

ON A SQUARE FUNCTION ESTIMATE RELATED TO THE KATO  
PROBLEM AND SOLVABILITY OF ELLIPTIC PDE'S

Pascal Auscher

Université Paris-Sud, France

`pascal.auscher@math.u-psud.fr`

**Abstract**

We consider the following theorem: If  $B$  is a bounded and strictly accretive multiplication operator on  $L^2(\mathbb{R}^n, \mathbb{C}^N)$  and  $D$  is a self-adjoint first order differential operator with constant coefficients on  $L^2(\mathbb{R}^n, \mathbb{C}^N)$  such that  $\|Df\|_2 \sim \|f\|_{\dot{W}^{1,2}}$  for  $f$  in the range and the domain of  $D$ , then  $DB$  has quadratic estimates.

If  $D$  is one-one then this goes back to Coifman-McIntosh-Meyer's work and can be proved by standard  $T(b)$  arguments. If  $D$  is not one-one, this is more difficult and was first proved by Axelsson, Keith and McIntosh as a consequence of their results on perturbed Dirac operators and elaboration of the local  $T(b)$  for square functions as in proof of the Kato square root problem. It turns out that this estimate implies the Kato square root problem. It also implies solvability of boundary value problems as explained in Alan McIntosh's lecture. Here we sketch a direct proof and give some further consequences. This is joint with Andreas Axelsson and Alan McIntosh.