

## Short talks

**Axelsson, Andreas** (Stockholm University, Sweden):

*Variational vs. singular integral solutions to boundary value problems.*

**Abstract:**

This talk concerns Dirichlet and Neumann boundary value problems for second order elliptic divergence form equations with non-symmetric, or more general complex, and measurable coefficients independent of the transversal coordinate in the upper half space. I shall discuss some recent results obtained which shows that, in the case of non-symmetric coefficients, the solutions obtained with the boundary equation method may differ from the solutions obtained with Lax–Milgram’s method. Surprisingly, for sufficiently non-symmetric coefficients, the boundary equation solutions for the Dirichlet problem are given by a signed harmonic measure. The variational solutions in the examples given, have earlier been studied by Kenig et al.

I shall also talk on recent joint work with Auscher and McIntosh. This gives perturbation results for well posedness of the boundary value problems for complex coefficients, and uniqueness of the solutions.

**Badr, Nadine** (Université Paris-Sud, France):

*Real Interpolation of Sobolev spaces on metric-measure spaces.*

**Abstract:**

We study the real interpolation of Sobolev spaces on metric-measure spaces. We prove that the non-homogeneous Sobolev spaces  $W_p^1$  (resp. homogeneous  $\dot{W}_p^1$ ) defined on a Riemannian manifold satisfying a doubling property and a Poincaré inequality, form a real interpolation scale on an interval of values of  $p$ . We extend this result to other geometric frames. By means of an example, we study the necessity of Poincaré inequality to the interpolation.

**Baklouti, Ali** (Faculty of Sciences of Sfax, Tunisia):

*On Hardy’s Theorem on Solvable Lie groups.*

**Abstract:** Let  $G$  be a solvable connected and simply connected Lie group. It is well known that Hardy’s Theorem holds when  $G$  is nilpotent. In this talk, we show that the result fails to hold for general exponential Lie groups. As such, we get a positive answer whenever the center of the group in question is not trivial. We finally show that the last condition is not necessary when  $G$  is merely solvable. This is a joint work with E. Kanuith.

**Ben Ali, Besma** (University of Paris XI, France):

*Maximal inequalities and Riesz transform estimates on  $L^p$  spaces for Schrödinger operators .*

**Abstract:** We give various estimates for the Schrödinger operator

$$H(\mathbf{a}, V) = \sum_{j=1}^n \left( \frac{1}{i} \frac{\partial}{\partial x_j} - a_j \right)^2 + V$$

in  $\mathbb{R}^n$ , with magnetic field  $B = \text{curl}(\mathbf{a})$  and a non negative electric potential  $V$ . Our main tools are improved Fefferman-Phong inequalities and reverse Hölder estimates for weak solutions of  $H(\mathbf{a}, V)$ .

**Ben Salah, Nour** (University of Sfax, Tunisia):

*Some uncertainty principles on nilpotent Lie groups.*

**Abstract:** Let  $G$  be a connected and simply connected nilpotent Lie group. It is well known that for  $2 \leq p, q \leq \infty$ , the  $L^p - L^q$  version of Hardy's theorem holds. The subject of the talk consists in presenting some new results concerning the case when  $1 \leq p, q \leq \infty$  for some restrictive situations.

**Bernicot, Frédéric** (Université Paris-Sud, France):

*On a bilinear pseudodifferential calculus.*

**Abstract:** In this talk, we want to present new results about the bilinear pseudodifferential calculus on  $\mathbb{R}$ . First we will define our pseudodifferential classes for the bilinear symbols. We look for largest classes of associated bilinear operators, which contains Coifman-Meyer operators, Marcinkiewicz multipliers and far more singular operators related to the bilinear Hilbert transforms. Mainly we study bilinear symbols  $\sigma(x, \xi_1, \xi_2) \in BS_{\rho, \delta}$ : that is meaning that  $\sigma$  satisfies for all  $a, b, c \geq 0$ ,

$$\left| \partial_x^a \partial_{\xi_1}^b \partial_{\xi_2}^c \sigma(x, \xi_1, \xi_2) \right| \lesssim (1 + d(\xi, \Gamma))^{\delta a} (1 + d(\xi, \Gamma_1))^{-\rho b} (1 + d(\xi, \Gamma_2))^{-\rho c},$$

where  $\Gamma, \Gamma_1, \Gamma_2$  are three cones of the frequency plane and  $d(\xi, \Gamma)$  corresponds to the distance in this plane from the point  $\xi$  to the cone. Then we will present recent results about  $L^p$  estimates for the operators of type  $BS_{1,0}$ . We will describe their continuities in Sobolev spaces and rules of symbolic bilinear calculus for the duality and composition operations. Then we will explain a work joint with R. Torres about more singular symbols: those of type  $BS_{1,1}$ . In this case, similarly to results about linear operators associated to the classical class  $S_{1,1}^0$ , we obtain the continuities in Sobolev spaces of positive order.

**Betancor, Jorge J.** (Universidad de La Laguna, Spain):

*Higher order Riesz transforms for Laguerre expansions.*

**Abstract:** In this note we present  $L^p$ -boundedness properties for the higher order Riesz transforms associated with Laguerre operators. Also we prove that the  $k$ -th Riesz transform is a principal value singular integral operator (modulo a constant multiple of the identity when  $k$  is even). We exploit a new identity connecting Riesz transforms in the Hermite and Laguerre settings. The results we present here are part of a joint paper with J.C. Fariña, L. Rodríguez-Mesa and A. Sanabria.

**Bez, Neal** (University of Birmingham, UK):

*Heat-flow monotonicity and Young's convolution inequality.*

**Abstract:**

We prove that if  $u_1, \dots, u_n : (0, \infty) \times \mathbb{R}^d \rightarrow (0, \infty)$  satisfy certain heat inequalities and  $p_1, \dots, p_n \in [1, \infty]$  then the  $r$ th power of the spatial convolution  $u_1^{1/p_1} * \dots * u_n^{1/p_n}$  also satisfies a heat inequality of a similar type provided  $1/p_1 + \dots + 1/p_n = 1/r + n - 1$ . An immediate corollary is that the  $L^r(\mathbb{R}^d)$  norm of  $u_1^{1/p_1} * \dots * u_n^{1/p_n}$  is nondecreasing in time. This in turn gives a direct heat-flow proof of the sharp Young convolution inequality. This is joint work with J. Bennett.

**Blasco, Óscar** (University of Valencia, Spain):

*The bilinear Riesz transform and independence of dimension.*

**Abstract:** The aim of this talk is to show the boundedness of the bilinear Riesz transform with norm independence of the dimension: If  $1 < p_1, p_2, p_3 < \infty$  and  $1/p_3 = 1/p_1 + 1/p_2$  then there exists  $C$  independent of  $n$  such that

$$\left\| \left( \sum_{k=1}^n |R_k(f, g)|^2 \right)^{1/2} \right\|_{L^{p_3}(\mathbb{R}^n)} \leq C \|f\|_{L^{p_1}(\mathbb{R}^n)} \|g\|_{L^{p_2}(\mathbb{R}^n)},$$

where

$$R_k(f, g)(x) = \frac{1}{d_n} \int_{\mathbb{R}^n} f(x-y)g(x+y) \frac{y_k}{|y|^{n+1}} dy$$

and  $d_n$  is chosen for  $\|R_k\|_{L^2(\mathbb{R}^n) \times L^2(\mathbb{R}^n) \rightarrow L^1(\mathbb{R}^n)} = 1$ .

**Boza, Santiago** (Universitat Politècnica de Catalunya, Spain):

*Weighted Hardy modular inequalities in variable  $L^p$  spaces for decreasing functions.*

**Abstract:** We study weighted modular inequalities with variable exponents for the Hardy operator restricted to non-increasing functions. We show that the exponents  $p(\cdot)$  for which these modular inequalities hold, must be either constant or extremely oscillating functions near the origin. Similarly to the constant case, we introduce the class of weights  $B_{p(\cdot)}$ , and prove some of the classical properties in this context. This is a joint work with Javier Soria from Universitat de Barcelona.

**Chae, Myeongju** (Hankyong National University, Korea):

*Global well-posedness of rough solutions to the  $L^2$  critical Hartree equation on  $\mathbb{R}^n$ ,  $n \geq 3$ .*

**Abstract:** In this talk we present global well-posedness of rough solutions to the  $L^2$  critical Hartree equation on  $\mathbb{R}^n$ ,  $n \geq 3$ . More precisely we show that a global solution exists for initial data in the Sobolev space  $H^s(\mathbb{R}^n)$  and any  $\frac{2(n-2)}{3n-4} < s < 1$ . We use the I-method, an interaction Morawetz estimate, and modified multilinear estimates.

**Chua, Seng-Kee** (National University of Singapore):

*Self-improving properties of Poincaré type inequalities.*

**Abstract:** We show that the self-improving nature of Poincaré estimates persists for domains in rather general measure spaces. We consider both weak type and strong type inequalities, extending techniques of B. Franchi, C. Pérez and R. Wheeden. As an application in spaces of homogeneous type, we derive global Poincaré estimates for a class of domains with rough boundaries that we call  $\phi$ -John domains, and we show that such domains have the requisite properties. This class includes John (or Boman) domains as well as  $s$ -John domains.

**Curbera, Guillermo P.** (Universidad de Sevilla, Spain):

*Sobolev type inequalities via optimal domains.*

**Abstract:** Sobolev type inequalities are presented which arise by substitution of the classical gradient with an alternative functional, arising from an inequality of Talenti. For spaces which are not rearrangement invariant this leads to new inequalities.

**Daher, Radouan** (University of Hassan II, Morocco):

*Generalized Hardy's theorem on Chebli-Trimeche hypergroups.*

**Abstract:** The classical Hardy's theorem on real line was generalized by Miyachi in 1997 and Bonami, Demange, Jaming in 2002. In this talk we consider the generalized Fourier transform on Chebli-Trimeche hypergroups and we show that Miyachi's theorem and Bonami-Demange-Jaming's can be reformulated for this generalized Fourier transform in terms of the heat kernel. This is joint work with Professor S. Fahaloui, University Moulay Ismail, Meknes, Morocco.

**Feuto, Justin** (University of Orléans, France):

*End points for the div-curl Lemma in Hardy spaces.*

**Abstract:** After the seminal paper of Coifman-Lions-Meyer-Semmes, many authors have been interested in generalizations of the div-curl lemma. We consider here the end-point  $q = \infty$  while  $p > \frac{n}{n+1}$ . We prove that if  $U$  has coefficients in the Hardy space  $H^p(\mathbb{R}^n)$ , with  $\operatorname{div} U = 0$ , while  $V$  has bounded coefficients, with  $\operatorname{curl} V = 0$ , then  $U \cdot V$  is in  $H^p(\mathbb{R}^n)$ . For  $p = 1$ , this is due to Auscher, Russ and Tchamitchian. In this case, it is also possible to take  $V$  with coefficients in  $BMO$  and find that  $U \cdot V$  belongs to some Hardy-Orlicz space.

This may be compared to the properties of the product of a distribution in the Hardy space  $H^p(\mathbb{R}^n)$  with a function in its dual space in the sense of distributions. We show that, for  $p < 1$  this product can be written as the sum of an integrable function with a distribution in  $H^p(\mathbb{R}^n)$ . For  $p = 1$ , it was proved by Bonami, Iwaniec, Jones and Zinsmeister that such a product is the sum of an integrable function and a distribution in some Hardy-Orlicz space. Compensations by cancellation for scalar products as  $U \cdot V$  allow to suppress the term in  $L^1$ . This is a joint work with Aline Bonami and Sandrine Grellier.

**Flores, Manuel** (Universidad de La Laguna, Spain):

*On Hypoellipticity for  $\bar{\partial}_b$  on some Nilpotent Groups.*

**Abstract:**

In this talk we will try to show that the canonical solution to the  $\bar{\partial}_b$ -problem fails to be hypoelliptic, in  $L^2$ -sense, on some class of finite type high codimension submanifolds of  $\mathbb{C}^n$ . More precisely if, for  $n \geq 2$  an even integer we consider

$$\Sigma_n = \left\{ z = (z_1, z_2, \dots, z_n) \in \mathbb{C}^n : \operatorname{Re} z_k = [\operatorname{Re} z_1]^k, k = 1, 2, \dots, n \right\},$$

$\Sigma_n$  turns out to be a step  $n$  nilpotent Lie group which generalizes the 1-dimensional Heisenberg group. If  $\bar{\partial}_b^*$  denotes the  $L^2$  adjoint of  $\bar{\partial}_b$  on  $\Sigma_n$  then, if for  $f \in \operatorname{Ran} \bar{\partial}_b$ ,  $u = G[f] \perp \operatorname{Ker} \bar{\partial}_b$  is the canonical (or Kohn's solution) to  $\bar{\partial}_b u = f$ , the operator  $\bar{\partial}_b^* G$  does not have a bounded extension to  $L^2$  as long as  $n > 2$ . In other words, the estimate

$$\|\bar{\partial}_b^* u\|_{L^2(\Sigma_n)} \lesssim \|\bar{\partial}_b u\|_{L^2(\Sigma_n)}, \quad u \perp \operatorname{Ker} \bar{\partial}_b$$

does not hold. This is in sharp contrast with the case  $n = 2$ , for  $\Sigma_2$  is just the classical 1-dimensional Heisenberg group in its polarized form, where the above estimate is known to hold.

**Folch-Gabayet, Magali** (Universidad Nacional Autónoma de México, México):  
*Toeplitz operators on Herglotz wave functions.*

**Abstract:**

In this work we study Toeplitz operators on  $\mathcal{W}^2$  (the space of Herglotz wave functions in  $\mathbb{R}^2$ ) for radial nonnegative symbols  $\rho$ .

Using precise estimates for the Bessel functions we prove that for  $\rho \in L^1([0, \infty), \frac{r}{1+r^3})$ , the boundedness of the Toeplitz operator  $T_\rho$  is equivalent to the boundedness of the sequence

$$\left\{ n^2 \int_0^\infty J_n^2(r) \rho(r) \frac{r}{1+r^3} dr \right\}_{n \in \mathbb{Z} - \{0\}}.$$

This is joint work with J.A. Barceló, S. Pérez-Esteve and A. Ruiz.

**Gatto, A. Eduardo** (DePaul University, USA):

*On the boundedness on inhomogeneous Lipschitz spaces of Fractional Integrals, Singular Integrals and Hypersingular Integrals associated to non-doubling measures on metric spaces.*

**Abstract:** In this talk we will show four “T1” Theorems on inhomogeneous Lipschitz spaces defined on a non-doubling measure metric space: A “T1” theorem for the boundedness of fractional integrals, a “T1” theorem for the boundedness of smooth truncations of singular integrals, a “T1” for principal value singular integrals and a “T1” theorem for hypersingular integrals.

Reference: Boundedness on Lipschitz spaces of singular integrals associated to non-doubling measures. CRM. Preprint #772, October 2007.

**Gigante, Giacomo** (University of Bergamo, Italy):

*Equiconvergence theorems for Chébli-Trimèche hypergroups.*

**Abstract:** We consider a Sturm-Liouville operator of the kind  $\frac{d^2}{dt^2} + \frac{A'(t)}{A(t)} \frac{d}{dt}$  on  $(0, +\infty)$  and the related eigenfunction expansion. We prove that, under suitable assumptions on  $A(t)$ , the partial sums of the Fourier integral associated to such expansion behave like the partial sums of the classical Fourier-Bessel transform. This implies an almost everywhere convergence result for  $L^p(A(t) dt)$  functions. Our methods rely on asymptotic expansions for the eigenfunctions and the Harish-Chandra function, that we prove under very weak hypotheses. This is a joint work with Luca Brandolini.

**Hagelstein, Paul** (Baylor University, USA):

*Recent Developments Regarding the Halo Conjecture.*

**Abstract:** The Halo Conjecture has long provided a fascinating open problem in the theory of differentiation of integrals. Recent progress towards the resolution of this conjecture will be discussed, in particular the theorem of Hagelstein and Stokolos that any density basis consisting of a homothety invariant collection of convex sets must necessarily differentiate  $L^p$  for sufficiently large  $p$ . Connections between this result, the recent work of Bateman and Katz on Kakeya sets and directional maximal operators, and improvements on the well-known theorem of Córdoba and Fefferman relating the  $L^p$  bounds of geometric maximal operators to those of certain multiplier operators will also be given.

**Hong, Sunggeum** (Chosun University University, Korea):

*Riesz means for conic and cylindric distance functions with convex polygons.*

**Abstract:** We consider convolution operators  $T^\delta$  and maximal operators  $S_*^\delta$  for conic and cylindric distance functions with a convex polygon, respectively. We show that there are  $p$  and  $\delta$  restrictions for sharp estimates on  $H^p(\mathbb{R}^3)$ ,  $p < 1$ . More precisely,  $T^\delta$  satisfies weak type  $(p, p)$  on  $H^p(\mathbb{R}^3)$ ,  $1/2 < p < 1$  for the critical value  $\delta = 2(1/p - 1)$ . On the other hand,  $S_*^\delta$  is of weak type  $(p, p)$  on  $H^p(\mathbb{R}^3)$  when  $2/3 < p < 1$  and  $\delta = 3(1/p - 1)$ , or when  $p = 2/3$  and  $\delta > 3(1/p - 1)$ . This is a joint work with Joonil Kim and Chan Woo Yang.

**Hytönen, Tuomas** (University of Helsinki, Finland):

*The vector-valued non-homogeneous Tb theorem.*

**Abstract:** In this talk I describe an effort to bring together two different directions in which the classical Calderón–Zygmund theory of singular integrals on  $L^p(\mathbb{R}^n)$  has been generalized:

- to allow for vector-valued functions with range in an infinite-dimensional Banach space, and
- to replace the underlying Lebesgue measure  $dx$  by a more general  $d\mu$ , especially one without the doubling property.

More precisely, a new  $Tb$  theorem is presented which (essentially) gives a simultaneous generalization of the vector-valued  $T1$  theorem due to Figiel (Proc. Conf. Strobl, 1990) and the non-homogeneous  $Tb$  theorem of Nazarov, Treil, and Volberg (*Acta Math.*, 2003). The proof involves, among other things, careful analysis of the non-homogeneous “Haar” functions and McConnell’s decoupling estimates for so-called tangent martingale differences which, although known in Stochastics, seem not to have been exploited in the Calderón–Zygmund theory before.

**Kim, Joonil** (Chung-Ang University, Korea):

*Multiple Hilbert Transforms Along Polynomial Surfaces.*

**Abstract:** Given  $\Lambda \subset \mathbb{Z}_+^n$ , with  $\mathbb{Z}_+$  the set of nonnegative integers, let  $P_\Lambda(t) = \sum_{m \in \Lambda} a_m t^m$  where  $a_m \in \mathbb{R} \setminus \{0\}$ . Associated with  $P_\Lambda$ , we define a multiple Hilbert transform by

$$\mathcal{H}_{P_\Lambda} f(x) = \text{p.v.} \int_I f(x_1 - t_1, \dots, x_n - t_n, x_{n+1} - P_\Lambda(t)) \frac{dt_1}{t_1} \dots \frac{dt_n}{t_n}$$

where  $I$  is a cube of  $\mathbb{R}^n$  containing the origin. For the case  $n \geq 3$ , we discuss a necessary and sufficient condition that given  $\Lambda \subset \mathbb{Z}_+^n$ , the operator  $\mathcal{H}_{P_\Lambda}$  associated with any polynomial  $P_\Lambda$  of the above form is bounded in  $L^p(\mathbb{R}^{n+1})$ .

**Lie, Victor** (University of California Los Angeles):

*The (weak- $L^2$ ) Boundedness of The Quadratic Carleson Operator.*

**Abstract:** We prove that the generalized Carleson operator with polynomial phase function of degree two is of weak type  $(2, 2)$ . For this, we introduce a new approach to the time-frequency analysis of the quadratic phase.

**Lifyand, Elijah** (Bar-Ilan University, Ramat-Gan, Israel):

*Fourier transforms and general monotone functions.*

**Abstract:** Introducing a new class of functions in a joint work with S. Tikhonov allowed us to obtain new results on integrability of Fourier transforms. Among them are solution of one Boas' problem and extension to the radial case.

**Luef, Franz** (University of Vienna, Austria):

*On a new phase-space calculus with applications to Landau quantization.*

**Abstract:** We present some results about phase-space calculus, which is based on new projective representation of phase space. The coorbit spaces in the sense of Feichtinger and Gröchenig turn out to be modulation spaces over phase space. A new type of Gabor frames is introduced to characterize those modulation spaces. Finally we briefly discuss the Weyl calculus for this projective representation and point out its relevance for Landau quantization. The talk is based on joint work with M. de Gosson.

**Maas, Jan** (TU Delft, Netherlands):

*Boundedness of Riesz transforms for elliptic operators on abstract Wiener spaces.*

**Abstract:** A classical result in Malliavin calculus, due to P.A. Meyer, says that the Riesz transform associated with the Ornstein-Uhlenbeck operator is bounded on  $L^p$  with Gaussian measure for  $1 < p < \infty$ .

We generalise this result to non-symmetric elliptic operators on abstract Wiener spaces. Our main tools are the  $H^\infty$ -functional calculus and randomised versions of gradients estimates.

As an application let  $-A$  generate an analytic  $C_0$ -contraction semigroup on a Hilbert space  $H$  and let  $-L$  be the  $L^p$ -realisation of the generator of its (bosonic) second quantisation. Our results imply that two-sided bounds for the Riesz transform of  $L$  are equivalent with the Kato square root property for  $A$ . This is joint work with Jan van Neerven.

**Mei, Tao** (University of Illinois at Urbana-Champaign):

*$H^1 - BMO$  duality associated with semigroup of operators.*

**Abstract:** We study Hardy and BMO spaces for “functions” on abstract “domains”  $\Omega$ . We do not assume any geometric/metric structure on the “domains”. Instead, we assume there exists a semigroup of positive operators on  $L^p(\Omega)$ . We will show an analogue of Fefferman's  $H^1 - BMO$  duality result in this setting. Our “functions” can be quite abstract objects, for example, elements of a Von Neumann algebra.

**Moen, Kabe** (University of Kansas, United States):

*Weighted Inequalities for Fractional Operators.*

**Abstract:** A weighted theory for multilinear fractional operators including fractional integrals and fractional maximal functions is presented. Sufficient conditions for the two weight inequalities of these operators are stated, including a “power-bump” condition and an  $A_\infty$  condition. The one weight theory from here is a consequence of the two weight inequalities.

**Morris, Andrew** (Australian National University, Australia):

*A tent space characterisation for local Hardy spaces.*

**Abstract:**

We will discuss a generalisation of the local Hardy spaces of Goldberg that is adapted to a class of bisectorial operators with certain off-diagonal estimates acting on a doubling metric measure space. The characterisation will be via a local analogue of the tent spaces of Coifman-Meyer-Stein and recent work of Auscher-McIntosh-Russ. A characterisation in terms of local molecules will also be given. This is joint work with Andrea Carbonaro and Alan McIntosh.

**Nowak, Adam** (Wrocław University of Technology, Poland):

*Riesz transforms for the Dunkl harmonic oscillator and the Dunkl Ornstein-Uhlenbeck operator.*

**Abstract:** In this talk we propose an approach to the theory of Riesz transforms in a framework emerging from certain reflection symmetries in Euclidean spaces. Relying on Rösler's construction of multivariable generalized Hermite functions and polynomials associated with a finite reflection group on  $\mathbb{R}^d$ , we define and investigate systems of Riesz transforms related to the Dunkl harmonic oscillator and the Dunkl Ornstein-Uhlenbeck operator. The talk is based on the recent joint papers with K. Stempak and L. Roncal:

- [1] A. Nowak, K. Stempak, *Riesz transforms for the Dunkl harmonic oscillator*, (2007), p. 1–18, submitted.
- [2] A. Nowak, L. Roncal, K. Stempak, *Riesz transforms for the Dunkl Ornstein-Uhlenbeck operator* (2008), p. 1–14, submitted.

**Nyström, Kaj** (Umeå University, Sweden):

*Boundary Behaviour of  $p$ -Harmonic Functions in Domains Beyond Lipschitz Domains.*

**Abstract:** In a sequence of papers John Lewis and I have proved a number of results concerning the boundary behaviour of positive  $p$ -harmonic functions,  $1 < p < \infty$ , in a bounded Lipschitz domain  $\Omega \subset \mathbf{R}^n$ . In particular, we have established the boundary Harnack inequality as well as Hölder continuity for ratios of positive  $p$ -harmonic functions,  $1 < p < \infty$ , vanishing on a portion of  $\partial\Omega$ . The purpose of this talk is to describe recent results by John Lewis and myself concerning extensions of the results mentioned to domains more general than Lipschitz domains. Currently we are unable to extend our results for  $p \neq 2$ ,  $1 < p < \infty$ , to the general setting of NTA-domains introduced by Jerison-Kenig. However in this talk I consider certain Reifenberg flat and Ahlfors regular NTA-domains, beyond Lipschitz domains, for which our methods are applicable. Neither class of domains is contained in the other and our proofs of the  $p$ -harmonic boundary Harnack results differ at key points for each class.

**Ombrosi, Sheldy** (Universidad Nacional del Sur, Argentina):

*$A_1$  bounds for Calderón-Zygmund operators related to a problem of Muckenhoupt and Wheeden.*

**Abstract:** The Muckenhoupt-Wheeden conjecture claims that any Calderón-Zygmund operator is of weak type  $(1, 1)$  with respect to the pair of weights  $(w, Mw)$ . This would imply that the weighted weak type  $(1, 1)$  of a Calderón-Zygmund operator holds with linear growth on the  $A_1$ -constant of  $w$ . As far as we know, the latter statement is also an open problem. In this talk we will present a recent progress on that question. This is a joint work with Andrei Lerner and Carlos Pérez.

**Ou, Winston** (Scripps College, US):

*Near Symmetry of  $A_\infty$  and Refined Jones Factorization.*

**Abstract:** The theory of the structure of  $A_\infty$  reached a peak in 1980 with the Jones factorization theorem, which stated that any  $A_p$  weight could be expressed as a product  $w_0 w_1^{1-p}$  of weights in the limiting class  $A_1$ . We will describe how some recent insights into the structure of the limiting weight classes have simplified the understanding of a later refinement (by Cruz-Uribe and Neugebauer) of Jones's result to simultaneously incorporate reverse Hölder as well as  $A_p$  information. (Paper to appear in *Proceedings of the AMS*)

**Paluszyński, Maciej** (University of Wrocław, Poland):

*Some remarks on the  $\varphi$  and  $\psi$  transforms of Frazier and Jawerth.*

**Abstract:** We present a new approach to the theory of  $\varphi$  and  $\psi$  transforms of Frazier and Jawerth which gives direct proofs of their boundedness in the case when either function  $\varphi$  or  $\psi$  is not band-limited. This uses the more recent characterizations of function spaces, which do not require the analysing function  $\varphi$  to be band-limited. We present an application to the problem of invertibility of these transforms, and we show that certain affine system generated by the Mexican hat function is complete in Lebesgue and Hardy spaces. This is joint work with H.-Q. Bui.

**Pandey, Dwijendra Narain** (Indian Institute of Technology Kanpur, India):

*On numerical solution of Sobolev-type integro-differential equation with nonlocal conditions.*

**Abstract:** We present a numerical approximate solution to Sobolev-type integro-differential equation subject to nonlocal initial boundary conditions. We use Laplace transform method to study the numerical solution of considered equation. Following Laplace transform of the original problem, an appropriate method of solving integro-differential equations is used to solve the resultant time-independent modified equation. At the end of the study we provide a few examples to show the accuracy of the proposed method.

**Portal, Pierre** (Université Lille 1, France):

*Functional calculus of differential operators in  $L^p$ .*

**Abstract:**

Following the solution of Kato's square root problem (Auscher, Hoffmann, Lacey, McIntosh, Tchamitchian 2002), harmonic analytic methods (of the local  $T(b)$  type) for  $H^\infty$  functional calculus problems have recently flourished. In this short talk, we will present one of these methods, developed in a joint work with T. Hytönen and A. McIntosh, which allows to characterize, in  $L^p$ , the functional calculus, of (some) differential operators in terms of (randomized boundedness) properties of their resolvents. This gives results on Kato's square root estimates in  $L^p$ , and also shows that the  $H^\infty$  functional calculus of a wide range of differential operators is stable under small perturbations.

**Sanmartino, Marcela** (Universidad Nacional de La Plata, Argentina):

*Weighted a priori estimates for Poisson equation.*

**Abstract:** Let  $\Omega$  be a bounded domain in  $\mathbb{R}^n$  with  $\partial\Omega \in C^2$  and let  $u$  be a solution of the classical Poisson problem in  $\Omega$ ; i.e.,

$$\begin{cases} -\Delta u = f & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \end{cases}$$

where  $f \in L^p_\omega(\Omega)$  and  $\omega$  is a weight in  $A_p$ . The main goal of this paper is to prove the following a priori estimate

$$\|u\|_{W^{2,p}_\omega(\Omega)} \leq C \|f\|_{L^p_\omega(\Omega)},$$

and to give some applications for weights given by powers of the distance to the boundary.

**Sawano, Yoshihiro** (Gakushuin University, Japan):

*Fractional integral operators for general measures  $\mu$ .*

**Abstract:** In this talk we place ourselves in the setting of  $\mathbb{R}^d$  coming with a Radon measure  $\mu$ . We construct a potential operator adapted to  $\mu$  and shall establish some boundedness.

**Schul, Raanan** (University of California, Los Angeles):

*Bi-Lipschitz decomposition of Lipschitz functions into a metric space.*

**Abstract:** We will outline the proof of a quantitative version of the following statement. Given a Lipschitz function  $f$  from the  $k$ -dimensional unit cube into a general metric space, one can decompose  $f$  into a finite number of Bi-Lipschitz functions  $f|_{F_i}$  so that the  $k$ -Hausdorff content of  $f([0,1]^k \setminus \cup F_i)$  is small. The case where the metric space is  $\mathbb{R}^d$  is a theorem of P. Jones (1988).

**Sidi Lafdal, Hamad** (University Hassan II, Morocco):

*An  $L^p-L^q$ -Version of Morgan's Theorem for the Jacobi-Dunkl transform.*

**Abstract:** In this talk we show recent progress on the famous hypothesis of G.Morgan, we give an  $L^p-L^q$ -version of Morgan's theorem for the Jacobi-Dunkl transform  $\mathcal{F}_{JD}$  on  $\mathbb{R}$ .

More precisely, we prove that given  $1 \leq p, q \leq +\infty$ , such that  $\frac{1}{p} + \frac{1}{q} = 1$ ,  $\alpha > 2$ ,  $\beta = \frac{\alpha}{\alpha-1}$ , and  $a, b > 0$ , then for all measurable functions on  $\mathbb{R}$ , the conditions  $e^{a|x|^\alpha} f \in L^p_{\alpha,\beta}(\mathbb{R})$ ,  $e^{b|y|^\beta} \mathcal{F}_{JD} f \in L^q_{\alpha,\beta}(\mathbb{R})$  and  $(a\alpha)^{1/\alpha} (b\beta)^{1/\beta} > (\sin(\frac{\pi}{2}(\beta-1)))^{1/\beta}$  imply  $f = 0$ , where  $L^p_{\alpha,\beta}(\mathbb{R})$  is the Lebesgue space associated with the Jacobi-Dunkl transform.

**Sifi, Mohamed** (University of Tunis, Tunisia):

*Linear and bilinear multiplier theorems for the Dunkl transform.*

**Abstract:** In this talk we prove the Hörmander multiplier theorem for the Dunkl transform in full generality by using the Hörmander's technique. In particular, we obtain the boundedness of the generalized Hilbert transform on  $L^p(\mu_\alpha)$  spaces for  $1 < p < \infty$ . Next, we develop a Littlewood-Paley theory for the Dunkl transform and we establish an  $L^p$ -theorem for the bilinear multiplier operator.

**Sjögren, Peter** (University of Gothenburg, Sweden):

*Endpoint results for the Laguerre function maximal heat operator.*

**Abstract:** The classical Laguerre functions  $\mathcal{L}_k^\alpha$  form for each  $\alpha > -1$  an orthonormal system on the half-line. There is a corresponding Laplacian and a heat semigroup, and this has an obvious extension to several dimensions. For  $-1 < \alpha < 0$ , the heat semigroup behaves badly in the sense that it consists of operators which are not bounded on all  $L^p$  spaces. The corresponding maximal operator is therefore also bounded only on certain  $L^p$  spaces. We examine the endpoint cases here. The sharp results turn out to be rather intricate and depend strongly on the dimension. The one-dimensional case was treated earlier by Macías, Segovia and Torrea. This is mainly joint work with A. Nowak.

**Suazo, Erwin** (Arizona State University, USA):

*Evolution operator for a particular one-dimensional Schrödinger equation with a time dependent Hamiltonian.*

**Abstract:** The fundamental solution of the time-dependent Schrödinger equation with the Hamiltonian of the form below will be presented

$$i \frac{\partial \psi}{\partial t} = -a(t) \frac{\partial^2 \psi}{\partial x^2} + b(t) x^2 \psi - i \left( c(t) x \frac{\partial \psi}{\partial x} + d(t) \psi \right) - f(t) x \psi + ig(t) \frac{\partial \psi}{\partial x}$$

where  $a(t)$ ,  $b(t)$ ,  $c(t)$ ,  $d(t)$ ,  $f(t)$ , and  $g(t)$  are given real-valued functions of time  $t$  only. The corresponding Green function (propagator) is given in terms of elementary functions and certain integrals of the fields with a characteristic function. This characteristic function is the solution of a second order differential equation with function coefficients that has been obtained after certain substitutions from a Riccati differential equation. This Riccati differential equation was obtained in the process of splitting the Schrödinger equation in ordinary differential equations. We discuss a particular solution of a related nonlinear Schrödinger equation and some special and limiting cases are outlined. This is joint work with Sergei Suslov.

**Trujillo González, Rodrigo** (Universidad de La Laguna, Spain):

*A new theory of weights for multilinear singular integral operators.*

**Abstract:** We present a complete weighted theory for multilinear maximal operators, singular integral operators and their commutators. This study allows us to build a new class of weights, intrinsically adapted to the multilinear Calderón-Zygmund theory. This is a joint work with A. Lerner, S. Ombrosi, C. Pérez and R. Torres.

**Ujlayan, Amit** (Indian Institute of Technology Kanpur, India):

*On numerical solution of partial integro-differential equation.*

**Abstract:** Consider a partial integro-differential equation of the following type:

$$u_t + auu_x = \int_0^t k(t-\tau) \frac{\partial}{\partial x} (g(u_x(x,\tau))) d\tau + f(x,t),$$
$$u(x,0) = \psi(x), \quad k \geq 0 \quad 0 \leq x \leq 1, \quad 0 \leq t \leq T.$$

We will study the Adomian Decomposition method to solve the above partial integro-differential equation. At the end of the study we will demonstrate the fruitfulness of the adopted method.

**Urbina, Wilfredo** (Universidad Central de Venezuela & DePaul University):  
*Markov semigroups associated to families of generalized orthogonal polynomials.*

**Abstract:** In this work we define operator semigroups associated to a family of generalized orthogonal polynomials with hermitian matrix argument. In order to do this we consider a Markov generator sequence, and therefore a Markov semigroup, for the family of orthogonal polynomials on  $\mathbb{R}$  related to the generalized polynomials. Under the hypothesis of diffusion we prove that this semigroup is also Markov. We also give expressions for the kernel of this semigroup in terms of the one-dimensional kernels and obtain some classical formulas for the generalized orthogonal polynomials from the correspondent formulas for orthogonal polynomials on  $\mathbb{R}$ . This is joint work with Cristina Balderrama and Piotr Graczyk.

**Uriarte-Tuero, Ignacio** (University of Missouri-Columbia, USA and Fields Institute, Canada):  
*Astala's conjecture on distortion of Hausdorff measures under quasiconformal maps and related removability problems.*

**Abstract:** In his celebrated paper on area distortion for quasiconformal maps in the plane, Astala proved sharp Hausdorff dimension distortion estimates for quasiconformal mappings. He asked (Question 4.4) whether such estimates held at the finer level of Hausdorff measures. In joint work with M. Lacey and E. Sawyer, we answer this question in the affirmative.

An affirmative answer to Astala's conjecture was known to be sharp in terms of Hausdorff gauge functions via some examples in previous independent work of the speaker. Those examples also have applications to related quasiregular removability problems for BMO (same work), and Hoelder maps (joint work with A. Clop.) In another previous joint work with K. Astala, A. Clop, J. Mateu, and J. Orobitg, Astala's conjecture was proven only when the target dimension is 1, and related applications to removability for bounded quasiregular maps were given.

I will briefly survey these results and then focus on Astala's conjecture.

**Viola, Pablo** (Instituto de Matemática Aplicada Litoral, Argentina):  
*T1 Theorem, in spaces of homogeneous type, for functions valued in a Hilbert space.*

**Abstract:** Our work can be divided into two parts.

First, we give a proof of a  $T1$  Theorem (in the case  $T1 = 0$ ) for operators with kernels taking values in a general Hilbert space, in the setting of spaces of homogeneous type. The Cotlar's Lemma approach is used and the Hille's Theorem for Bochner integrals is fully employed. Also, we give an application of our result to the Littlewood-Paley theory.

Secondly, we study singular integral operators with Hilbert-valued kernels in the setting of  $\mathbb{R}^n$  with non-doubling measures. We obtain an explicit formula for these operators following the same approach as in [MST]. By using this formula and a result due to Krein we get a  $T1$ -theorem (with  $T1 = 0$ ) in this context. Finally, we develop a theory for antisymmetric kernels and we apply the results to the variation operator associated with the Laplace semigroup.

[MST] Roberto A. Macías, Carlos Segovia, and José L. and Torrea. Singular integral operators with non-necessarily bounded kernels on spaces of homogeneous type. *Adv. Math.*, 93(1):25–60, 1992.

**Wall, Treven** (University of Edinburgh, UK):

*The sharp  $A_p$  constant for weights in a reverse-Hölder class.*

**Abstract:** Coifman and Fefferman established that the class of Muckenhoupt weights is equivalent to the class of weights satisfying the “reverse Hölder inequality”. In a recent paper V. Vasyunin presented a proof of the reverse Hölder inequality with *sharp* constants for the weights satisfying the usual Muckenhoupt condition. In this talk, based on joint work with Martin Dindoš, we present the inverse, that is, we use the Bellman function technique to find the sharp  $A_p$  constants for weights in a reverse-Hölder class on an interval; we also find the sharp constants for the higher-integrability result of Gehring.

Additionally, we find sharp bounds for the  $A_p$  constants of reverse-Hölder-class weights defined on rectangles in  $\mathbb{R}^n$ , as well as bounds on the  $A_p$  constants for reverse-Hölder weights defined on cubes in  $\mathbb{R}^n$ , without claiming the sharpness.

The talk will emphasize the Bellman function technique as well as the connections the results have with solving non-divergence-form elliptic PDEs.

**Whitehouse, J. Tyler** (University of Minnesota, USA):

*$d$ -dimensional Menger-type Curvatures.*

**Abstract:** We define a discrete Menger-type curvature of  $d + 2$  points in a real separable Hilbert space  $H$  by an appropriate scaling of the squared volume of the corresponding  $(d + 1)$ -simplex, and we form a continuous curvature of an Ahlfors regular measure  $\mu$  on  $H$  by integrating the discrete curvature according to products of  $\mu$  (or its restriction to balls). The essence of this work is to estimate multiscale least squares approximations of  $\mu$  by the Menger-type curvature. More formally, we show that the continuous  $d$ -dimensional Menger-type curvature of  $\mu$  is comparable to the “Jones-type flatness” of  $\mu$ . The latter quantity adds up scaled errors of approximations of  $\mu$  by  $d$ -planes at different scales and locations, and is commonly used to characterize uniform rectifiability. We thus obtain a characterization of uniform rectifiability by using the Menger-type curvature. Our techniques combine discrete and integral multiscale inequalities for the polar sine with the “geometric multipoles” construction, which is a multiway analog of the well-known method of fast multipoles.