We analyze the integrability of $\widetilde{A}^{t,x}$. This plays an important role in Lemma 4.2.

Proposition 2.5. *For any $\theta > 0$, $t > 0$ and $x \in \overline{\Omega}$,*

$$E\,[\exp(\theta \widetilde{A}_t^{t,x})] < \infty\,. \tag{2.25}$$

Moreover, for any k,

$$\lim_{u\to 0} \sup_{\substack{0\le t\le k\\ x\in\overline{\Omega}}} E\,[\widetilde{A}_u^{t,x}] = 0\,. \tag{2.26}$$

Remarks. 1) In dimension 1, for the reflected Brownian motion, recall that the local time at 0, L_t^0, has exponential moments, since $L_t^0 \overset{(\mathrm{d})}{=} \sqrt{t}\,|N|$, where N is a centered, unit variance, Gaussian random variable.

2) A similar estimation can be found in [S.V.].

Proof of Proposition 2.5. 1) Let $\lambda > 0$ be fixed.

We choose $\gamma : \overline{\Omega} \longrightarrow \mathbb{R}$, a function of class C^2 such that,

$$\gamma(x) \ge 1\,, \qquad \text{for all } x \in \overline{\Omega}\,. \tag{2.27}$$

$$\begin{aligned} &\text{i)}\ \frac{\partial\gamma}{\partial n}(x) = 2\,\lambda\,,\\ &\text{ii)}\ \gamma(x) = 2\,,\ \text{ for any } x \in \partial\Omega\,. \end{aligned} \tag{2.28}$$

A straightforward calculation based on the Itô formula and (2.28) shows that $(U_s\,;\,0 \le s \le t)$ is a bounded martingale, where $\widetilde{X}_s = \widetilde{X}_s^{t,x}$, $\widetilde{A}_s = \widetilde{A}_s^{t,x}$ and

$$U_s = \gamma(\widetilde{X}_s)\exp(\lambda\widetilde{A}_s) - \int_0^s H(r)\exp(\lambda\widetilde{A}_r)\,dr\,, \tag{2.29}$$

$$H(r) = \frac{1}{2}\,\Delta\gamma(\widetilde{X}_r) - u(t-r,\widetilde{X}_r)\,\nabla\gamma(\widetilde{X}_r)\,.$$

γ being of class C^2, $H(r)$ is a bounded process, then there exists a positive constant k such that $|H(r)| \le k$ for any r in $[0,t]$. (2.27) implies that

$$H(r) \le |H(r)| \le k\,\gamma(\widetilde{X}_r)\,, \qquad \text{for all } r \in [0,t]\,.$$