

THE MATHEMATICAL FOUNDATIONS OF COMPRESSED SENSING

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Abstract

Compressed Sensing is a new area of signal and image processing which has as its goal to minimize the number of measurements (samples) that we need to take of a function/signal/image for accurate reconstruction. For example, it replaces the Shannon model of bandlimited signals by a model of signals as having a sparse representation in some dictionary of wave forms. The ideas for compressed sensing have their origin in certain constructions in approximation and finite dimensional geometry which showed the optimality of random sampling. This course will develop compressed sensing from its mathematical origins to its current implementation in signal and image processing.

The course will focus on the discrete sensing problem where we are given a vector in $x \in \mathbb{R}^N$ with N large and we wish to capture it through a small number n of measurements given by inner products with fixed vectors. Such a measurement system can be represented by an $n \times N$ matrix Φ . The vector $y = \Phi x$ is the vector of n measurements we make of x . The information that y holds about x is extracted through a decoder Δ . So $\Delta(\Phi x)$ should be designed to be a faithful approximation to x .

The lectures will introduce different ways to evaluate a sensing system (Φ, Δ) and then describe which such systems are best. The best systems will depend on the criteria to measure performance and in particular if we ask for accuracy with certainty or only with high probability.