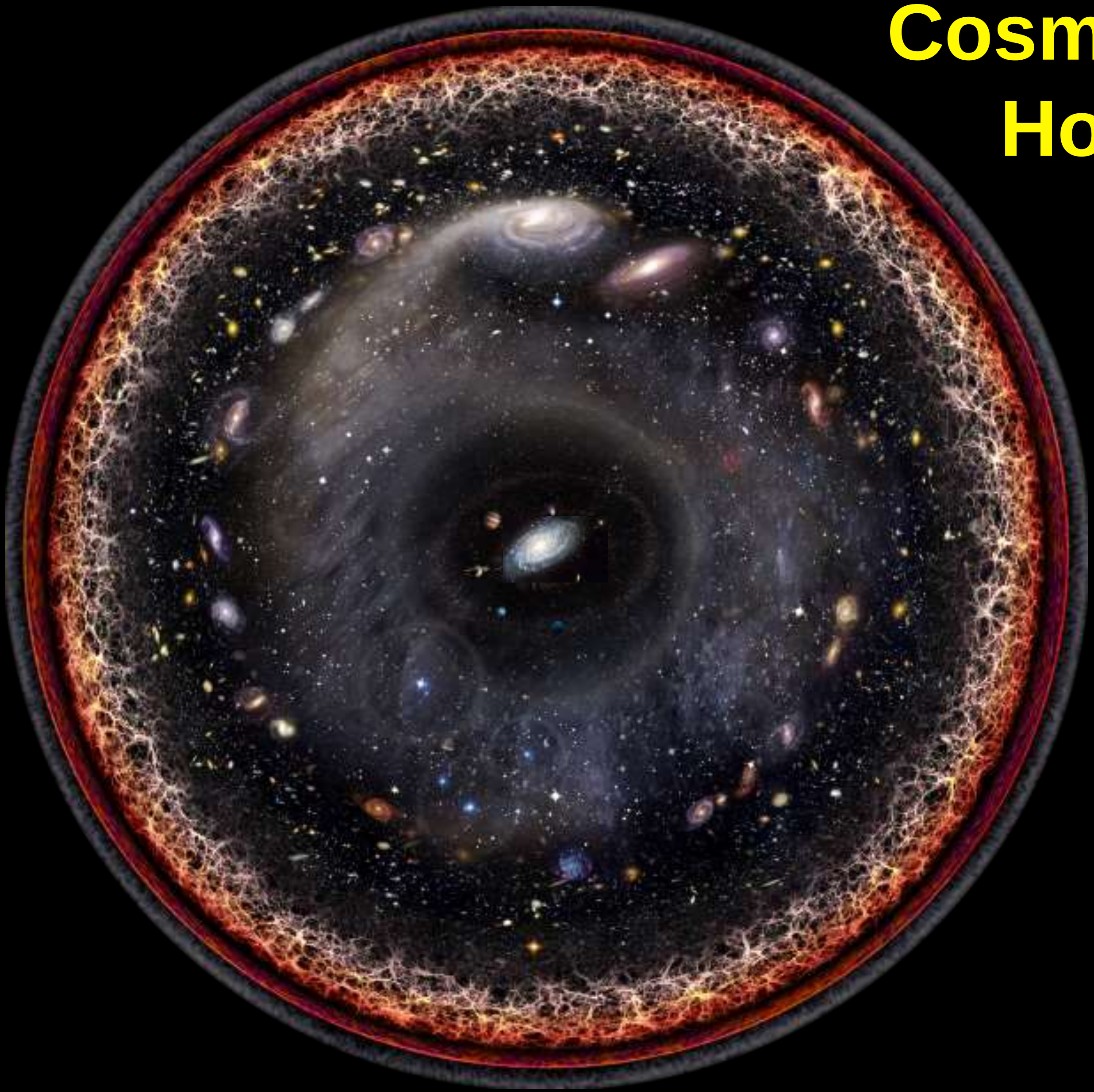


Universe with only Dark Energy

$$ds^2 = -\left(1 - R^2/L^2\right) dt^2 + \frac{dR^2}{1 - R^2/L^2} + R^2 d\Omega^2$$

Cosmological Horizon

$$L = \frac{c}{H_0}$$



Entropy

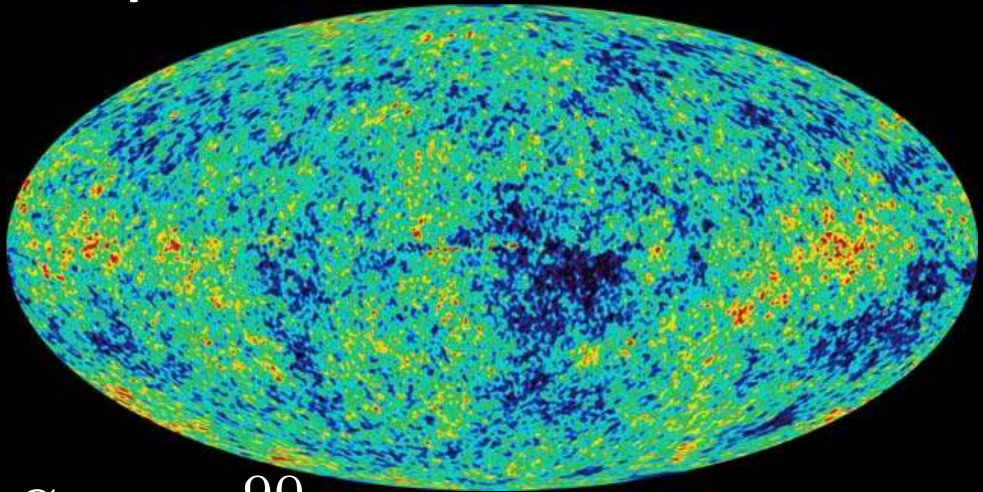
$$S(L) = k_B \frac{A(L)c^3}{4G\hbar}$$

Temperature

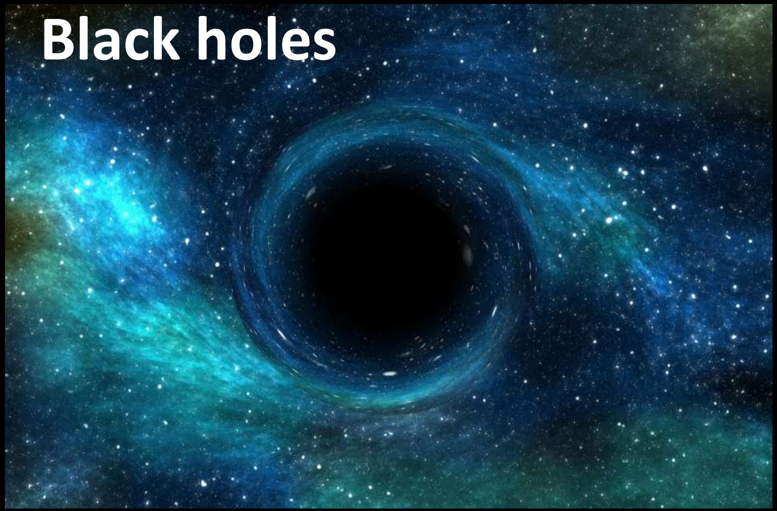
$$k_B T = \frac{\hbar H_0}{2\pi}$$

Entropy and Temperature are due to positive dark energy.

CMB-photons

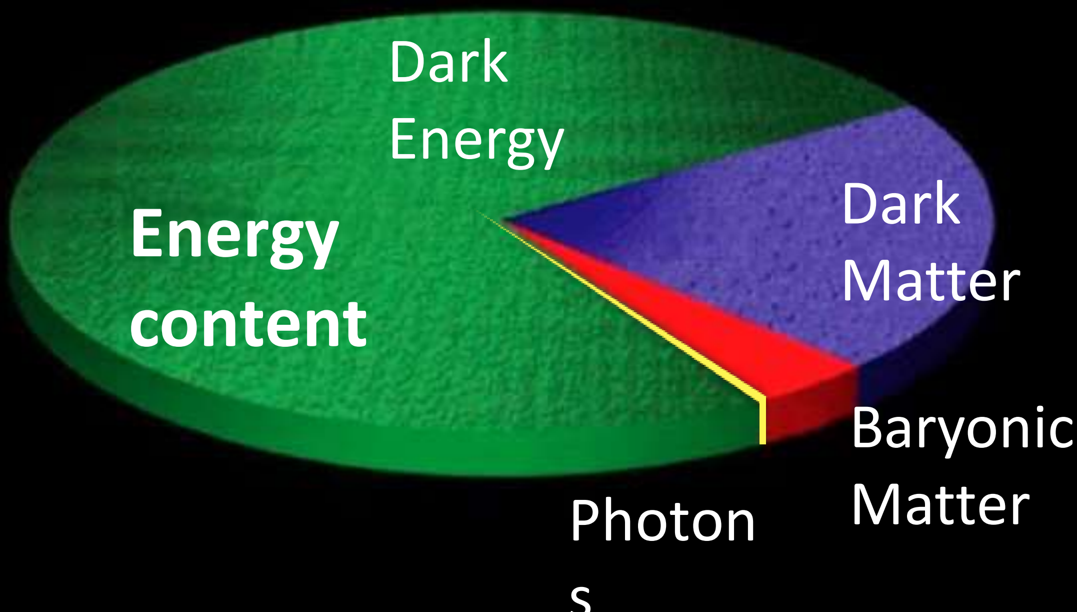


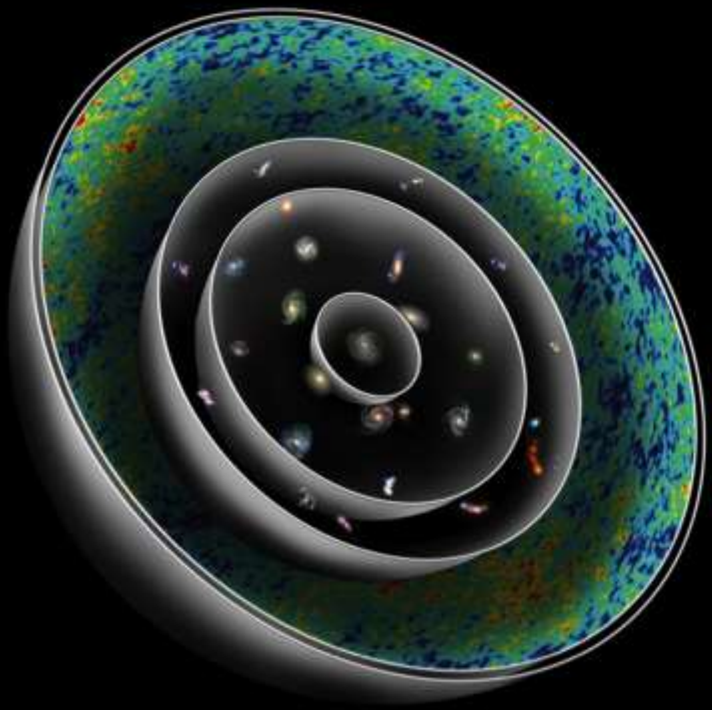
$S \approx 10^{90}$



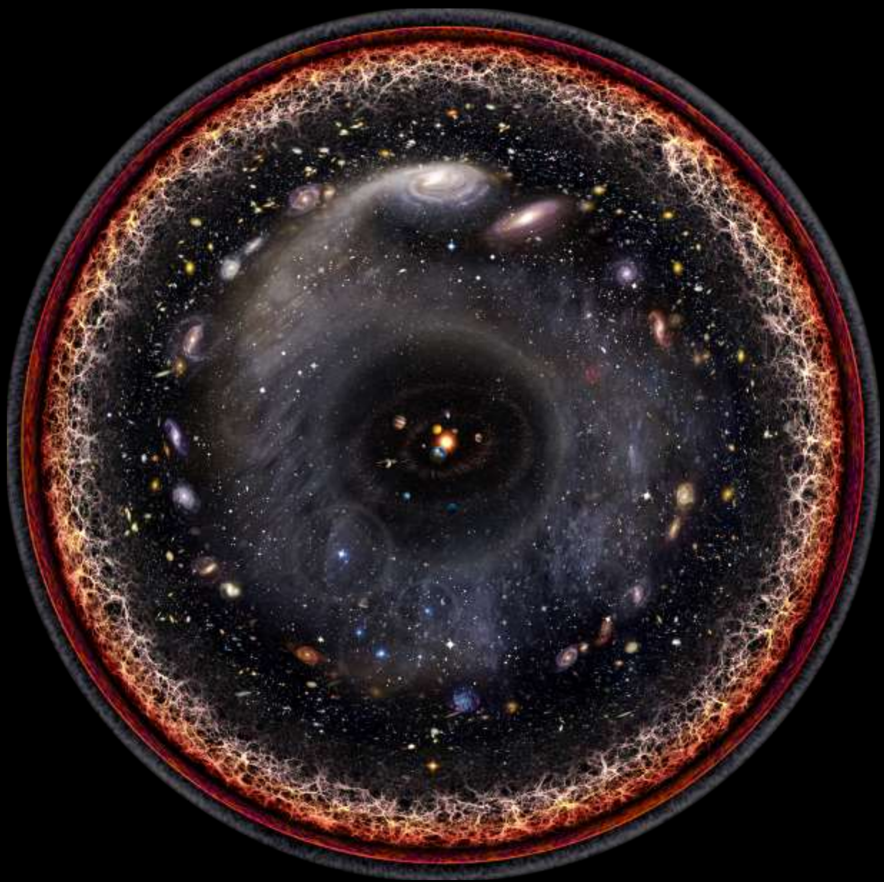
$S \approx 10^{100}$

What is the entropy content of the Universe?



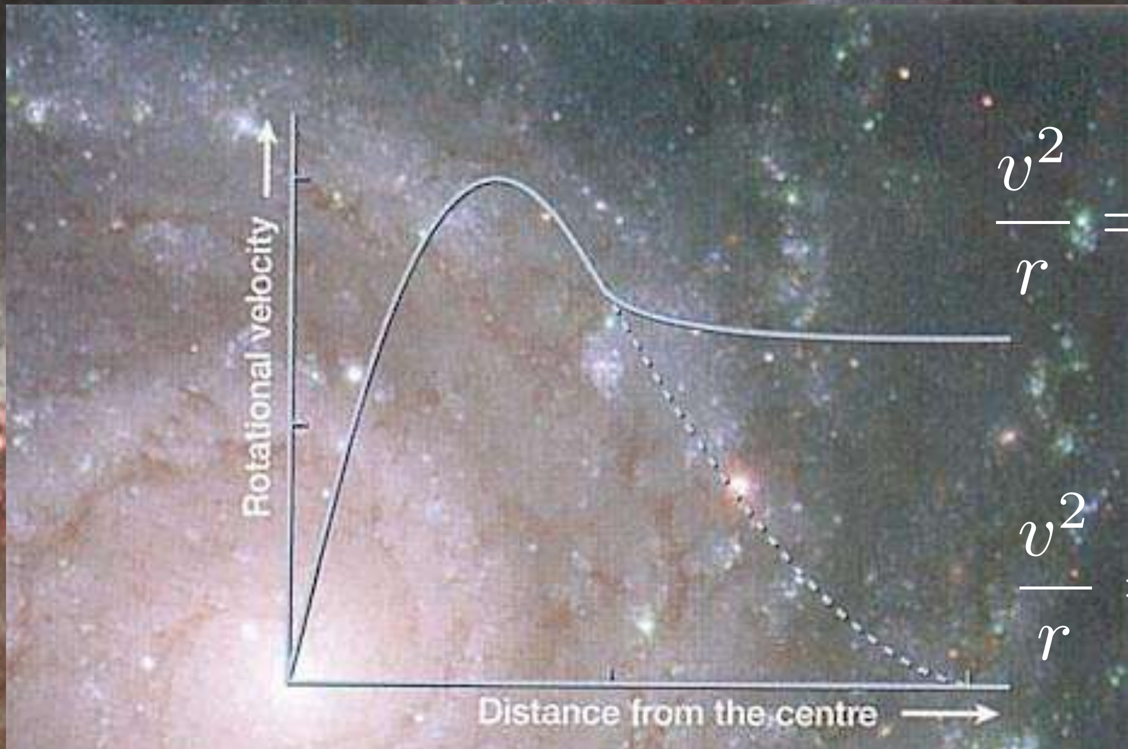


The Cosmos viewed from our perspective fits within one Hubble radius.



I will argue that most of the entropy in our universe is contained in the dark energy.

$$S \approx 10^{120}$$



$$\frac{v^2}{r} = \frac{GM_B}{r^2} + \frac{GM_D}{r^2}$$

$$\frac{v^2}{r} = \frac{GM_B}{r^2}$$

Empirically DM-effects appear when

$$\frac{GM_B}{r^2} < \frac{cH_0}{2}$$

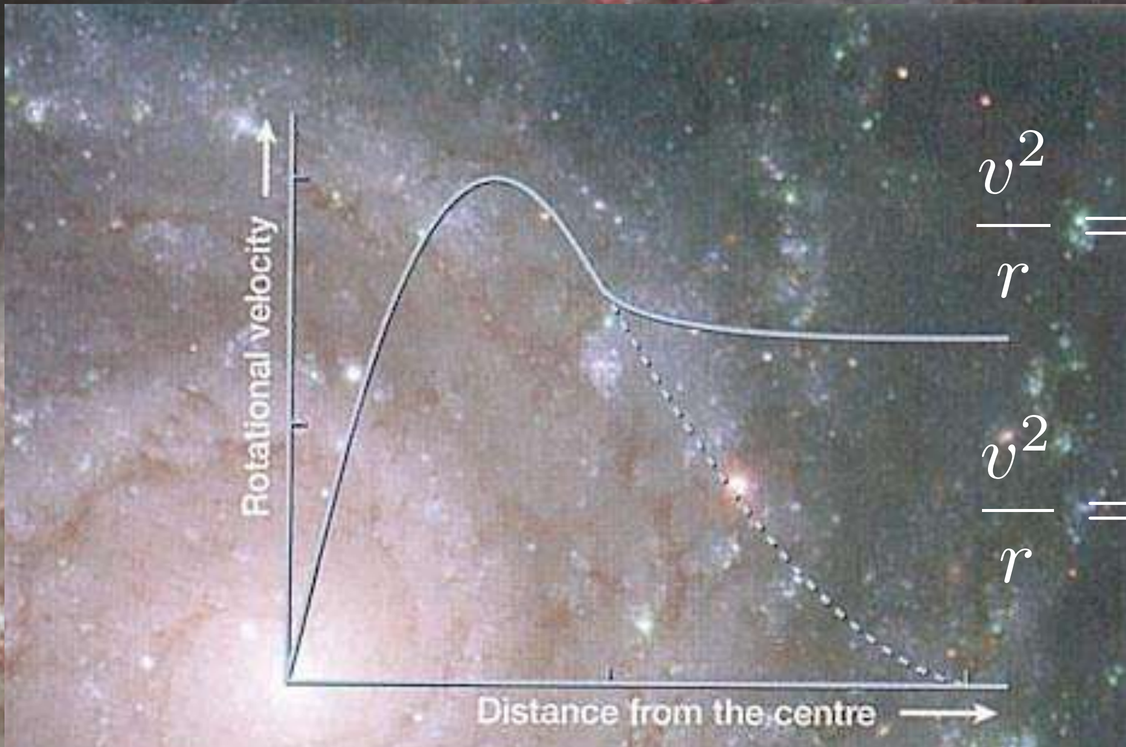
Adding mass to de Sitter space changes its entropy

The horizon radius obeys

$$1 - \frac{R^2}{L^2} - \frac{GM}{c^2 R} = 0$$

Hence the horizon area is reduced by

$$\frac{\Delta A\, c^3}{4G\hbar} = -\frac{Mc^2}{\hbar H_0/2\pi}$$



$$\frac{v^2}{r} = \frac{GM_B}{r^2} + \frac{GM_D}{r^2}$$

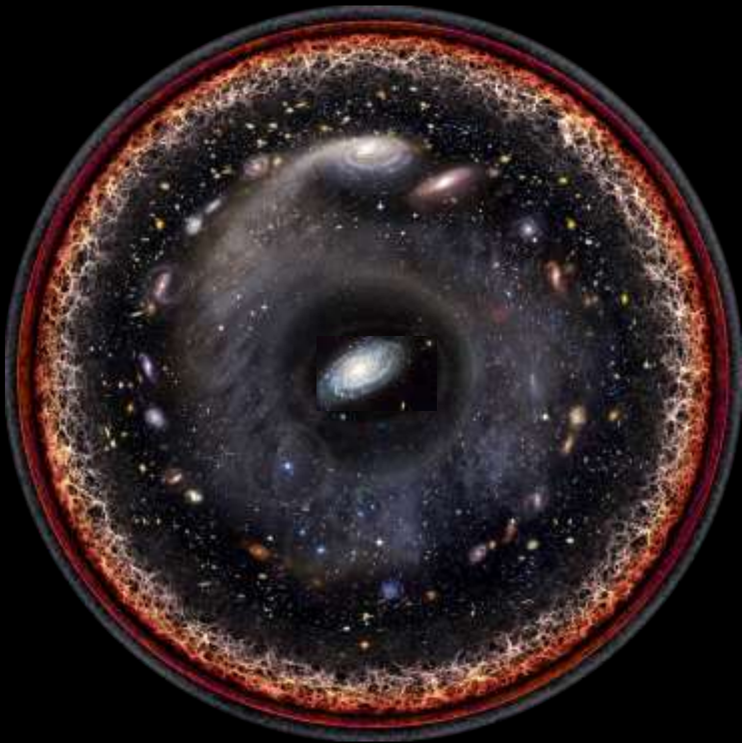
$$\frac{v^2}{r} = \frac{GM_B}{r^2}$$

Empirically DM-effects appear when

$$\frac{GM}{r^2} < \frac{cH_0}{2} \qquad \frac{Mc^2}{\hbar H_0/2\pi} < \frac{Ac^3}{4G\hbar}$$

de Sitter Horizon

$$L = \frac{c}{H_0}$$



$$S(L) = \frac{A(L)}{4G\hbar}$$

$$T = \frac{\hbar H_0}{2\pi}$$

Hypothesis:

de Sitter entropy + temperature are due to positive dark energy.

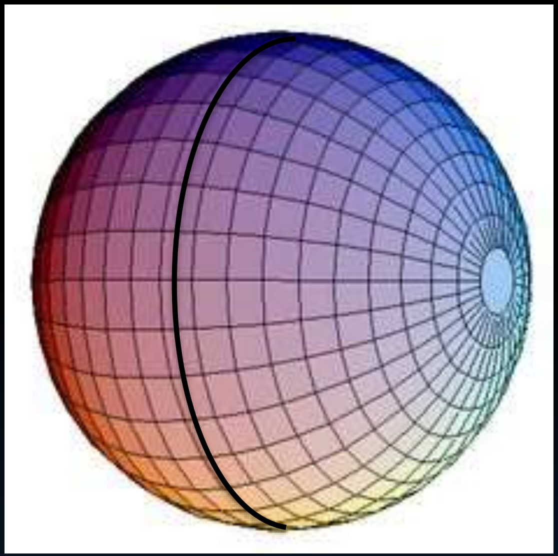
The entanglement entropy contains volume law contribution

$$S(R) = \frac{A(R)}{4G\hbar}\frac{R}{L}$$

$$R < L$$

Dark Energy Carries Entropy

De Sitter space



Microscopic dS-state

$$|vac\rangle_{dS} = \frac{1}{\sqrt{Z}} \sum_i |E_i\rangle_L |E_i\rangle_R e^{-\beta E_i/2}$$

Entanglement due to Dark Energy

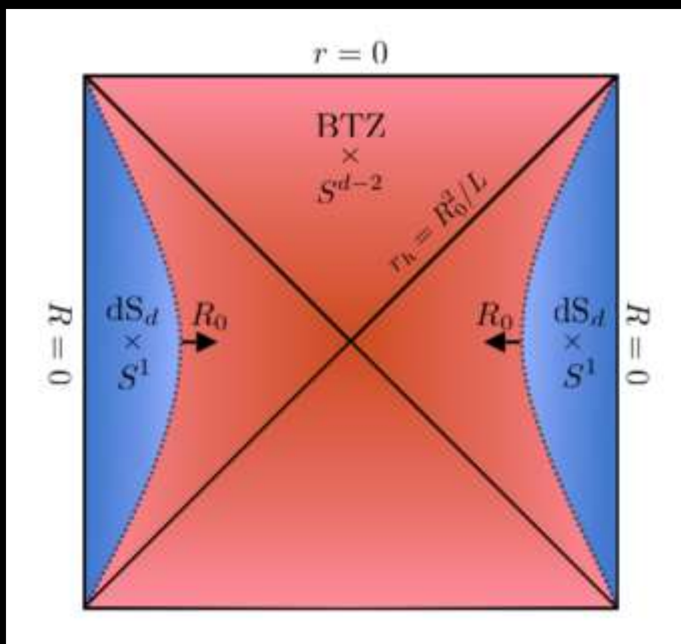
$$S_{ent} = \frac{A}{4G\hbar}$$

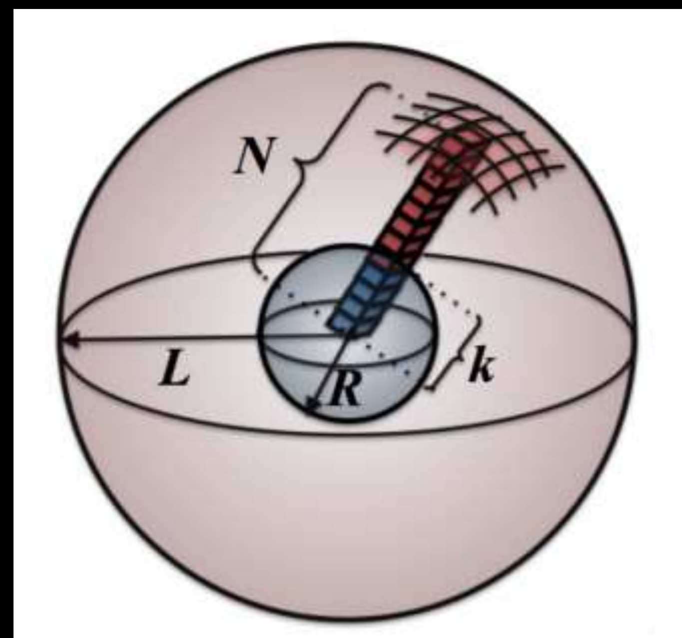
Can be derived from `non-AdS holography'

Towards non-AdS Holography via the Long String Phenomenon

arXiv:1801.02589

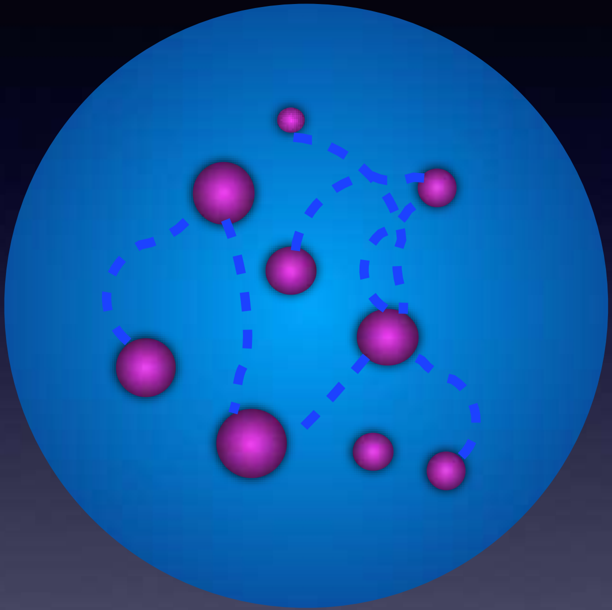
Sam van Leuven[1], Erik Verlinde[2] and Manus Visser[3]



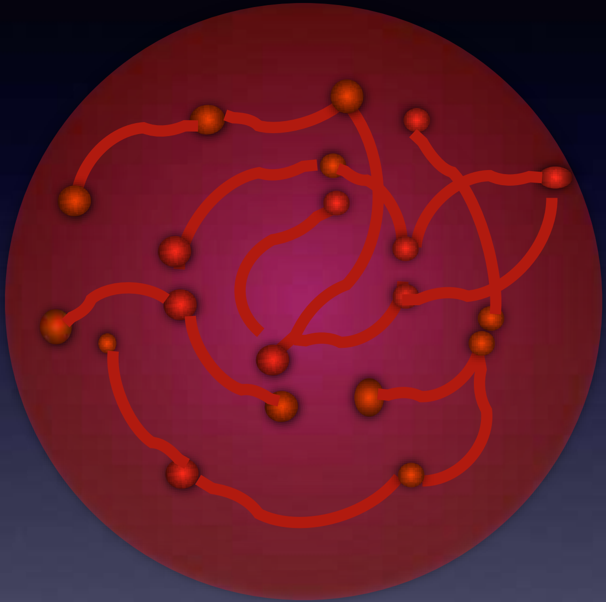


$$S = \frac{A(R_0)}{4G_d}\frac{R_0}{L} = \frac{V(R_0)}{V_0} \qquad \text{where} \qquad V_0 = \frac{4G_d L}{d-1}\,. \tag{4.40}$$

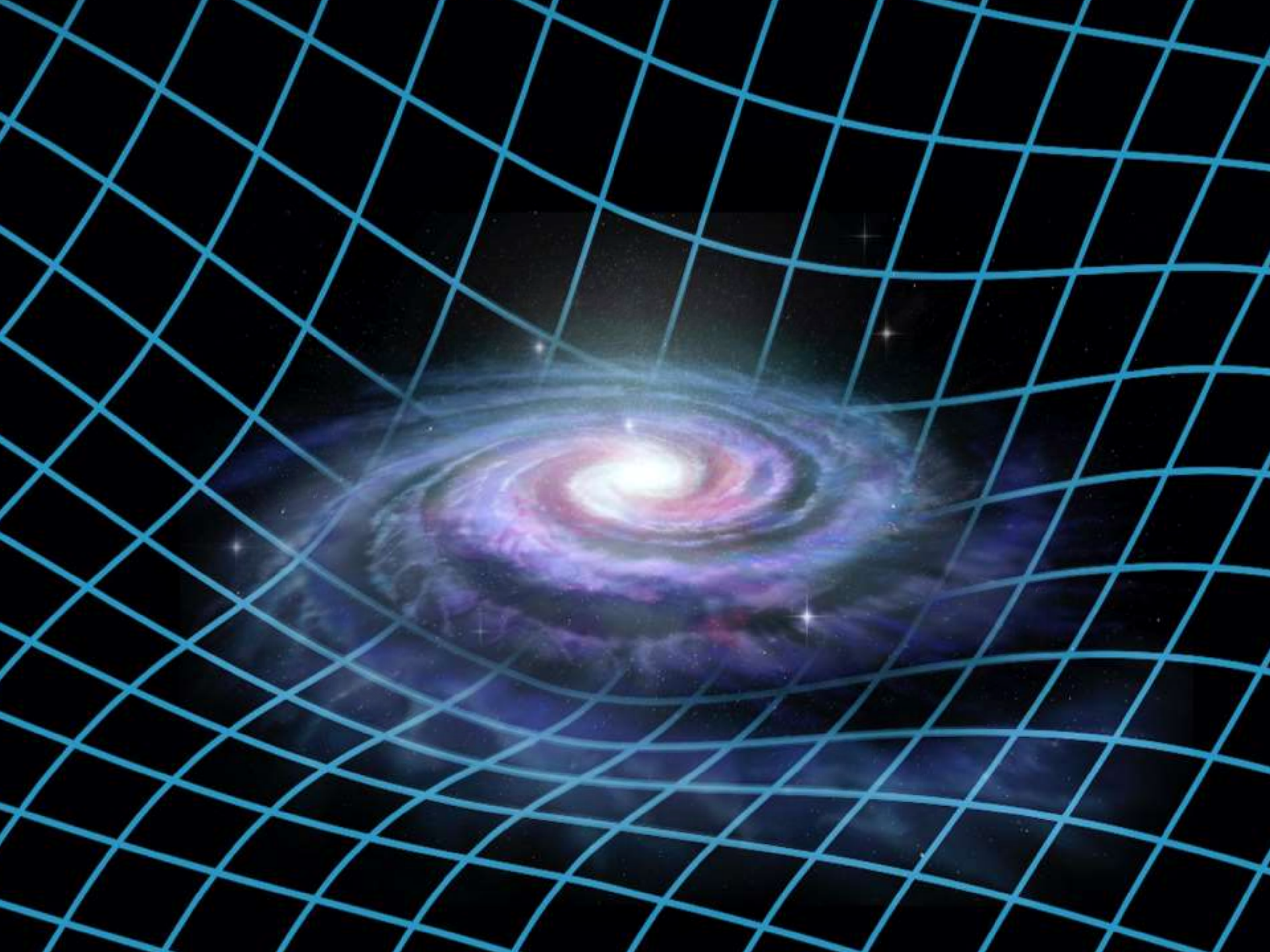
Two Phases of String Theory

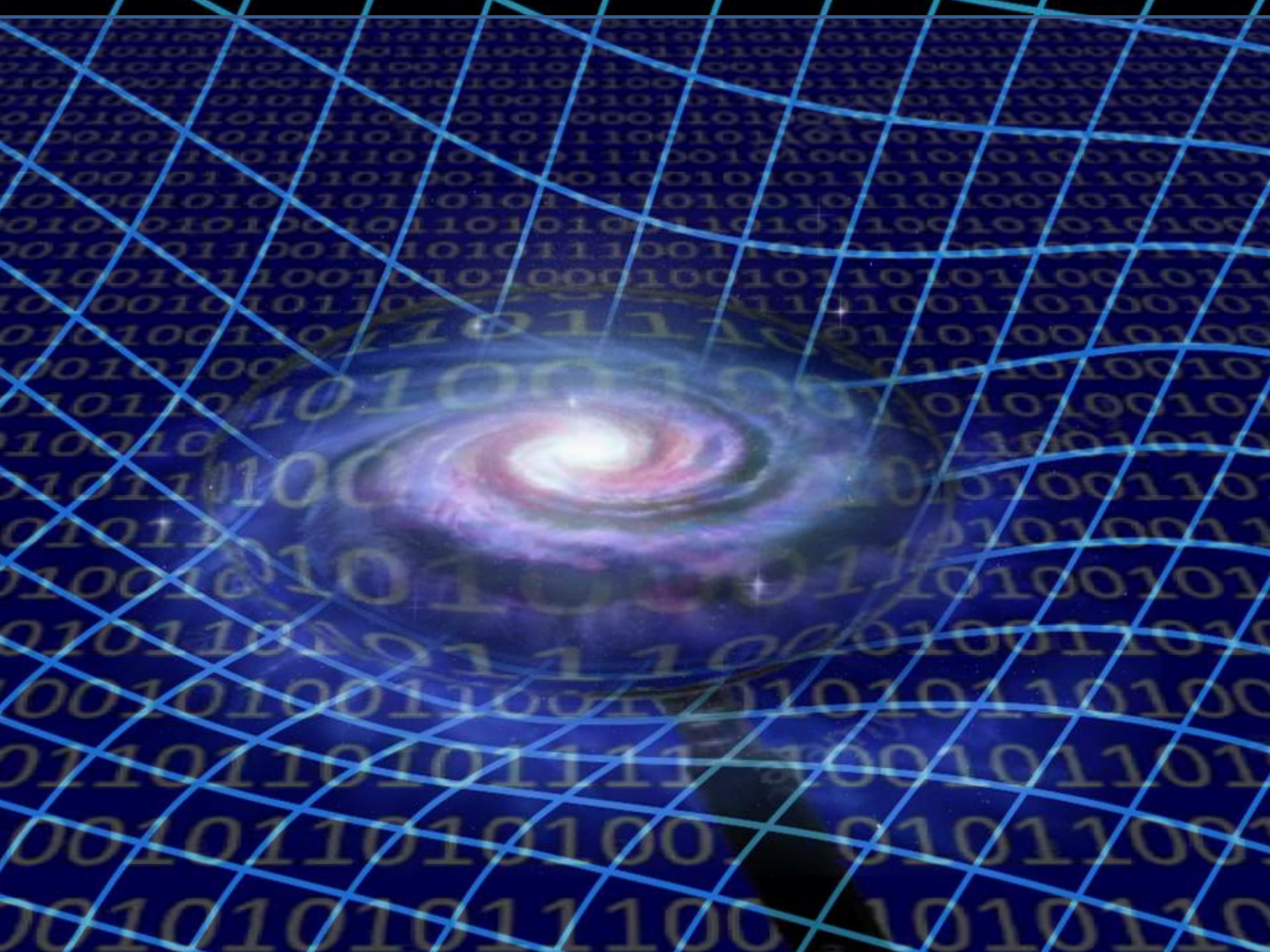


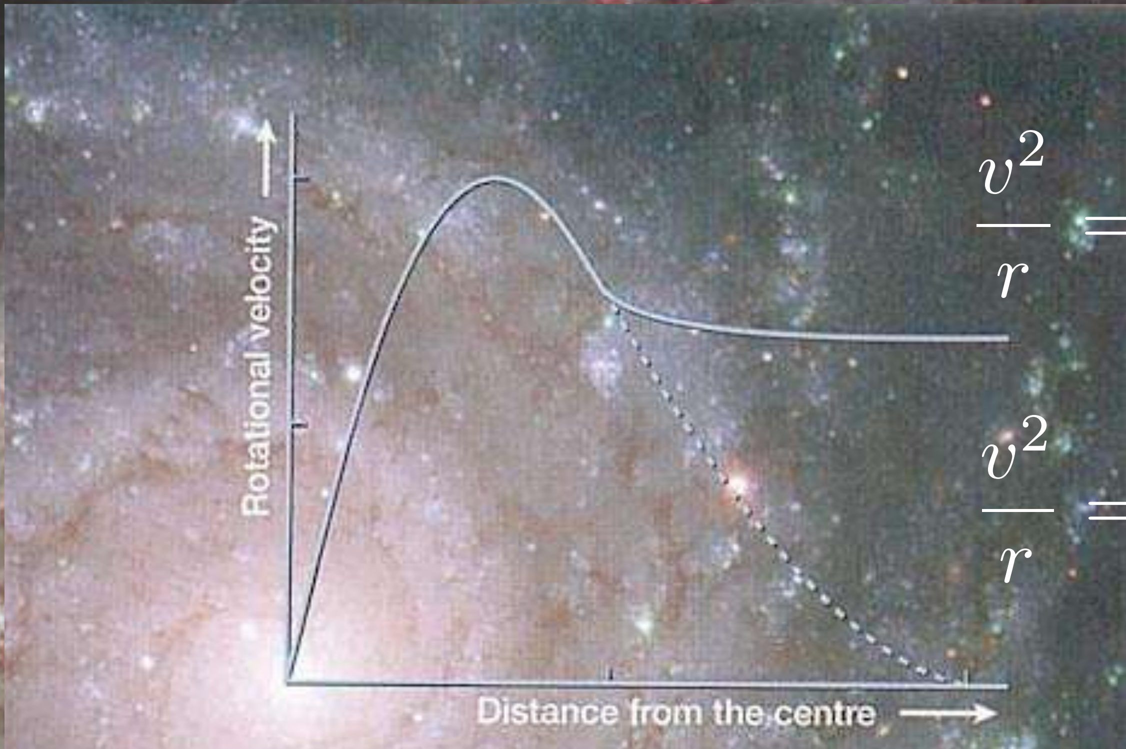
- "Coulomb branch"
- Vacuum phase
- localized excitations
- off-diagonal modes
in vacuum state



- "Higgs branch"
- Entropic phase
- thermalized state
with high entropy
- off diagonal modes
are excited: long strings







$$\frac{v^2}{r} = \frac{GM_B}{r^2} + \frac{GM_D}{r^2}$$

$$\frac{v^2}{r} = \frac{GM_B}{r^2}$$

$$L = \frac{c}{H_0}$$

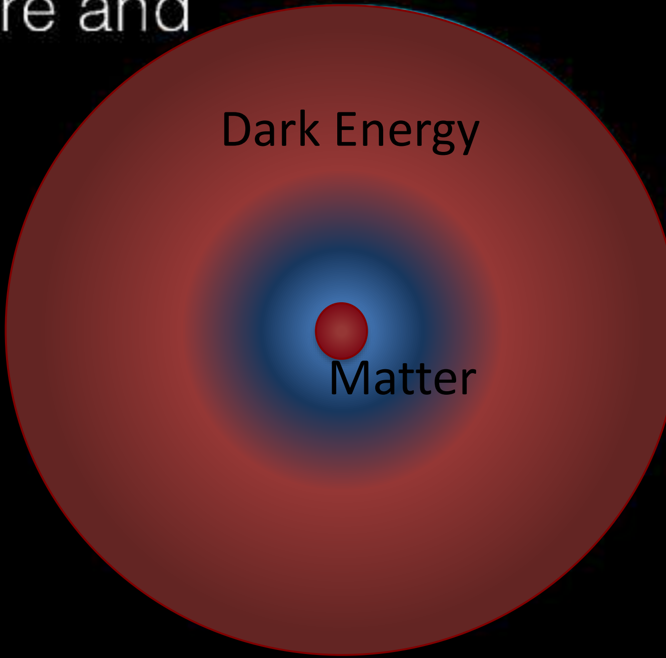
Empirically DM-effects appear when

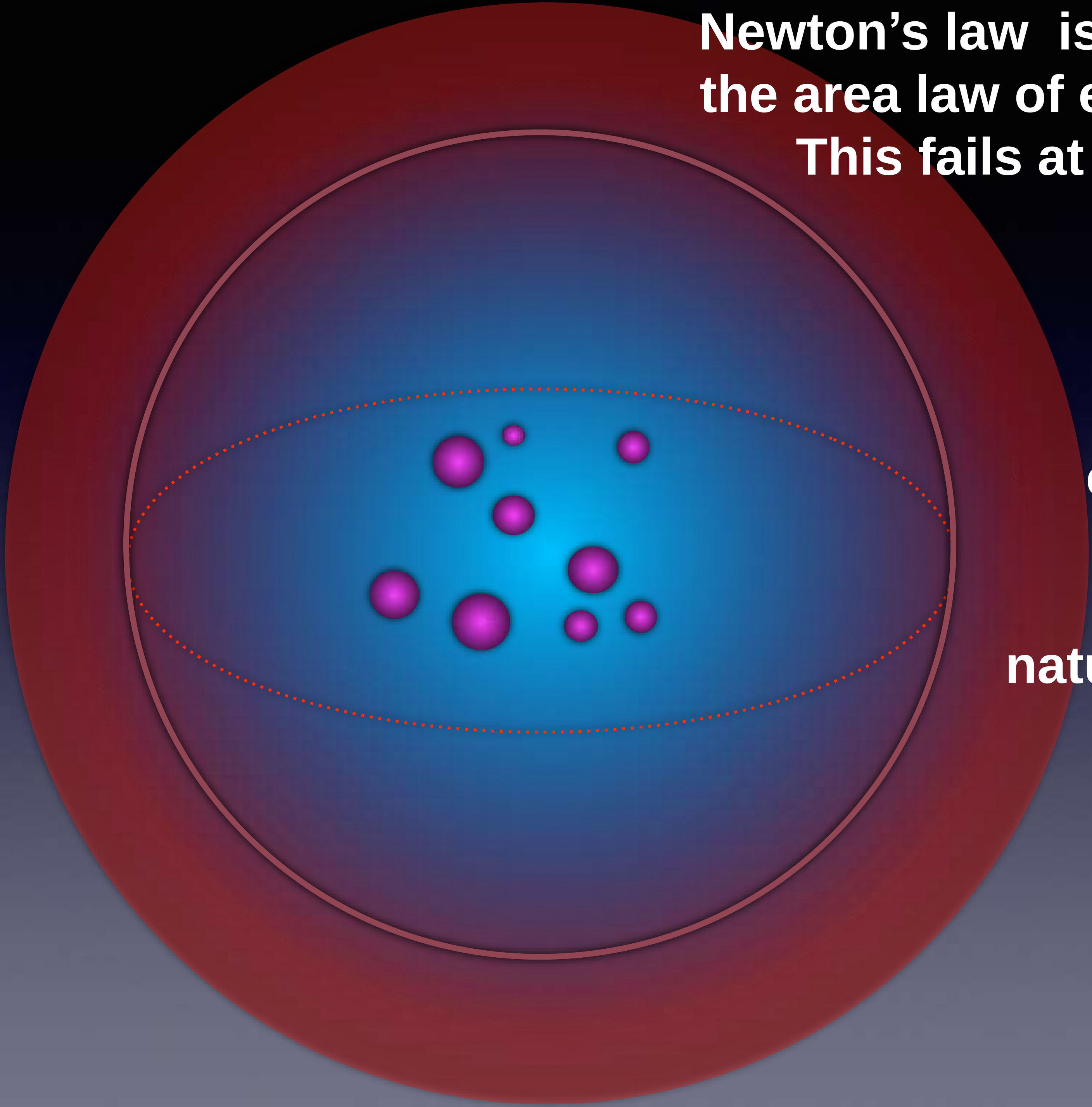
$$\frac{GM}{r^2} < \frac{cH_0}{2}$$

$$2\pi \frac{McR}{\hbar} < \frac{A(R)c^3}{4G\hbar}\frac{R}{L}$$

Assumptions

- Gravity emerges from entanglement entropy using the 1st law.
- GR requires entanglement to obey an area law: there is no thermal entropy contribution.
- de Sitter space corresponds to an excited state with a finite entropy, temperature and energy density: glassy state.
- The principles of emergent gravity still go through but need to be generalized to case with thermal entropy density (~elasticity).



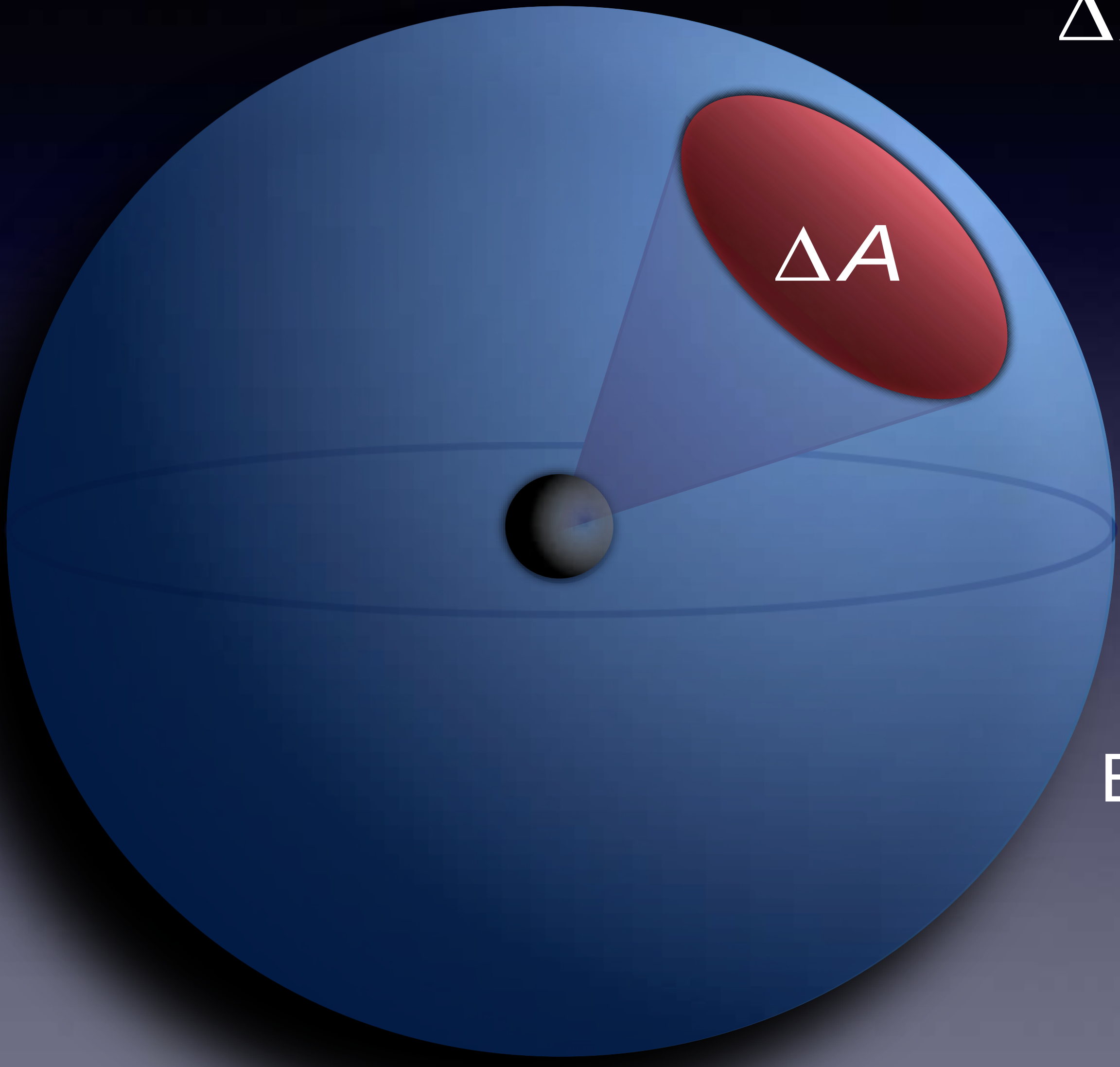


Newton's law is derived from the area law of entanglement. This fails at cosmological scales.

The volume entropy takes over and changes the nature of gravity.

It becomes analogous to elasticity!

Einstein equation => Mass leads to area deficit



$$\Delta A = \int_0^R \Phi \frac{dA}{dr} dr$$

(R. Feynman)

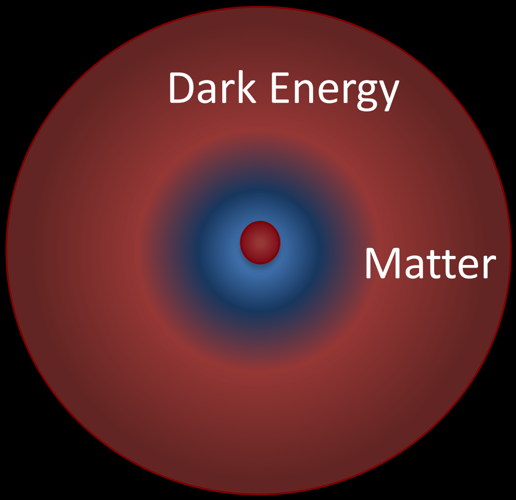
$$\frac{\Delta A c^3}{4G\hbar} = -2\pi \frac{McR}{\hbar}$$

Change in
Entanglement Entropy
=
Bekenstein bound

The empirical fact that Dark matter effects appear when

$$2\pi M c R < \frac{A(R)c^3}{4G\hbar}\frac{R}{L}$$

Naturally follows from Emergent gravity



The left hand side is the entropy associated with matter.

The right hand side is the entropy associated with dark energy

Radial Acceleration Relation

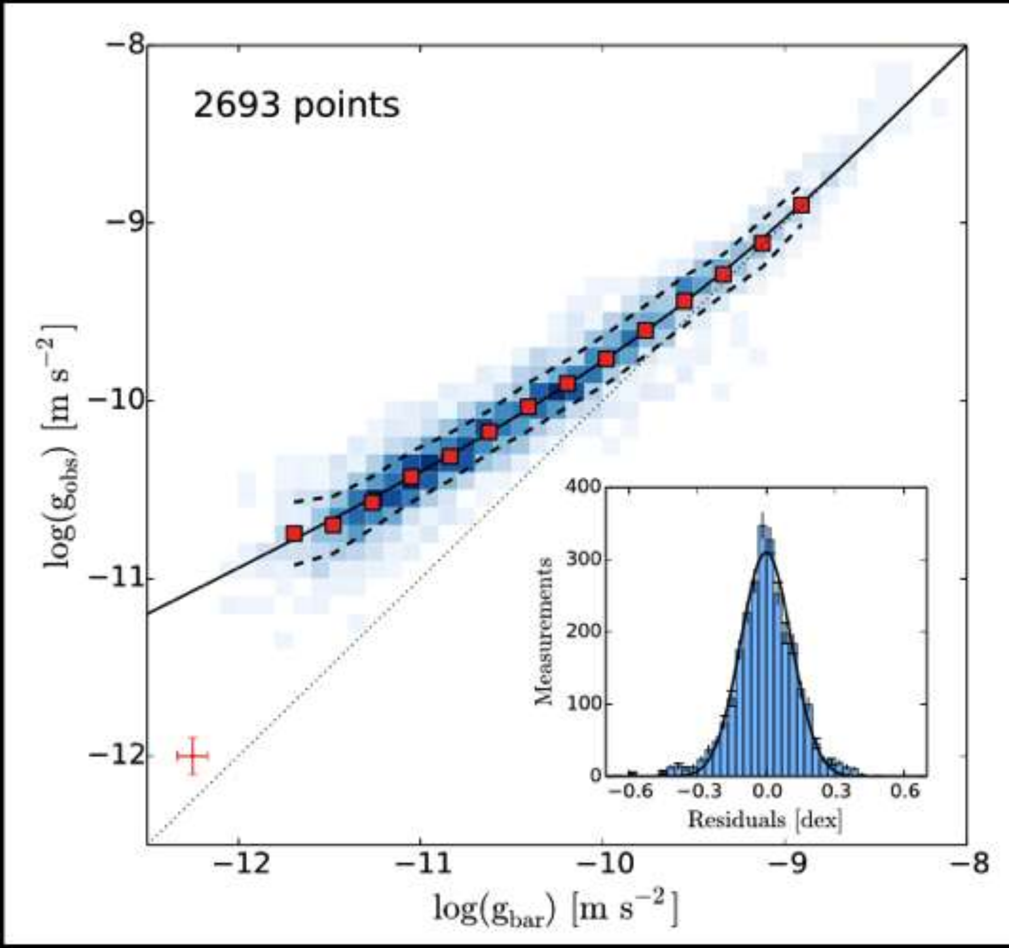
McGaugh, Lelli, Schombert. (2016)

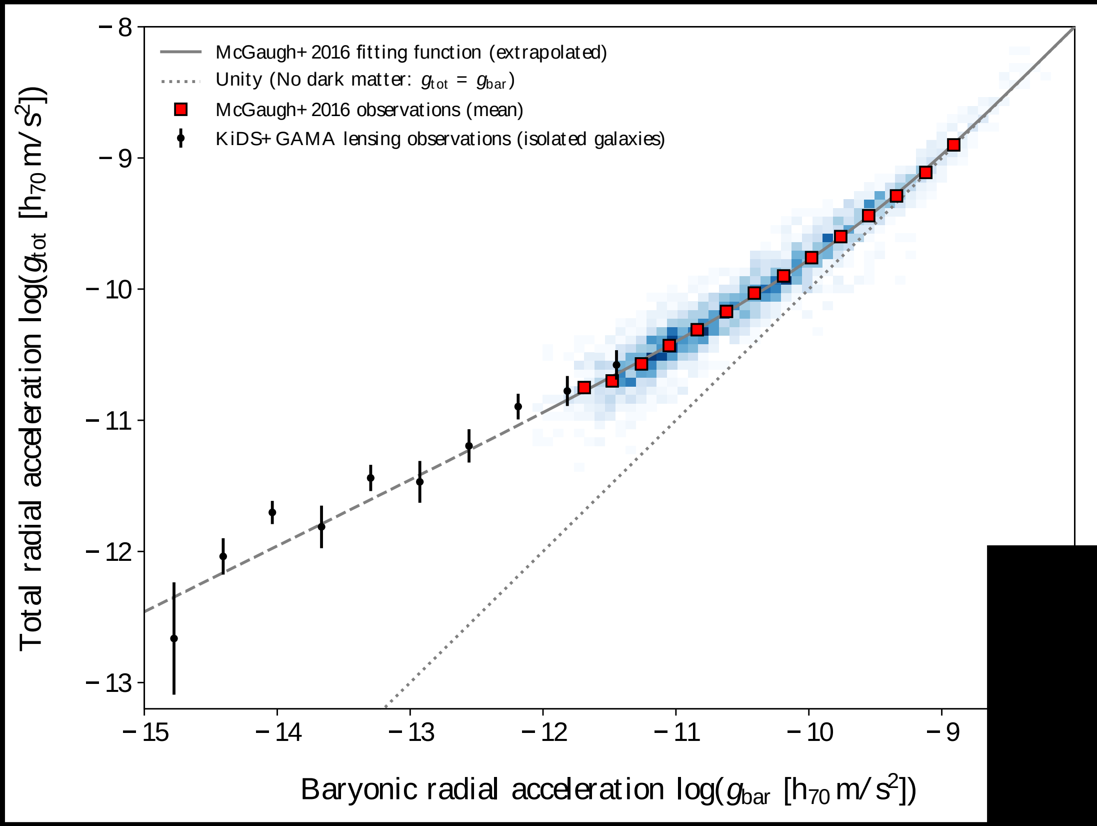
$$g_{obs}(r) = \frac{GM_B(r)}{r^2} + \frac{GM_D(r)}{r^2}$$

$$g_{bar}(r) = \frac{GM_B(r)}{r^2}$$

for large ***r*** :

$$\boxed{g_{obs}^2(r) \approx g_{bar}(r) cH_0/6}$$



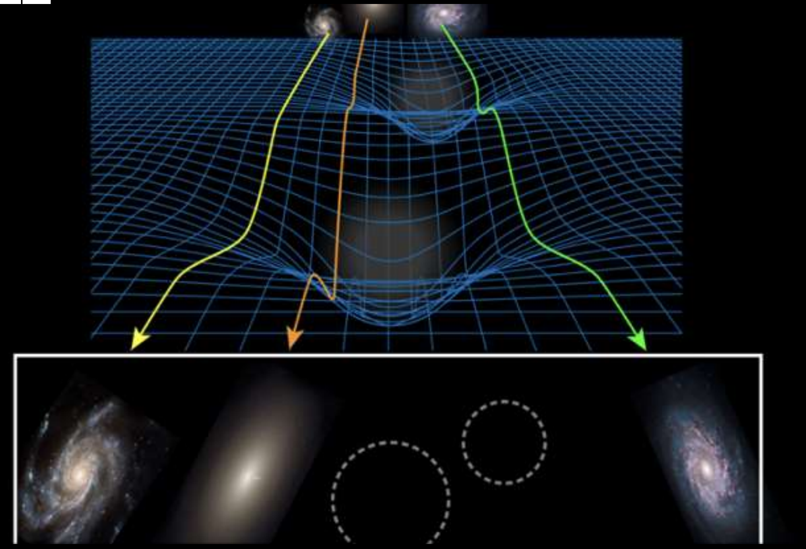


Radial acceleration relation

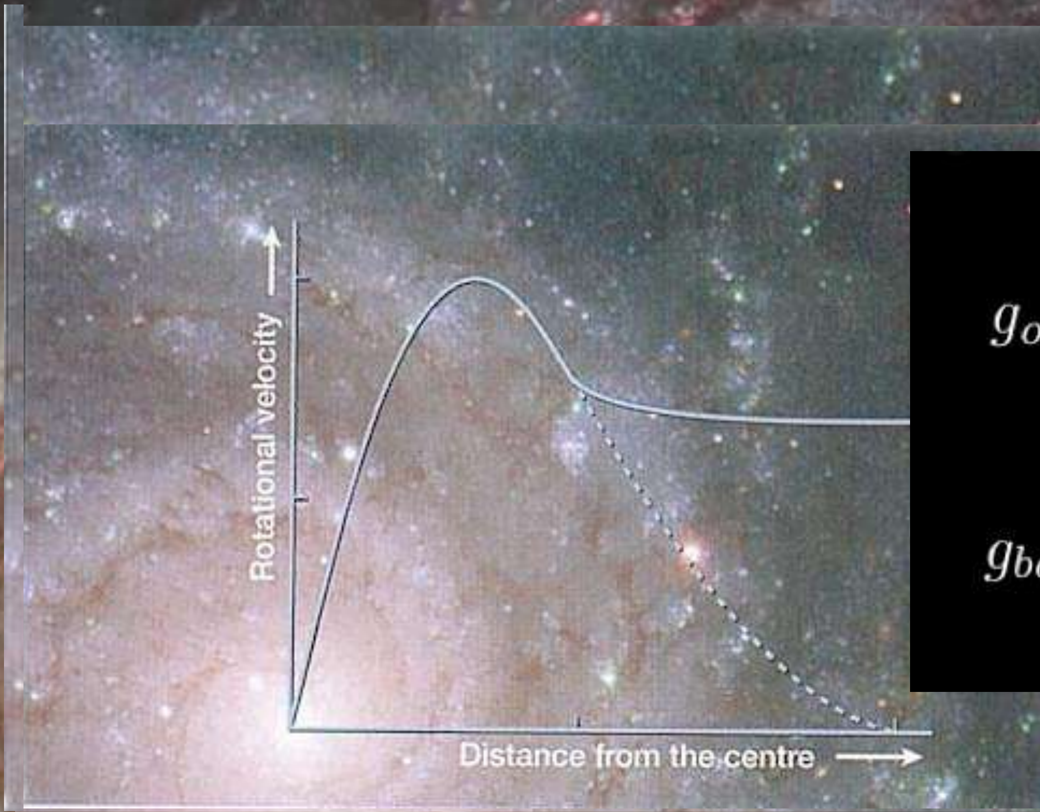
$$g_{obs}^2 \approx g_{bar} c H_0 / 6$$

holds over 4 decades .

Weak Lensing Observations



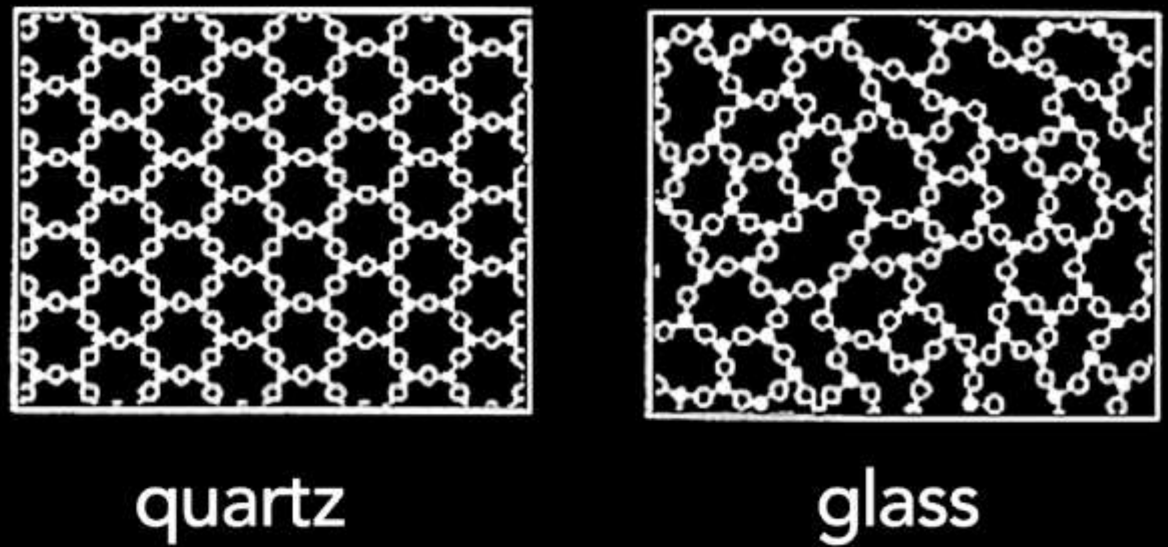
Margot Brouwer
+KIDS-collaboration



$$g_{obs}(r) = \frac{GM_B(r)}{r^2} + \frac{GM_D(r)}{r^2}$$

$$g_{bar}(r) = \frac{GM_B(r)}{r^2}$$

$$\frac{1}{8\pi G}\int_{r\leq R} g_i^2\, dV = \frac{M_B c R}{\hbar}\,\frac{\hbar H_0}{6}$$



Large entropy density may be very hard to detect:

High degeneracy close to groundstate: out of equilibrium

Extremely slow `glassy dynamics’ leads to memory effects

Dark Energy turns the spacetime vacuum in a glassy state.

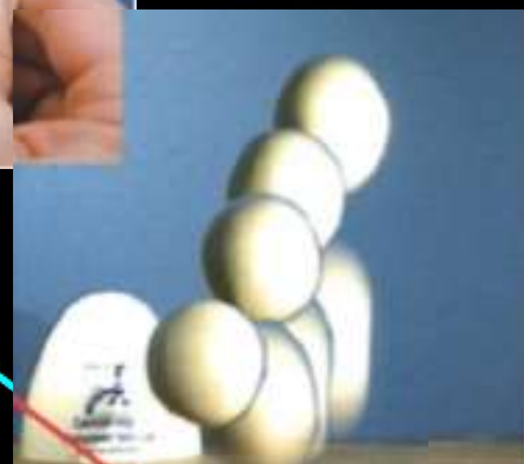
and leads to slow dynamics and memory effects

If you're to scale the history of the universe to a single year, humans wouldn't appear till December 31, 11.59 pm on New Year's Eve!

On that same timescale we have observed the Universe for only a fraction of a second.

At very long times the material behaves like a liquid spreading out onto a flat surface

At moderate times the Silly Putty™ stretches like a plastic solid

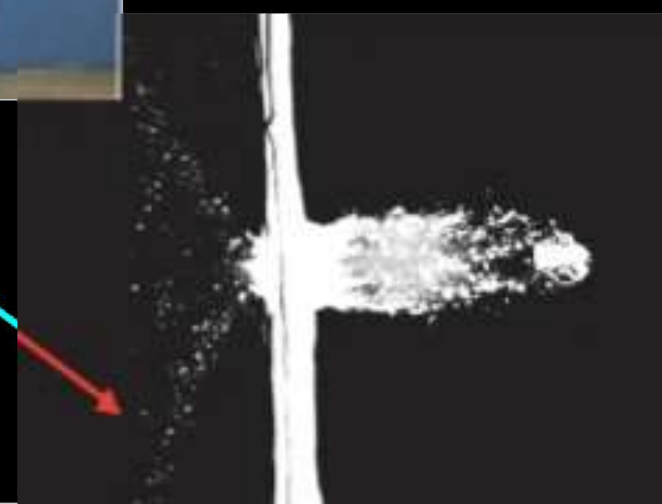


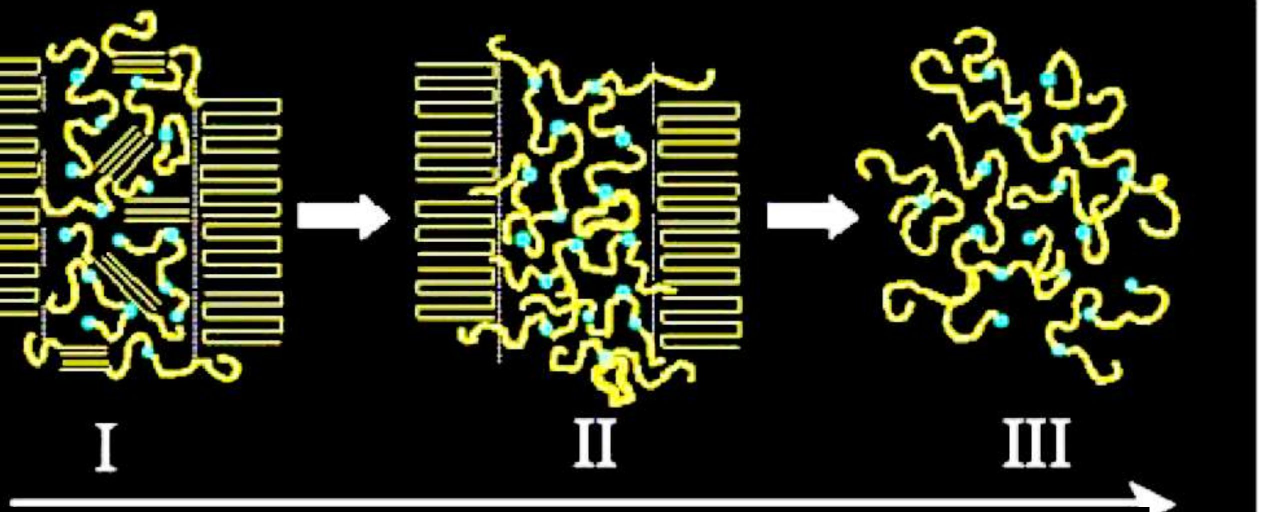
At short times the Silly Putty™ bounces like an elastic solid

Increasing Deborah Number

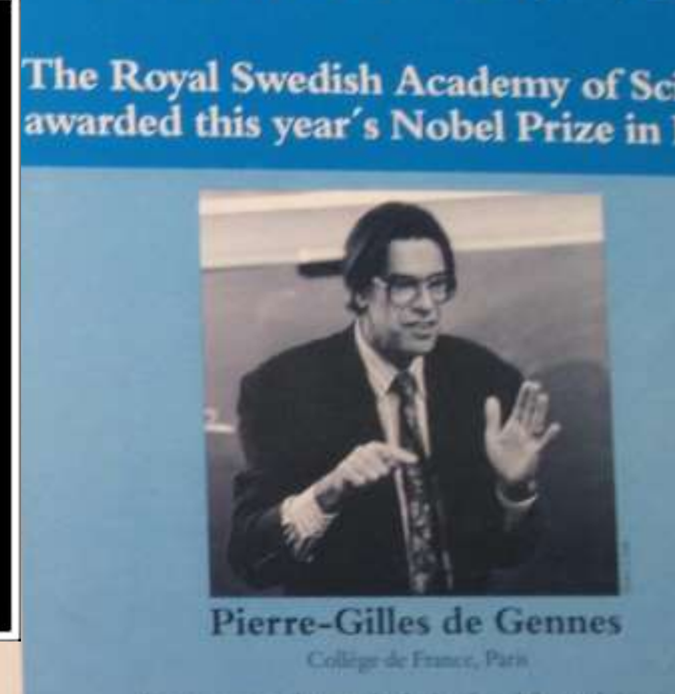
$$De = \frac{\lambda_{material}}{t_{flow}}$$

At very short times (the impact of a bullet) the Silly Putty™ shatters (courtesy MIT Edgerton Strobe Laboratories).



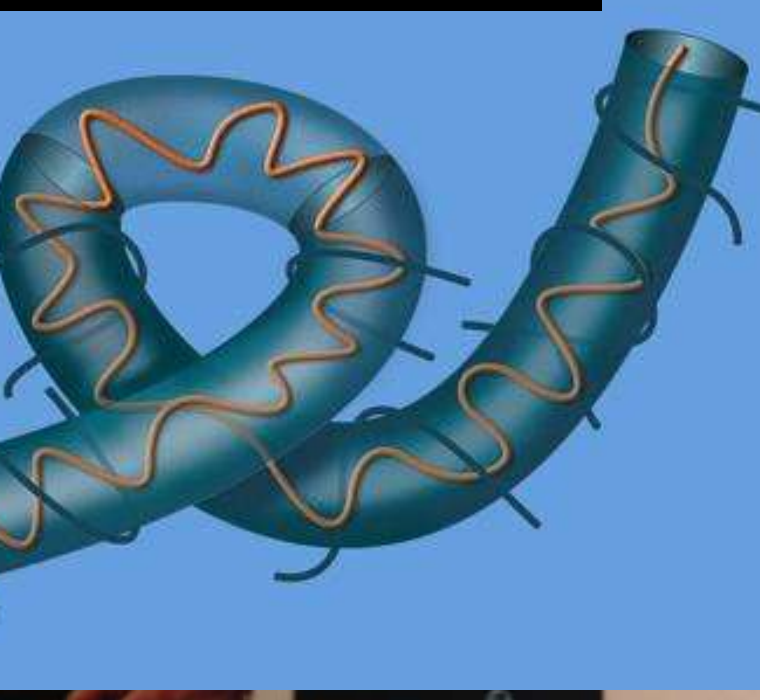


Polymer Melts



Reptation Model

Blob model



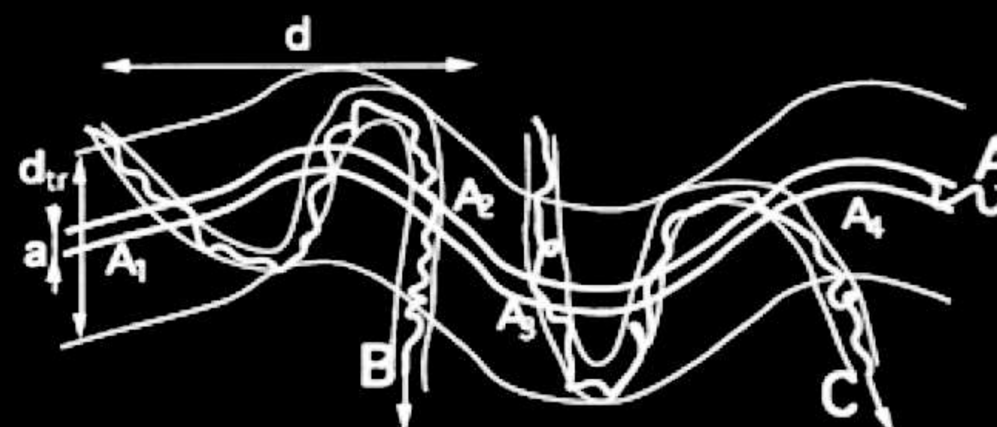


FIG. 1. As chain *A* reptates out of its tube, the nei
chains *B* and *C* move into the region and partially re
memory left by the chain *A* in the form of elastic dist
the entanglement net.

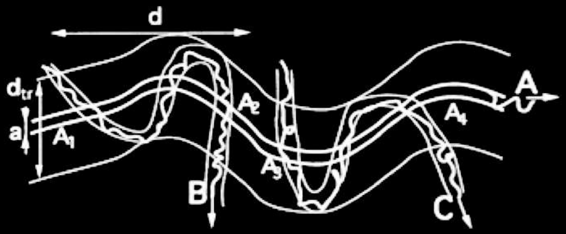


FIG. 1. As chain A reptates out of its tube, the neighboring chains B and C move into the region and partially recover the memory left by the chain A in the form of elastic distortions of the entanglement net.

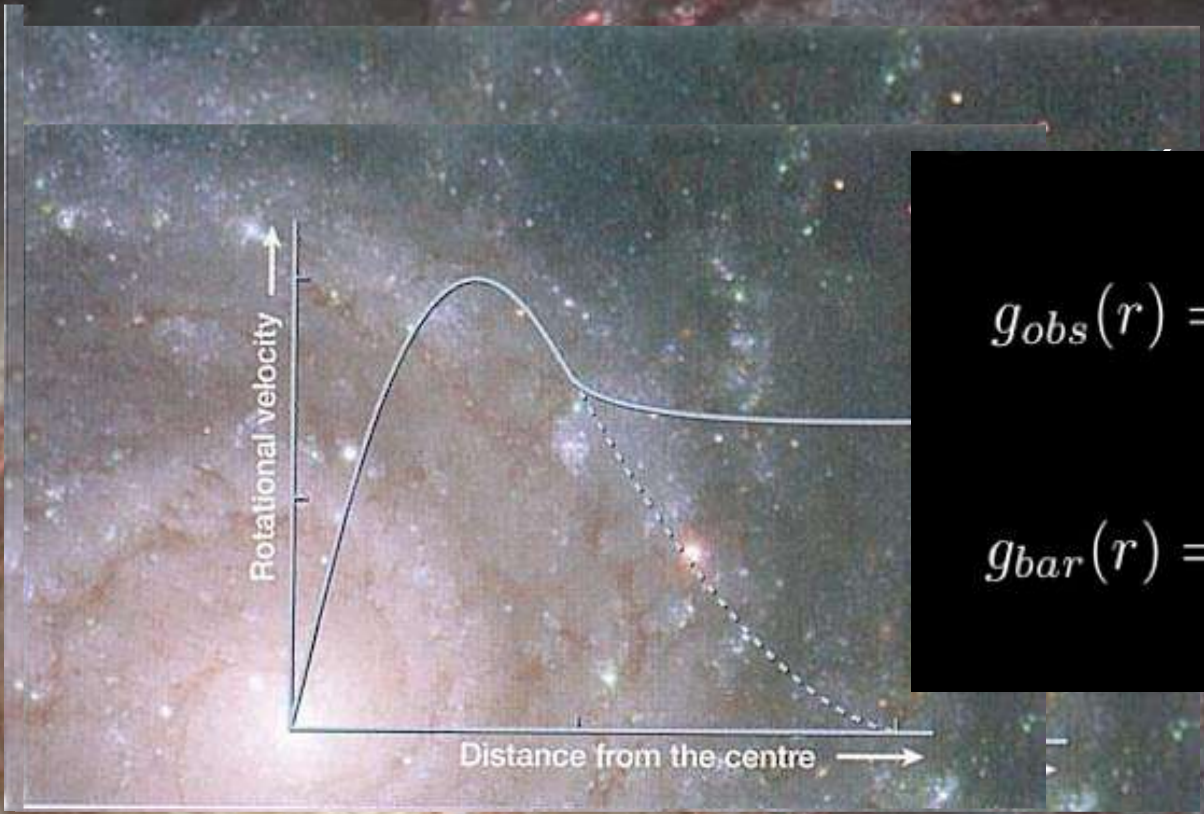
Memory Effects in Entangled Polymer Melts

Michael Rubinstein
te Research Laboratories, Eastman Kodak Company, Rochester, New York 14650-2110

S. P. Obukhov

A simple estimate of the energy of elastic deformation of an entanglement network with modulus $G \sim kT/N_e v_0$ due to displacement of N_e monomers from one end of the tube to the other can be made. Here N_e is the degree of polymerization between entanglements and v_0 is the volume of a monomer. The extra volume V_e appearing at the end of the tube is proportional to the number N_e of displaced monomers $V_e = N_e v_0$. This results in the displacement δr_e of elastic media distance r away from the tube end $\delta r_e \approx V_e/4\pi r^2$, leading to the strain ε_e in the neighborhood of the tube end of the order of $\varepsilon_e = \partial(\delta r_e)/\partial r \approx -V_e/2\pi r^3$. The elastic energy can be estimated as

$$E_e \approx \int (G\varepsilon_e^2/2) d^3 r \approx G V_e^2/a^3 \approx kT v_0 N_e/a^3 . \qquad (1)$$

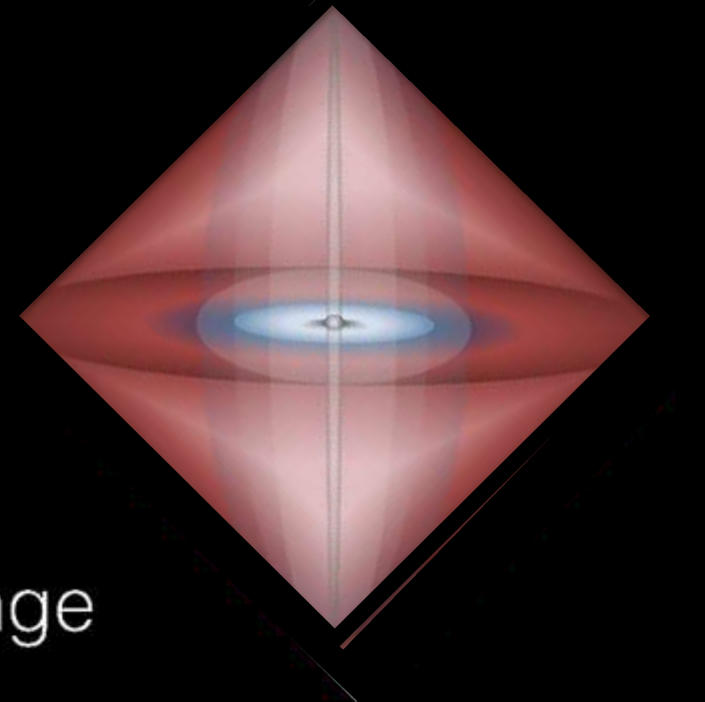


$$g_{obs}(r) = \frac{GM_B(r)}{r^2} + \frac{GM_D(r)}{r^2}$$

$$g_{bar}(r) = \frac{GM_B(r)}{r^2}$$

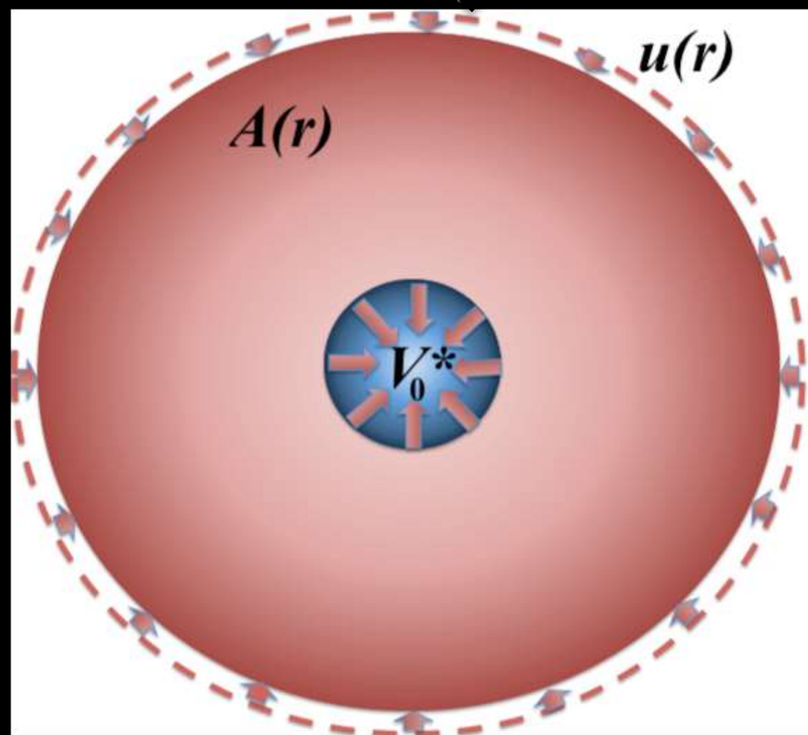
$$\frac{1}{8\pi G}\int_{r\leq R} g_i^2\, dV = \frac{M_B c R}{\hbar}\frac{\hbar H_0}{6}$$

Mass leads to an area change



$$\boxed{\frac{\Delta A(r)}{4G\hbar} = \frac{2\pi M}{a_0}\frac{r}{L}}$$

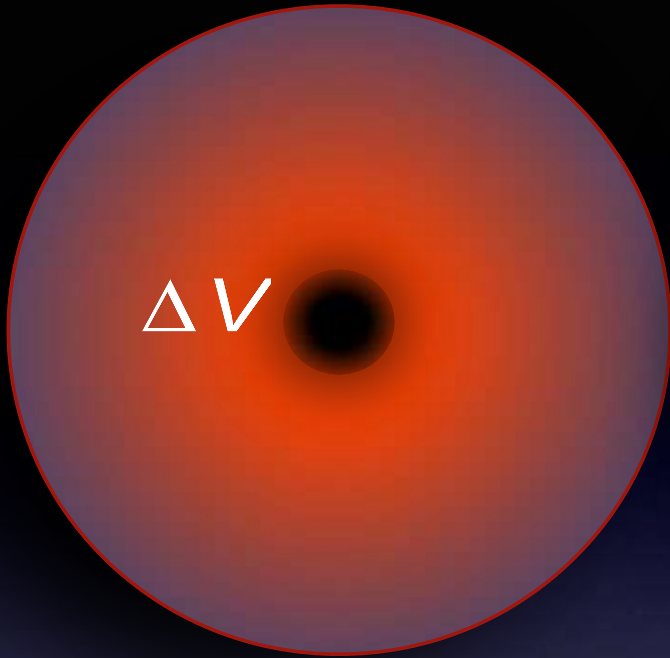
But it also leads to a volume change

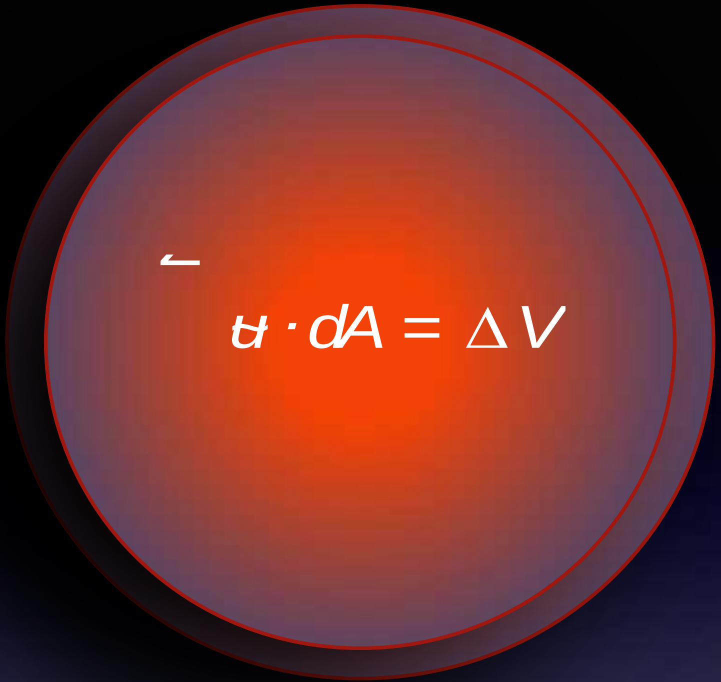


$$u_i(r) = \frac{\Phi(r)}{a_0} n_i$$

Total removed volume is

$$V^*(r) = \oint u_i(r) dA_i$$





displacement

$$u(r) \propto \frac{\Delta V}{A(r)}$$

strain

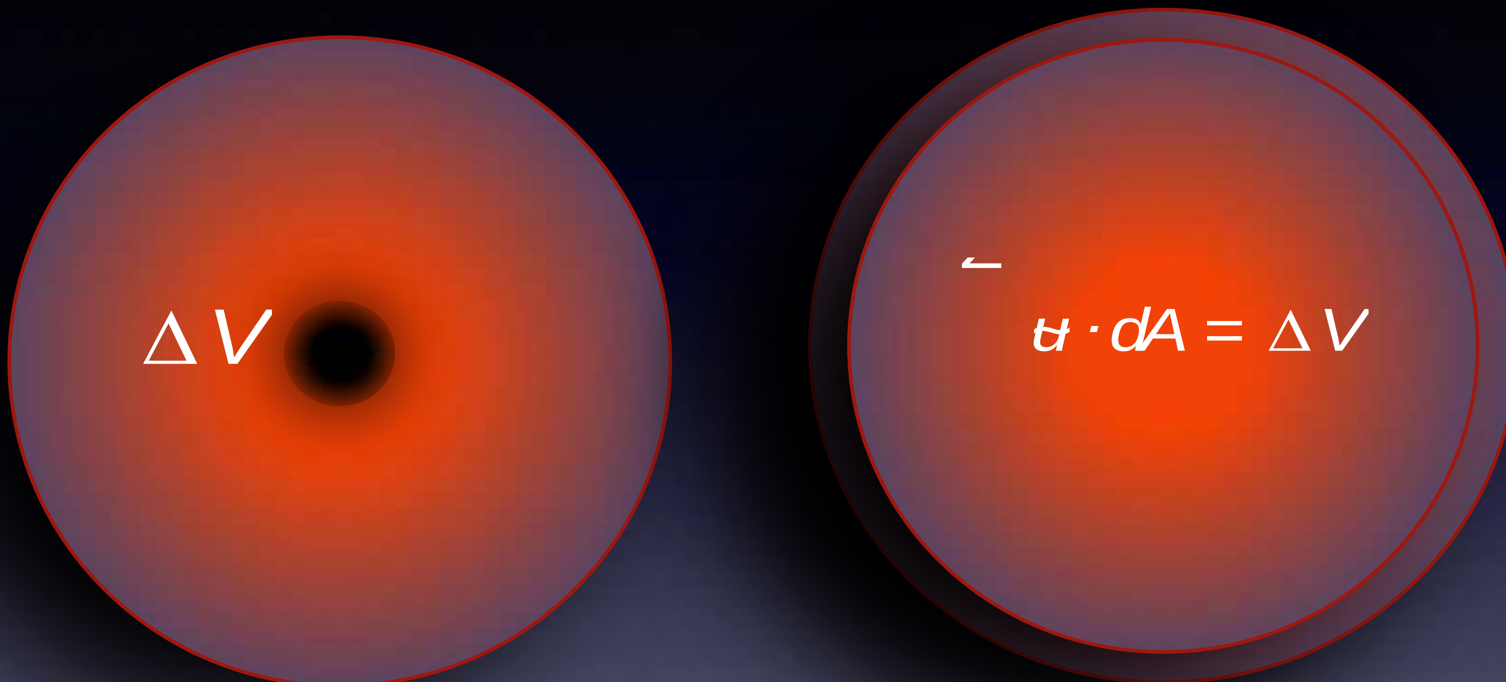
$$\epsilon(r) = \frac{\Delta V}{V(r)}$$

elastic energy is proportional to

$$\int \epsilon^2 dV = \Delta V$$

$$\epsilon > 1$$

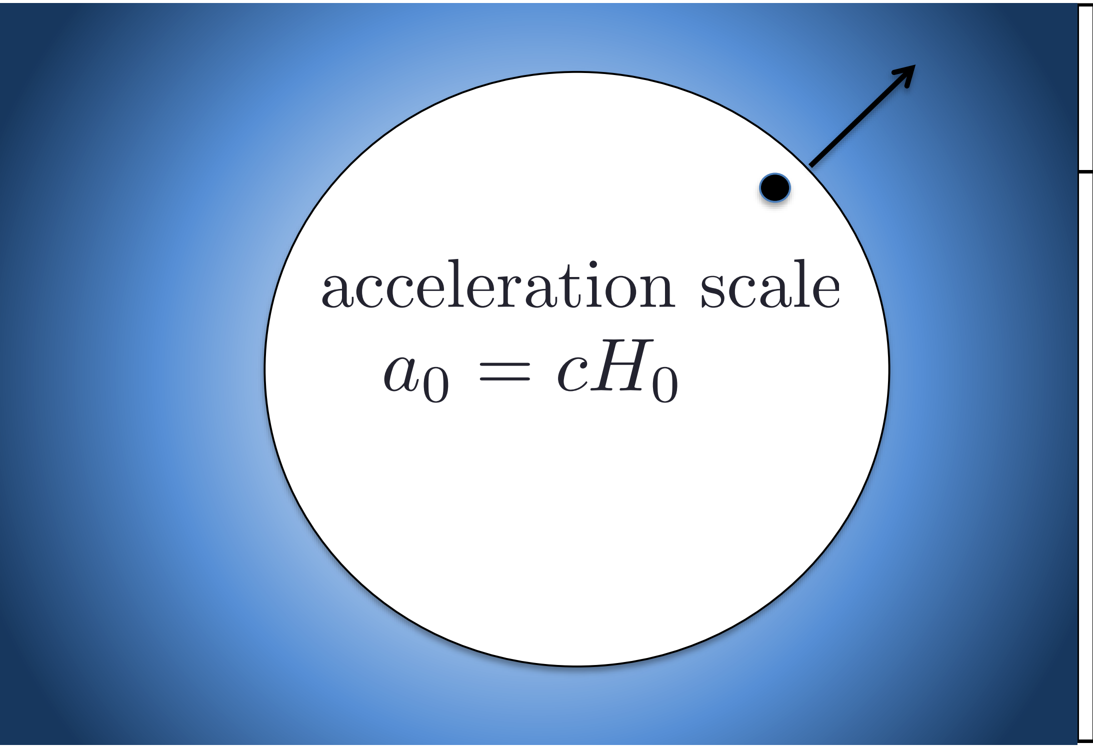
Removing entropy from the volume law entanglement entropy leads to an elastic respons.



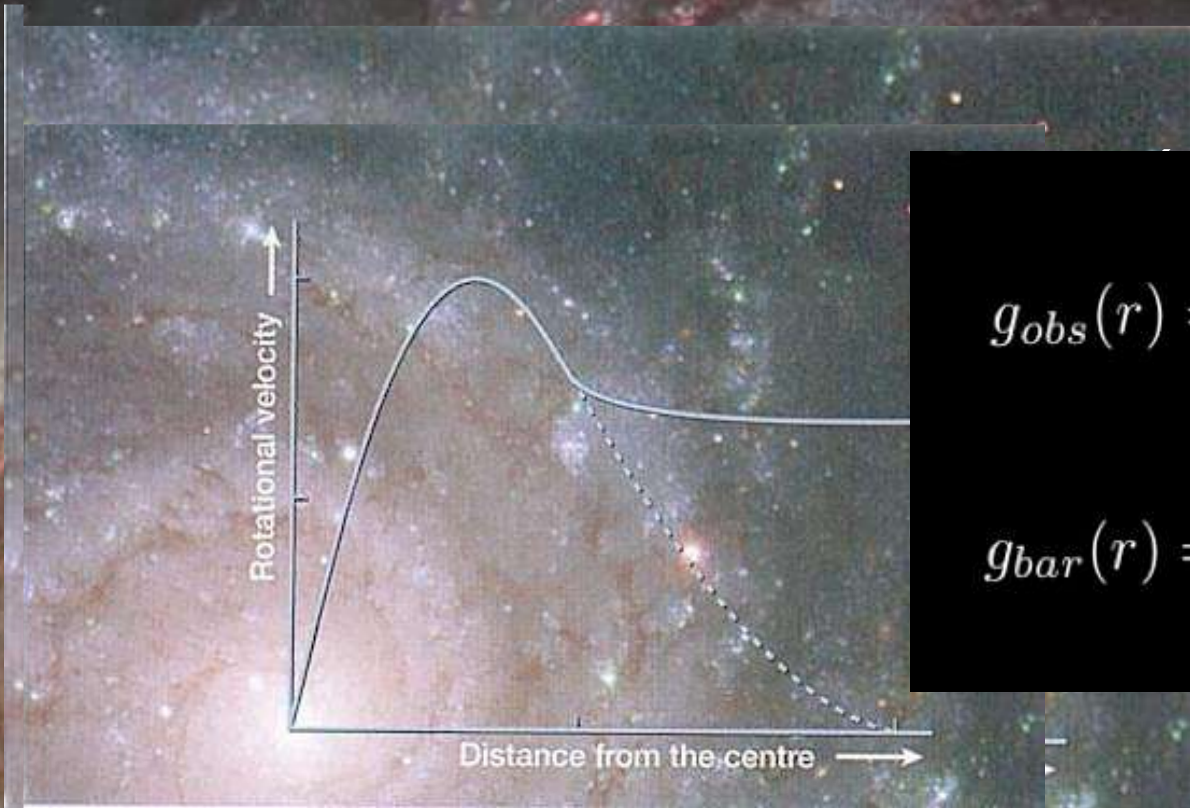
Standard theory of elasticity relates the elastic energy to the removed volume => determined by removed entropy

$$\int \varepsilon_{ij}^2 \, dV = \Delta V = \frac{4\pi GML}{c^2} R$$

Gravitational quantity		Elastic quantity	
Newtonian potential	Φ	displacement field	u_i
gravitational acceleration	g_i	strain tensor	ε_{ij}
surface mass density	Σ_i	stress tensor	σ_{ij}
mass density	ρ	body force	b_i
point mass	m	point force	f_i



Correspondence		
$\Phi\, n_i$	$=$	$a_0 u_i$
g_i/a_0	$=$	$\varepsilon_{ij} n_j$
$\Sigma_i a_0$	$=$	$\sigma_{ij} n_j$
$\rho\, n_i$	$=$	b_i/a_0
$m\, n_i$	$=$	f_i/a_0



$$g_{obs}(r) = \frac{GM_B(r)}{r^2} + \frac{GM_D(r)}{r^2}$$

$$g_{bar}(r) = \frac{GM_B(r)}{r^2}$$

$$\frac{1}{8\pi G}\int_{r\leq R} g_i^2\, dV = \frac{M_B c R}{\hbar}\frac{\hbar H_0}{6}$$

Alternative form of the result: (spherical symmetry)

$$\int_0^R \frac{G M_D(r)^2}{r^2} dr = \frac{M_B(R) c H_0 R}{6}$$

Express the masses in terms of average densities

$$M_B(R) = \frac{4p}{3} R^3 \bar{r}_B(R)$$

$$M_D(R) = \frac{4p}{3} R^3 \bar{r}_D(R)$$

$$H_0^2 = \frac{8pG}{3} \bar{r}_{crit}$$

and differentiate with respect to *R*

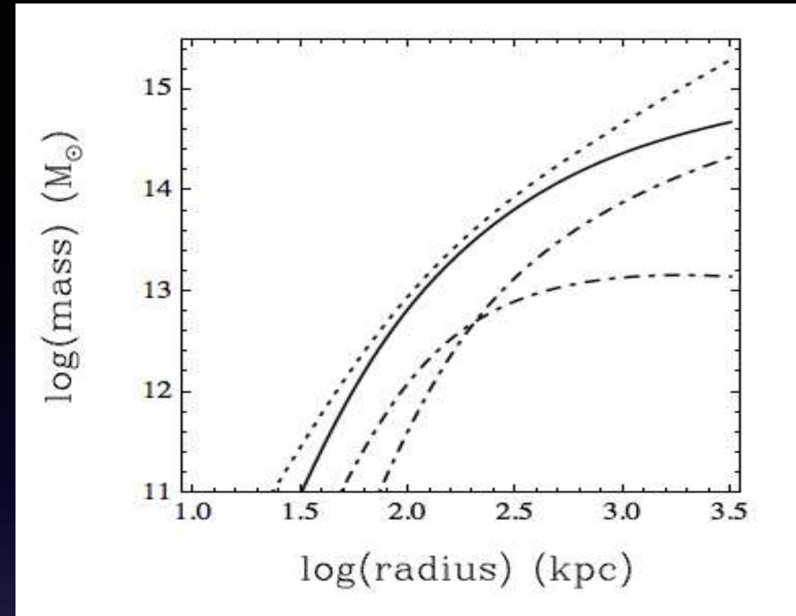
Universal formula for equivalent dark matter density

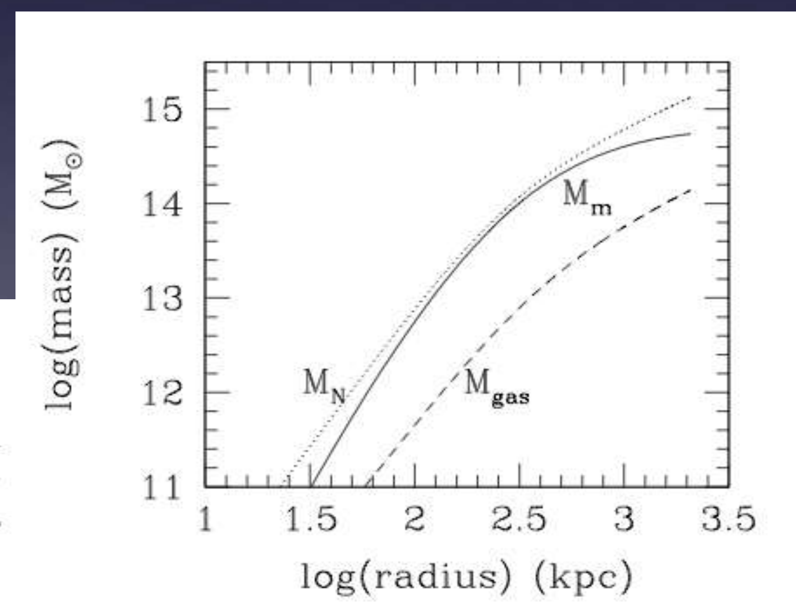
$$\boxed{\frac{\overline{\rho}_B(R)\overline{\rho}_{crit}}{\overline{\rho}_D^2(R)} = \frac{3H_0R}{4+\alpha_B(R)}}$$

$$M(R) = \frac{4\pi\overline{\rho}(R)R^3}{3}$$

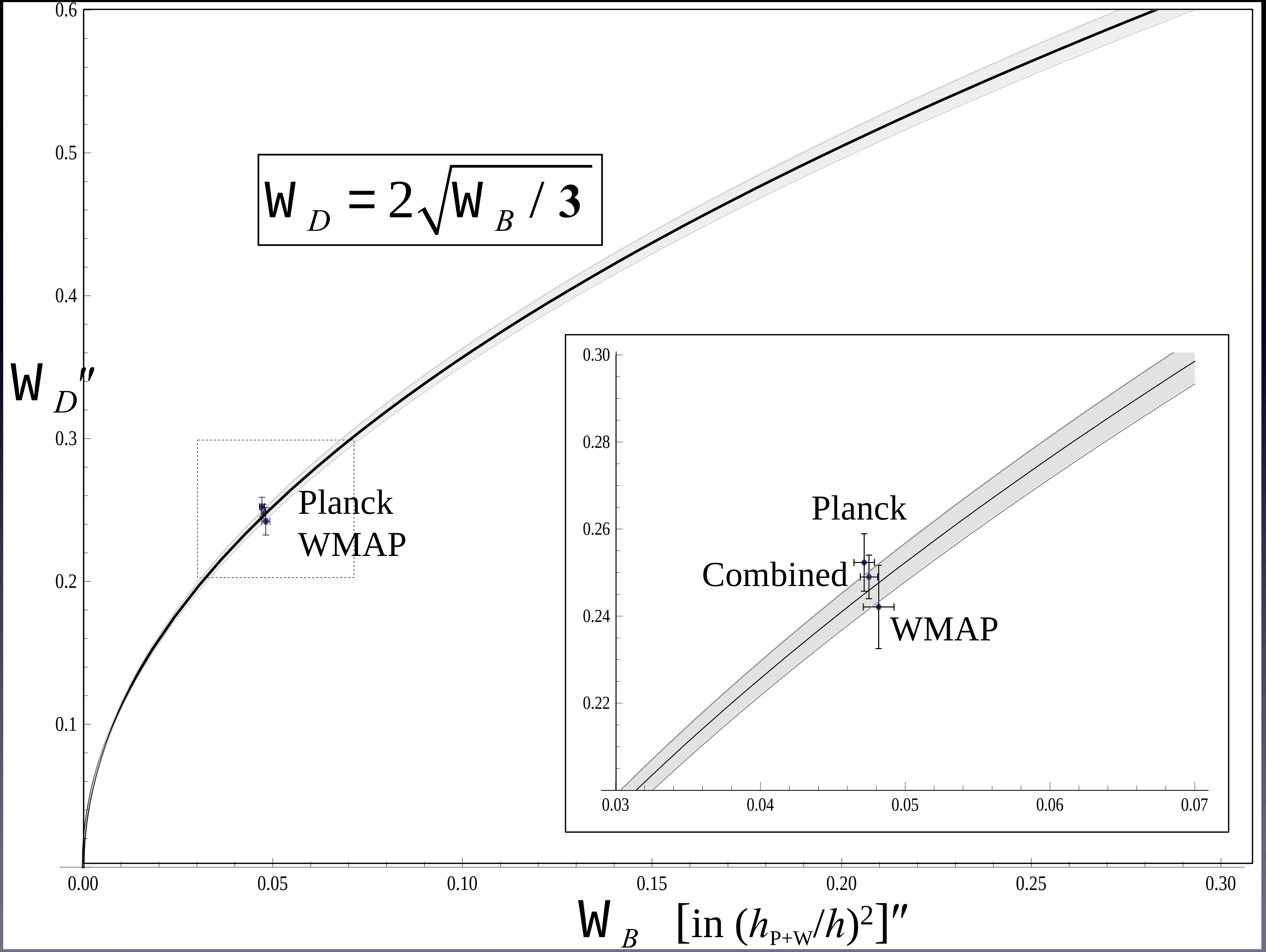
$$\alpha_B(R) = \frac{d\log\overline{\rho}_B(R)}{d\log R}$$

Coma Cluster





However, problems do arise when one attempts to apply MOND to the large clusters of galaxies. The and White (1988) first noted that, to successfully account for the discrepancy between the observed mass and the traditional virial mass in the Coma Cluster, the MOND acceleration parameter, supposedly a universal constant, should be about a factor of four larger than the value implied by galaxy rotation curves.



$\Omega_D = 2\sqrt{\Omega_B / 3}$
Planck
WMAP
Combined
0.6
0.5
0.4
0.3
0.2
0.1
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.30
0.28
0.26
0.24
0.22
0.03
0.04
0.05
0.06
0.07
"Ω_D"
"Ω_B [in $(h_{P+W}/h)^2$]"

Conclusions

- Gravity emerges from entanglement entropy using the 1st law.
- GR requires entanglement to obey an area law: there is no thermal entropy contribution.
- de Sitter space corresponds to an excited state with a finite entropy, temperature and energy density: glassy state.
- The principles of emergent gravity still go through but need to be generalized to case with thermal entropy density (~elasticity).