

VECTOR-VALUED AND NONCOMMUTATIVE ASPECTS OF LITTLEWOOD-PALEY THEORY

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Abstract

We discuss in this talk two aspects of the classical Littlewood-Paley theory. Let f be a function on \mathbb{R} . The Littlewood-Paley g -function of f is defined by

$$G(f)(x) = \left(\int_0^\infty \left| \frac{\partial}{\partial t} P_t * f(x) \right|^2 \frac{dt}{t} \right)^{1/2}, \quad x \in \mathbb{R},$$

where P_t denotes the Poisson semigroup on \mathbb{R} . The classical Littlewood-Paley inequalities read as

$$c_p \|G(f)\|_p \leq \|f\|_p \leq C_p \|G(f)\|_p, \quad 1 < p < \infty.$$

Here P_t can be replaced by the Poisson semigroup subordinated to any symmetric Markovian semigroup on a measure space. This is the so-called Littlewood-Paley-Stein theory.

The first aspect discussed in this talk concerns the vector-valued case of the preceding inequalities. Now f takes values in a Banach space X and $G(f)$ is defined as above just with the norm of X in place of the absolute value. Then it is not hard to show that the previous two-sided inequality holds iff X is isomorphic to a Hilbert space. However, the validity of only one of the two inequalities is a much subtler matter and is equivalent to the existence of an equivalent 2-uniformly convex or smooth norm of X . We also discuss the extreme case $p = \infty$ where *BMO* and Carleson measures are involved.

The second aspect deals with the noncommutative setting, where the usual L_p -spaces are replaced by noncommutative L_p -spaces and P_t is a quantum symmetric Markovian semigroup of maps preserving a given trace (or state). The resulting theory is closely related to noncommutative martingale/ergodic inequalities recently developed in noncommutative probability. It is also related to the noncommutative aspect of McIntosh's H_∞ -functional calculus.