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Abstract

The theory of direct perception holds that perception is specific to properties of ambient energy arrays. The present article explores a similar approach to perceptual and perceptual-motor learning: Change due to learning is portrayed as specific to properties of ambient energy arrays. This type of learning is labeled direct learning. It is argued that a theory of direct learning explains a wide range of phenomena in ecologically relevant and informationally rich situations as well as in simpler experimental situations. Previous experimental results are reanalyzed and reinterpreted from the perspective of direct learning. To achieve this, the notion of information space is introduced. In information spaces, points represent ambient energy patterns, paths represent change due to learning, and vector fields represent information that guides learning. These concepts also allow the theory of direct learning to be connected to and to take advantage of the theory of ordinary differential equations.

Keywords: Calibration, Direct Learning, Direct Perception, Education of Attention, Ecological Psychology, Information for Learning, Perceptual Learning, Perceptual-Motor Learning, Specification
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The ecological approach to learning is based on the understanding that a wealth of information resides in ambient energy arrays, especially in natural task ecologies. As a consequence, the aim of ecological learning theories is to explain how perceivers come to take advantage of the informational richness of ambient energy arrays (e.g., Gibson, 1969; Gibson & Pick, 2000). The ecological approach to learning originated with the rejection of enrichment theories of learning (Gibson & Gibson, 1955). Enrichment theories are based on the belief that stimulus variables are necessarily ambiguous with respect to the environment; perceivers are said to resolve the ambiguity, or to enrich information-poor stimuli, by processes such as inference or bringing to bear memories. Similarly, enrichment theories portray the emergence of expertise as an increase in the sophistication of the enrichment processes.

Theories resulting from the above-described approaches can often intuitively be distinguished as adding-to theories versus mere-change theories (Michaels & Carello, 1981). Enrichment theories might hold, for instance, that expert perceivers outperform novices because of the knowledge they have added to their memories during the process of learning, allowing them to make more accurate inferences. Ecological theories, in contrast, hold that learning entails mere changes, for example, changes in which properties of ambient energy arrays perceptual systems respond to. Thus, in the ecological view, the sophistication of expert performance derives from the improved fits of experts to their environments, rather than from an increased complexity of
computational and memorial processes. The learning theory that we explore in this article would indeed come under the heading of mere-change theories.

We begin the article with a brief review of the ecological approach to perception, often referred to as the direct-perception approach (e.g., Michaels & Carello, 1981). The message of the direct-perception approach that most influences the subsequent section on perceptual and perceptual-motor learning is that perception is said to be a single-valued function, or one-to-one mapping, of ambient energy patterns, sometimes referred to as informational variables. The introductory discussion of learning considers what adaptive changes are possible given the portrayal of perception as a single-valued function. These changes are described as the education of intention, the education of attention, and calibration. The subsequent and most extensive part of this article further addresses these processes and introduces the concept of direct learning.

Given that scientists should aim to reveal regularities with wide validity, theories should indicate what type of regularities can be expected. Ecological psychology has traditionally provided the expectation that particular perceptions or actions are informed by particular properties of ambient energy arrays. However, individual differences and changes within individuals in the use of informational variables, which are a main concern of the present article, weaken this traditional strength of the ecological approach. The final sections of this article aim to provide the theory with new regularities—namely, ambient energy patterns that inform learning—which, we hope, have a wider validity.
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A first question that any approach to perception might want to answer is what properties of their environments do animals perceive. The general expectancy of the ecological approach is that these properties include nonelementaristic ones (Gibson, 1979; Runeson, 1977). This view is most clearly illustrated by contrast with the opposite view, elementarism.

*The breakaway from elementarism*

Environments of animals can be described in many ways. With respect to a particular descriptive system, certain properties can be said to be simple and others complex. For instance, with respect to standard physical systems, the length of an object can be said to be a simple property because it can be described with a single standard physical unit, the meter. Descriptions of other properties, say the throwableness of an object, would minimally require a few such units. Thus, if one uses standard physics as reference, properties such as throwableness might be called complex.

In the history of psychology, perceptual theories have often been based on concepts from standard physics or on other systems that are to a large extent arbitrary with regard to perception. On the basis of such choices, one might conjecture that properties such as length are easier to perceive than properties such as throwableness. Or, more extremely, one might even claim that perception is limited to apparently simple properties, and, thus, that cognitive processing is required for the apprehension of apparently complex properties. For instance, perceivers might be said to be able to perceive the size, weight, and shape of an object, but not its throwableness, which would
have to be inferred from the perceived properties. Such theories—which take elementary variables of the descriptive system also to be elementary variables of the system to be described—are referred to as **elementaristic**.

The ecological approach explicitly objects to elementarism (e.g., Runeson, 1977; 1994, 1995; Stoffregen & Bardy, 2001). Rather than assume which variables are elementary for perceptual systems, ecological psychologists take as one of their major aims the discovery of variables that are elementary for such systems. Prominent among attempts to discover candidate descriptive variables is the concept of **affordance** (Gibson 1977, 1979; Michaels & Carello, 1981; Turvey, 1992). An affordance for a particular animal is a property of the environment that permits that animal to engage in some action. An affordance might be difficult for scientists to describe, but that does not mean that it is not an elementary property for a perceiver-actor. One might even expect affordances to be primary properties because they are the properties that are relevant to behavior, and, thus, the ones that exerted evolutionary pressure on the design of perceptual systems.¹

Also relevant for what follows, the breakaway from elementarism implies that learning to perceive properties that appear simple to the scientist might, in fact, be more difficult than learning to perceive properties that do not appear simple. In the next section, we addresses how the perception of such possibly nonelementaristic properties should be portrayed.

**The methodological doctrine of information-perception specificity**

As mentioned above, one of the main themes of ecological psychology is the rejection of the traditional thesis that stimulus variables are necessarily ambiguous with
respect to the environment, and therefore must be disambiguated, or enriched, by
cognitive processes such as inference (e.g., Fodor & Pylyshyn, 1981). If, however,
detected properties of ambient energy arrays specify environmental properties, nothing
needs to be disambiguated. Such informational richness would thereby imply that
perception could be portrayed as a single-valued function, or one-to-one mapping, of
information-rich ambient energy patterns (e.g., Turvey, Shaw, Reed, & Mace, 1981).
Indeed, a fundamental assumption underlying the ecological approach is that perception
is a single-valued function of a single informational variable. In other words, perception
is portrayed as direct (e.g., Michaels & Carello, 1981).

The assumed specificity of perception to a single informational variable should
not be seen as an empirically verifiable statement, but as a methodological doctrine, or a
strategic commitment of scientists (Turvey, 2001). Let us illustrate this with an example.
We first remind the reader that the optical variable tau is defined as the ratio of optical
size to the rate of optical expansion and that, under certain conditions, tau specifies the
time to contact of an approaching object (e.g., Lee & Reddish, 1981). Consider, then, the
claim that the perception of the time to contact is a function of a single variable, tau, and
the related but different claim that this perception is a function of two variables, namely,
optical size and rate of optical expansion. Following Ullman (1980), one might argue that
describing perception as a function of a single variable would be appropriate only if a
single mechanism were to detect tau, and this mechanism were not decomposable in any
meaningful way, whereas the apparently less parsimonious portrayal using two variables
would be appropriate if two distinct mechanisms were to exist for the detection of optical size and the rate of expansion, and a third mechanism for the division of these variables.

It is our belief, however, that even if evidence for multiple detection and combination mechanisms were found, this could be interpreted as indicating how variables such as \( \tau \) are detected. That is, the function with a single variable would remain a valid description of how a particular optical variable is related to perception, ignoring detail at the level of detection mechanisms. Thus, which portrayal is more appropriate depends on the level at which one chooses to analyze. One should choose a level of analysis that is appropriate to reveal the phenomena under study. Although fine-grained analyses of detection mechanisms as embodied in biological tissue can be expected to reveal other phenomena, the ecological approach expects that an understanding of perceiving, action, and learning might benefit equally or even more from analyses at coarser grains. In the present article we therefore choose to portray perception as a single-valued function of a single ambient energy pattern.

*Specificity granted by local constraints: a prelude to perceptual learning*

In natural situations, perception and action are often successful, which implies that any approach to perception should be able to explain veridical perception. If, indeed, perception is at least sometimes veridical, and perception is a single-valued function of properties of ambient energy arrays, then at least some environmental properties must be specified by such ambient energy properties. Ecological studies therefore often aim to identify informational variables that are specific to relevant environmental properties.
A first requirement for the identification of useful information-environment specificities is the widely accepted refutation of elementarism at the level of ambient energy arrays (Gibson, 1966, 1979). Given a nonelementaristic ontology, apparently complex informational variables, which might be defined over considerable spatiotemporal intervals and over different sensory arrays, are on equal footing with apparently simpler variables (Stoffregen & Bardy, 2001). Consequently, perception could equally well be based on apparently complex variables as on apparently simpler variables. Typically, however, identifying informational variables that are specific to relevant environmental properties is feasible only if one considers apparently complex informational variables.

An equally crucial requirement for the identification of information-environment specificities is to restrict analyses to the natural ecologies of particular animals (Gibson, 1979). In addition to universal constraints such as those captured by natural laws, ecological psychologists therefore consider ecological constraints, which might hold only in the ecologies of particular animals (Runeson, 1988; Runeson, Jacobs, Andersson, & Kreegipuu, 2001). Gibson demonstrated, for instance, that the constraint of a regular distribution of objects in the environment grants to texture gradients the status of information about the slant of surfaces. Likewise, global optical flow provides information about the direction of self-movement, given the constraint that the terrestrial environment does not spontaneously deform.

Yet additional constraints, including game rules and local conventions, prevail in environments narrowed even further, such as experimental settings (cf. Runeson, 1989).
We use the term *local constraints* to refer to constraints that hold throughout a specific task situation but not throughout the ecology of the animal (Jacobs, Runeson, & Michaels, 2001; Jacobs, Runeson, & Andersson, 2001). It seems improbable, however, that evolution has endowed perceivers with perceptual systems that are attuned to properties of ambient energy arrays whose specificities are granted by constraints on restricted, contrived, and often novel task situations. One of the questions addressed below, therefore, is how perceivers *learn* to take advantage of information-environment specificities furnished by local constraints.

**Change in perception and perceptually guided action**

The previous section reviewed the direct-perception approach. The present section considers change over time in perception and perceptually guided action. The assumption that perception is a single-valued function of a single informational variable implies two types of change. First, change might occur in which informational variable is detected. In the present view, such change is said to be due to *the education of intention* or to *the education of attention*. Second, the particular single-valued function that carries the operative informational variable into perception might change. Such change may be due to *calibration* and, again, also to the education of intention. We now describe these processes in more detail.

*Education of intention*

Many perceptions and actions are possible in any situation. In the case of an approaching object, for instance, a perceiver or actor might perceive whether the object is useful or harmful, whether he or she could catch, hit, or avoid the object, or perhaps also
the speed or size of the object. Similarly, an animal might intend to escape from, or to prey on a particular other animal. Certain perceptions and actions are more beneficial than others and, with experience, humans and other animals might improve in choosing which of the possible perceptions and actions they intend to actualize. We have previously referred to this process as the education of intention (Jacobs & Michaels, 2002; see, for instance, Shaw & Kinsella-Shaw, 1988, for a more detailed understanding of intentions and their role in ecological psychology).

Different intentions are presumed to organize perceptual or perceptual-motor systems differently. For instance, if the intention—perhaps motivated by an experimenter—is to perceive the time remaining until a ball reaches one’s eye plane, an expert perceptual system might be set up so that it detects an optical variable such as $\tau$. If, on the other hand, the intention is to perceive the lateral distance at which a ball will cross the eye plane, the system might be set up so that it detects the ratio of optical velocity to expansion (Bootsma & Peper, 1992; Michaels, Jacobs, & Bongers, 2006). Thus, the education of intention might have an important influence on which particular variable perception is a single-valued function of.

Intentions can be expected to depend on many factors—needs, wishes, hopes, beliefs, and also external influences such as instructions—which makes the education of intention difficult to study. Ecological studies therefore often proceed by choosing situations in which a particular intention can be assumed. In catching experiments, for instance, it is generally assumed that perceivers intend to catch balls, and not to hit or to avoid balls. Such assumptions are necessary because intentions define task situations. For
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instance—and this is important for the remainder of the article—assuming a particular intention is required to define and identify specifying and nonspecifying variables.

Consider, for example, the ambient energy arrays in natural environments. Each property of such arrays probably specifies some property of the environment (Michaels, Withagen, Jacobs, Zaal, & Bongers, 2001). The specified property, however, might not be the one that a perceiver intends to perceive. In the following, the term specifying variable is therefore used only for a variable that specifies the property that a perceiver intends to perceive, and the term nonspecifying variable for a variable that does not specify the property that a perceiver intends to perceive, regardless of whether a so-called nonspecifying variable might specify other properties.

**Education of attention**

In the preceding, it was argued that one of the functions of intentions is to set up perceptual or perceptual-motor systems to detect a particular informational variable. Over time, however, the particular variable whose detection is entailed by a particular intention might change; that is, the same intention might come to entail the detection of another variable. The process of coming to attend to the more useful variables is referred to as the education of attention (Gibson, 1966, 1979; cf. Michaels & de Vries, 1998). Thus, even if the intention does not change, one expects that, with experience, perceivers come to attend to the more useful variables. Attention is optimally educated if perceivers detect a variable that specifies the property that they intend to perceive.

Evidence for the education of attention has been obtained with learning studies that present perceivers and actors with tasks that are to a certain extent novel (e.g., Fajen
Such studies indicate that after practice with feedback, perceivers and actors come to rely on the more useful informational variables. Let us describe one of these studies. In Jacobs et al. we asked whether the education of attention to the more useful informational variables might proceed faster if the nonspecifying variables that are initially used by novice perceivers are rendered not at all useful in a practice phase. To test this, a series of experiments was performed in which participants practiced with feedback to judge the relative mass of simulated colliding balls in a pretest-practice-posttest design. The practice phases given to different groups of perceivers differed with regard to which of the nonspecifying variables were rendered useless as well as to how this was achieved.\(^2\)

One of the ways in which nonspecifying variables were rendered useless was by creating sets of practice displays in which the considered nonspecifying variables (i.e., the exit-speed difference between the balls, their scatter-angle difference, and combinations of these) were uncorrelated with the to-be-perceived property (i.e., relative mass). The effects of these practice conditions strongly depended on whether or not the variable that a perceiver had used before practice was the variable that was rendered useless. Perceivers who initially relied on a variable that was made useless in practice came to rely on other variables. In the end, all of these participants attended to the more useful variables, and they continued to do so in the posttest. On the other hand, performance of perceivers who relied on nonspecifying variables that were not rendered
useless, and could thus yield reasonably accurate performance, was essentially not affected by this practice condition.

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Insert Figure 1
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Figure 1 presents the results of two observers who both appeared to use the exit-speed difference before practice (i.e., in Block 1). In the practice phase (Blocks 2, 3, and 4), the exit-speed difference correlated fairly highly with the to-be-perceived mass ratio for the observer represented in the right panel, but it did not correlate at all with mass ratio for the observer represented in the left panel. As described above, this resulted in the left observer's use of information that specifies mass ratio, while the right observer continued to use the speed difference. This performance continued in a posttest (Block 5), which was identical for both participants. We will return to these results in a later section.

**Calibration**

Thus far, we have considered the education of intention and the education of attention. The present section concerns *calibration*, defined as the process by which the single-valued function itself becomes adjusted. These notions might become more intuitive by analogy with measurement devices, often intended to measure standard physical properties (Runeson, 1994). Unfortunately for the measurer, measurement devices are always to some extent sensitive to so-called context variables. A barometer, for instance, intended to measure air pressure, might typically also react to changes in, say, temperature and humidity. Abandoning elementarism, one could say that the output
of such devices is a function not of the property to be measured, but of a higher-order variable that is also related to some context variables.

Bad instruments, then, might perhaps be modified so that they come to respond more to the to-be-measured variable and less to other variables. This process would be analogous to the education of attention, that is, to changes in which informational variable perception is a single-valued function of. The observation to be made here, however, is that different measurement systems might generate different outputs even if they respond to the same input variable. Likewise, a measurement system might be changed in what response it generates given a particular input variable. Such change is referred to as *calibration*.

Assume, for instance, that a perceiver intends to perceive the length of an unseen, hand-held rod, and also assume that perception is a single-valued function of the rod’s first principal moment of inertia ($I_1$), as reported by Solomon and Turvey (1988). For a collection of homogeneous rods of a constant diameter, values of $I_1$ are related one-to-one to the lengths of the rods. However, even if perception is a single-valued function of $I_1$, and thereby of the length of the rods, perceivers still differ and change in aspects of the single-valued function itself, such as the slope and intercept (Withagen & Michaels, 2004).

To give an action-related example we briefly consider a model for lateral catching proposed by Peper, Bootsma, Mestre, and Bakker (1994; cf. Bootsm, Fayt, Zaal, & Laurent, 1997), which we modified to study the education of attention and calibration
The modified version of the model relates an optical variable, $O$, to a lateral acceleration of the hand, $A_{\text{hand}}$, using two calibration parameters, $c_1$ and $c_2$:

$$ A_{\text{hand}} = c_1 \left( \frac{c_2 O - X_{\text{hand}}}{(\dot{\varphi}/\varphi - \dot{\rho}/\rho)^{\gamma}1 + V_{\text{hand}}} \right). \quad (1) $$

In the model, $\varphi$ is the angular size of the ball, $\rho$ the angle between the fronto-parallel plane and a line from the eye to the ball, $X_{\text{hand}}$ the lateral hand position, and $V_{\text{hand}}$ the lateral hand velocity. Calibration parameter $c_2$ indicates how the optical variable $O$ is calibrated with respect to the presumably kinesthetic variable that specifies hand position, and parameter $c_1$ indicates how the higher-order variable on the right side of the equation is calibrated with respect to hand acceleration. We computed the best-fitting calibration parameters and observed significant change with practice; catchers indeed change in calibration (Jacobs & Michaels, 2006).

Hence, in addition to the education of intention and the education of attention, calibration is essential to perception and perceptually guided action (cf. Bingham & Pagano, 1998; Bingham, Zaal, Robin, & Shull, 2000). Although our inventory of learning processes is not meant to be exhaustive, our discussion so far is sufficient for the present purpose—to address direct learning.

Loans of intelligence and direct learning

Above we have shown that the direct-perception approach naturally leads one to describe perceptual and perceptual-motor learning as the education of intention, the education of attention, and calibration. Ironically, then, these learning processes might
seem to undermine a cornerstone of the direct-perception approach—they might seem to entail loans of intelligence and/or inferential-like processes. The problem is: Who is in charge of the learning? Does one need to suppose intelligence on the part of a homunculus to explain, for instance, which new variables are detected, or when changes in variable use occur? This problem is the crux of the present article. As a solution we propose that change due to learning is a single-valued function, or one-to-one mapping, of higher-order properties of ambient energy arrays, which is to say, we suggest the possibility that learning is direct, rather than based on inferential or other cognitive processing. Several concepts are required to further develop this proposal. We start with the concept of information space.

Information spaces: A common language for learning

The present section introduces ways to describe and analyze those properties of ambient energy arrays that are relevant to perception and perceptually guided action. We thereby introduce the tools that will be used to formulate concepts and proposals in following sections.

The richness of ambient energy arrays makes it inconceivable to consider the full variation of such arrays; researchers have to limit themselves to the parts of the arrays that are relevant to the process under study. The most concise way to describe relevant parts of ambient energy arrays is by considering a single informational variable. Examples of this research strategy can be found in the early work on the perception of time to contact (e.g., Lee & Reddish, 1981) which, one could argue, addressed mainly whether or not the perception of time to contact was based on the variable $\tau$. Likewise,
early research on dynamic touch addressed mainly whether length perception was or was not based on the largest principal moment of inertia (Solomon & Turvey, 1988). Concentrating on a single candidate variable, however, is inappropriate for our view on learning, because it does not permit one to understand change in variable use, and thus not to address the education of attention.

A second way to describe relevant parts of ambient energy arrays is to consider several candidate variables, instead of one. In later studies on the perception of time to contact, for instance, variables such as rates of change of binocular convergence, optical size, and rate of expansion, were considered, in addition to tau (e.g., van der Kamp, Savelsbergh, & Smeets, 1997), and more recent studies on length perception by dynamic touch considered all three principal moments of inertia (e.g., Fitzpatrick, Carello, & Turvey 1994) and yet other variables (e.g., Kingma, van de Langenberg, & Beek, 2004). Considering several variables is more appropriate for our view on learning because it allows one to consider changes in which variables are used. Consequently, most of the more recent ecologically-motivated learning studies, including the example from the colliding-balls paradigm addressed earlier, considered several candidate variables (e.g., Michaels & de Vries, 1998; Runeson et al., 2000).

Considering several variables, however, still limits possible change in which variable is used to discrete changes, or jumps, from the use of one candidate variable to the use of another. As we will see later, the view presented here entails continuous change in variable use; considering several candidate variables is therefore not sufficient. As an alternative, we suggest a more general concept, the concept of information space,
to describe the relevant properties of ambient energy arrays. An information space is a space each point of which represents a (higher order) property of ambient energy arrays. Let us illustrate what information spaces might look like using a few simple examples and the analogous (but more intuitive) concept of *calibration space*.

Consider Figure 2. As analogies to perceptual systems, the thermometers couple an ambient energy property, temperature, to an output variable, fluid height on the scale. Suppose that this “perception” is governed by a linear single-valued function, say, \( P = c_1 + c_2 \cdot E \), in which \( E \) stands for the ambient energy property, temperature; \( P \) stands for perceived temperature; and \( c_1 \) and \( c_2 \) are calibration parameters. The thermometer in Figure 2a does not change; while it “perceives,” the coupling is always described by the same ambient energy property and the same calibration parameters (note, though, that the temperature can change). An information space for this case could thus consist of a single point, representing the variable temperature, and a calibration space could also consist of a single point, representing the constant values of both \( c_1 \) and \( c_2 \).

Instead of being made of a single tube, the thermometer in Figure 2b is made of two tubes, one that is graduated and can slide along the outside of the other. It is easy to see that given the linear single-valued function described above, change as represented in Figure 2b would go together with change in calibration parameter \( c_1 \), without affecting either the ambient energy variable \( E \) or calibration parameter \( c_2 \). In other words, an
appropriate calibration space for this system would consist of an interval of points, each point describing a height of the outer tube or, equivalently, a value of parameter $c_1$.

Given that neither of the thermometers changes in a way that affects calibration parameter $c_2$, a two-dimensional calibration space would not be useful in this example. An example of a two-dimensional calibration space can be found in the catching studies described in the section on calibration. The two calibration parameters in Equation 1 might form the coordinate axes of such a two-dimensional space. The points in the space would then represent a calibration described by the coordinates of the points with respect to the axes.

For an example of an information space, rather than a calibration space, we reconsider an experimental situation from Experiment 4 of Michaels & de Vries (1998). In that experiment, perceivers were asked to estimate the peak pulling force achieved by a stick figure that bimanually pulled on a handle using a full-body motion. We choose this paradigm because virtually all of the variance in perceivers’ judgments was accounted for by two kinematic variables, maximal velocity and maximal displacement of the puller’s center of mass. Rather than thinking of these as separate variables, as was done in the original article, we portray them as mere points in a one-dimensional space. As coordinate variable we used $\varphi$, ranging from $-\pi/2$ to $\pi/2$. Each value of $\varphi$, or each point in this interval, represents a different property of the ambient energy array, $E_{\varphi}$.

More precisely, we defined the space such that each point $\varphi$ represents the informational variable $E_{\varphi} = \cos(\varphi) \cdot \text{velocity} + \sin(\varphi) \cdot \text{displacement}$. Obviously, this space includes the
variable *velocity* at $\varphi = 0$, the variable *displacement* at $\varphi = \pi/2$, and variables related both to *velocity* and *displacement* at other values of $\varphi$.\(^3\)

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Insert Figure 3
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Thus far we considered empty calibration and information spaces. As a first attribute, information spaces can be endowed with a measure of the usefulness of the variable that each point represents (see Figure 3). The horizontal axis of Figure 3 is the just-described information space for the Michaels and de Vries (1998) experiment. The vertical axis and the solid line represent the usefulness of each variable in the space, as measured by its correlation with the property to be judged, relative pulling force. So, for example, if a perceiver were to exploit a variable described by a point with coordinate $\varphi = \pi/2$, performance would appear to be random; displacement alone was not correlated with force. If the perceiver were to use the variable at the point $\varphi = 0$ (i.e., velocity), performance would be good, but a variable leading to the best performance would be achieved in the region around $\varphi = -.5$, using a variable with a velocity coefficient of about .9 and a displacement coefficient of -.5.

In our next example we construct a two-dimensional information space. We use the colliding-balls experiment already described in the section on the education of attention. Candidate variables that have been shown to explain a large proportion of the variance in judgments are scatter-angle difference, exit-speed difference, and a mass-specifying invariant. As a first step we construct a space using these variables as
coordinate axes (see the upper part of Figure 4). A point in this space, \((a_0, b_0, c_0)\), represents the informational variable

\[ E = a_0 \cdot \text{scatter-angle} + b_0 \cdot \text{exit-speed} + c_0 \cdot \text{invariant}. \]

An undesirable property of spaces such as the one in the upper part of Figure 4 is that all points that lie on the same line through the origin represent variables that are scalar multiples of each other, which means that they are equivalent with regard to their predictive values as computed with, for instance, correlation and regression analyses. Rather than being interested in the collection of points in the space, we are therefore interested in the collection of lines through the origin. This collection can essentially be represented by a two-dimensional space: Considering a plane that does not include the origin, any point of the plane can be said to represent the line that intersects the plane at that point.

As shown in Figure 4, we chose to consider a plane that passes through the points \((1, 0, 0), (0, 1, 0), (0, 0, 1)\); that is, through the points representing scatter-angle difference, exit-speed difference, and a mass-specifying invariant. We projected this slanted plane on a horizontal one, obtaining a two-dimensional information space. In the lower part of Figure 4, the coordinates of this two-dimensional space are referred to as \(a\) and \(b\). Given how the space is constructed, it is easy to see that a point in the space, \((a_0, b_0)\), represents the informational variable

\[ E = a_0 \cdot \text{angle} + b_0 \cdot \text{speed} + (1 - a_0 - b_0) \cdot \text{invariant}. \]

In this space, the variables scatter-angle difference, exit-speed difference, and the mass-specifying invariant are represented by the points \((1, 0), (0, 1),\) and \((0, 0)\), respectively.
All of the points in the space, except the origin, are variables that do not specify relative mass. This means that the correlations between relative mass and these variables depend on the particular set of collisions that one considers (see Footnote 2). Remember that two sets of collisions were used in Jacobs et al. (2001); one in which the exit-speed difference did not correlate with relative mass and one in which scatter-angle difference did not correlate with relative mass. Figures 5 shows the just-described information space as the horizontal planes, and the usefulness of the variables represented by points in the space by the vertical axes and the curved surfaces, one surface for each set of collisions.

Obviously, the figures reflect the constraints on the sets of collisions. The correlation with the specifying invariant (i.e., the origin) is 1.0 in both sets of collisions. The correlation for the point (1, 0), or scatter-angle difference, is 0.0 in the angle-correlation zero condition and .75 in the speed-correlation zero condition. Conversely, the correlation for the point (0, 1), or exit-speed difference, is .75 in the angle-correlation zero condition and 0.0 in the speed-correlation zero condition.

We want to emphasize that we accord the same ontological status to variables described as compounds of other variables, such as $a_0 \cdot \text{angle} + b_0 \cdot \text{speed} + (1 - a_0 - b_0) \cdot \text{invariant}$, and to variables not described as compounds, such scatter-angle difference, exit-speed difference, or the invariant alone. Hence, we do not suggest that perception entails detecting coordinate variables and combining these, say, according to the weights $a_0$ and $b_0$. The argument against such weighted cue combining is the traditional
ecological one: The existence of smart perceptual devices that directly detect apparently complex combinations is as feasible as the existence of devices that detect apparently simple coordinate variables (Runeson, 1977). Attributing a privileged ontological status to coordinate variables would be falling back into elementarism. So far, the only requisite for the coordinate variables is that the resulting space is sufficiently rich in information. Otherwise the coordinates can, at this point, be chosen arbitrarily and should thus not feature in perceptual theories.

Let us briefly mention a more general method to construct information spaces. Imagine a task commonly analyzed with $n$ non-collinear variables: (1) Construct an $n$-dimensional space using the $n$ variables as coordinate axes. A 3-dimensional example of such an $n$-dimensional space is illustrated in the upper part of Figure 4. (2) Restrict the $n$-dimensional space to an $n-1$ dimensional space consisting of the equivalence classes of proportionality. In Figure 4 we did this by using an $n-1$ dimensional hyperplane that did not include the origin. To obtain the information space illustrated in Figure 3, on the other hand, we used half of the $n-1$ dimensional hypersphere (in this case, half a circle). As in the case of using a plane, each line through the origin was represented by the intersection of the line and the half of the circle, which led to the trigonometric definition of the space. (3) Represent the data of the to-be-considered experiment in the $n-1$ dimensional space. We will later illustrate this step with several examples (look ahead to Figures 9-11).

To summarize, the goal of this section is to set on the agenda the search for information spaces. Such spaces respect the continuity of ambient energy arrays, which
we think is important to perceptual and perceptual-motor learning. We have also outlined a method to obtain information spaces, but we do not claim that this method is unique. To the contrary, our hope is that more ecologically plausible information spaces will eventually be derived by methods that rely less on the use of lower-order variables. At present, however, the information spaces described above suffice to introduce the direct learning view. We are now in a position to consider the nature of information for learning.

Information for learning

Consider an expert pistol shooter. As long as the shooter does not act, nothing reveals that he or she is an expert, but when the shooting starts, the shooter’s expertise reveals itself in the match between the targets and the shooting. Now imagine the same pistol shooter with prism goggles. One expects the prisms to cause a discrepancy between the targets and the shooting. More generally, non-optimalities of perception-action systems might reveal themselves in the consequences of action and perception embedded in actions, which is to say, in observable properties of the environment-actor system (see Bootsma, 1998, for the term environment-actor system). The present section argues that to achieve a direct-learning approach one has to consider higher-order properties of the environment-actor system that are, most notably, extended over time.

Imagine a distance-perception experiment in which a participant reports a distance of, say, 30 m, followed by feedback indicating that the actual distance was, say, 25 m. Such a single judgment-feedback pair is ambiguous with respect to the origin of possible errors. An error on a single trial might be due to the non-optimality of one or more types
of calibration or to attending to a nonspecifying variable. Unambiguous properties of the environment-actor system (in this case the judgment-feedback relation) will be revealed only over at least several trials. If the error were due to a non-optimal calibration of only the intercept, to give an example, one would expect the same underestimation on all trials, as indicated by a combination of a nonzero constant error and a high correlation between judgments and feedback. A high correlation together with an average underestimation would therefore specify that an increase in the intercept would improve the adaptation of the perceiver to its environment (cf. Jacobs, 2001; Withagen & Michaels, 2005).

Of course, concepts such as correlations and constant errors can be used in this particular situation only because of the nature of the task and feedback. In the case of lateral catching (look back to the section on calibration), information related to a non-optimal calibration, for instance, would be considerably more analytically complex. The point of this section, then, is that the breakaway from elementarism required for direct learning is more dramatic than the breakaway from elementarism required for direct perception. In particular, informational richness useful to learning might be discovered only on longer time scales (cf. Fowler & Turvey, 1978). Most boldly, our assumption is that there exist higher-order properties of environment-actor systems that are specific to changes that reduce non-optimalities of perception-action systems, at least in ecologically relevant environments. We also assume that there exists detectable information in ambient energy arrays that specifies such properties. We refer to this information as
information for learning. We next address how this type of information can be represented as vector fields in information and calibration spaces.

Let us start with a simple example: the calibrating thermometer depicted in Figure 2b (look back to the section on information spaces). Suppose that the position of the outer tube of the thermometer is low with regard to its optimal position. “Perceiving” with this non-optimal calibration would result in a constant overestimation of temperature. The overestimation contains information about how to calibrate the thermometer, namely, a constant overestimation should be coupled to an increase in tube height and, likewise, an underestimation of temperature should be coupled to a decrease in tube height. An increase or decrease in tube height goes together with an increase or decrease in $c_1$, and can thus be represented as a vector in the calibration space that points to the right or left, respectively, with the norm of the vector indicating how fast the change should be. Given that different values of $c_1$, or different heights of the outer tube, lead to different over or underestimations, the information for calibration, as represented by the information vector, depends on the point in the calibration space. As shown in Figure 6, information for calibration can hence be represented as a vector field. In this particular field, the information vector, $v_{\text{info}}$, is defined as $v_{\text{info}} = k (c_{\text{optimal}} - c_1)$, in which $k$ is a constant and $c_{\text{optimal}}$ is the value of $c_1$ that goes together with the optimal tube height.

The existence of information for learning is far from trivial. The thermometer analogy, for instance, is in this sense limited. Given that the “perception” of the
thermometer is passive and not embedded in an ecologically relevant situation, no information is available to the “perceiver” about the relation between “perceived” and actual temperature. This illustrates that one can represent the information in a calibration space (or information space) independently of whether the information, in fact, exists as an ambient energy pattern. Before we address the more important question of whether information for learning exists in ambient energy arrays, we illustrate with a few examples how one might construct information fields in information spaces, rather than in a calibration space.

------------Insert Figure 7------------

In the experiment of Michaels and de Vries (1998) we have seen that the most useful region of the information space for the experiment lies around the point $\phi = -0.5$ (look back to Figure 3), which leads to the expectation that perceivers converge toward that region and, as thus, that the information field leads them to it. Analogous to the information field in the previous calibration example, we could define an information field as $v_{info} = k (\phi_{optimal} - \phi)$, in which $k$ is a constant and $\phi_{optimal} = -0.5$. This field is shown in Figure 7. An apparently more complicated field, for the by now familiar colliding balls example, could be

$$v_{info} = \left( (1 - r^2)\frac{b - a}{\sqrt{a^2 + b^2}}, (1 - r^2)\frac{-b - a}{\sqrt{a^2 + b^2}} \right), \quad (2)$$

where $a$ and $b$ refer to the coordinates of the space as depicted at the bottom part of Figure 4, and $r$ refers to the correlation between relative mass and the variable defined by
the point \((a,b)\). Remember that we illustrated in Figure 5 that the correlation between a variable represented by a point in the information space and relative mass depends on the set of collisions. Because the information field depends on this correlation, then, the field also depends on the set of collisions. Figure 8 shows the information fields associated with the two sets of collisions described in the earlier sections.

Note again that the preceding does not prove that the information fields are embodied in ambient energy arrays; it does not prove that the relevant non-optimalities of perceptual and perceptual-motor systems reveal themselves during perceiving and acting. Nevertheless, the existence of information for learning is the cornerstone of the direct learning theory. We argue that the typical complexity of behavior in natural environments should motivate ecological psychologists to start from the assumption that information for learning exists. In this sense, the case of information for learning is analogous to the case of information for perception. Assuming that information for perception exists—even without being able to demonstrate which ambient energy patterns are involved—has often proven to be useful, and has often led to subsequent discoveries of such information. In our view the assumption that information for learning exists might be similarly useful, and it might lead to searches for and discoveries of the information.

On the perceiver’s side of the lawful learning chains

The present article portrays learning as direct, or information-based, much like Gibson’s (1979) direct-perception approach. The direct-perception approach portrays
perceiving as establishing a lawful chain from properties to be perceived, to information for perception, to perceived properties, and vice versa, from intentions to perceive, to information for perception, to properties to be perceived. Likewise, the here-presented direct-learning approach portrays learning processes as lawful chains, in this case from higher-order properties of the environment-actor system, to information for learning, to change in perceptual or perceptual-motor systems. The previous section addressed the environmental side of this chain; we now address the perceiver’s side.

We aim to provide some further intuitions about the change that occurs over time in perceptual and perceptual-motor systems, using as an analogy the types of change that might occur in the calibrating thermometer of Figure 2. First, the actual temperature might change. Because the “perceptual system” couples the actual temperature to the fluid height, this change goes together with a change in the fluid height. Analogously, a first type of change in perceptual or perceptual-motor systems reflects a change in the ambient energy patterns themselves, which is to say, in the value of the operative informational variable. One could refer to this as change in the state of the system.

Second and more interesting, the thermometer can change with regard to the height of the outer tube. One could label this as a change in the set-up of the system. Assume, again, that perception is governed by the linear single valued function \( P = c_1 + c_2 E \). Change in the state of the system would be described by change in the value of \( E \), and a corresponding change in the value of \( P \), whereas change in the set-up of the system would be described by change in which informational variable \( E \) is operative and by change in the calibration parameters. Using a traditional ecological wording (Runeson,
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1977), change in set-up means change in the smart perceptual device, making the device
detect other variables or have a different calibration.

Perceptual learning, then, can be said to affect the set-up of systems. In the
thermometer analogy, it is easy to understand the change in the set-up in terms of
mechanism (i.e., change in the height of the outer tube) and the corresponding change in
the single-valued function (i.e., change in $c_{1}$). The change in set-up of human perceptual
systems is considerably more difficult to describe at the level of mechanism, or biological
tissue. We therefore analyze change in set-up as change in the single-valued function.
However, with regard to the following section it is important to keep in mind that change
in the single-valued function goes together with (or is dual to) change in biological tissue.

We now present a few examples of how change due to learning can be observed
in calibration and information spaces. Consider as a first example the single-dimensional
calibration space associated to calibration parameter $c_{1}$ of the previously discussed model
for lateral catching. In Jacobs and Michaels (2006) we computed the calibration
parameters for each of six blocks of trials. The locations of the numerals in Figure 9 show
the values of $c_{1}$, averaged over all monocular participants, with the number itself
indicating the corresponding block of trials. Note how the figure illustrates the change
due to calibration as movement through a calibration space.

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Insert Figure 9
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In a second example we use data of Michaels and de Vries’s (1998) Experiment 4.
In that experiment, participants practiced with feedback to judge force during six blocks
of trials. In Figure 3 we presented an information space for this example, along with the usefulness of the variables represented by points in the space. We now show that it can be used to illustrate changes in perceptual systems due to the education of attention. Let us examine the results from three participants (5, 7, and 8). Consider Figure 10. On each of the three plots are six performance points, identified by the numerals 1-6, representing performance on the six successive blocks of trials in the experiment. The horizontal position represents the variable in the space that correlated most highly with the participant’s judgment on that block, and the height represents performance—the correlation of judgment and force.

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Insert Figure 10
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To the extent that this particular information space is suited for the task, performance should not be above the usefulness curves (the solid lines), and to the extent that learning involves movement through the space, successive blocks should tend to follow the contours. For the participant whose results are plotted in the top graph, Block 1 shows a use of a variable represented by the point $\phi = \pi/2$ (i.e., very near displacement). Over blocks, the participant moved through the space, and ended near the point $\phi = 0$ (representing the use of velocity). The middle participant was already in the region with the highest correlation with force ($\phi = -.5$) during Block 1, and performance by Block 6 simply became less variable, without moving through the space. In the bottom panel, the participant begins near the point representing velocity, gets less variable, and then proceeds toward the locus in the space where the best performance is permitted. One
wonders if, with more practice, the top participant might have continued on the route of this bottom participant, who started where the top participant left off.

To summarize, change due to the education of attention and calibration, here referred to change in set-up, can be described as paths through information and calibration spaces, respectively. Hence, a first challenge in studying a particular task is to define an information space that includes the variables relevant to the task (as points in the space), and to define appropriate calibration spaces. A subsequent and perhaps more important challenge is to explain how perceivers traverse the paths that they traverse. This second challenge is addressed in the next section.

Specificities of learning

In the previous sections we have argued that the education of attention and calibration are guided by information for learning and information for calibration, respectively, and that these processes effect change in the set-up of perceptual and perceptual-motor systems. In this section it is suggested that the change in set-up is a single-valued function of information for learning and calibration, which is to say, that the change is specific to the information. In yet other words, we propose the possibility that perceptual and perceptual-motor learning are direct.

Given that information for learning (an ambient energy pattern) and change due to learning (change in biological tissue) can both be illustrated in information and calibration spaces, one can expect that specificity between these properties can also be illustrated in such spaces. That is indeed the case; in the language of information spaces, we conjecture that, at each point, the direction and speed of learning paths are equal to
the direction and speed of the information vector at that point. With this observation we can use the direct learning theory to analyze data.

Note first that a vector field—in this case an information field—can of course be interpreted as a description of a system of ordinary differential equations. If the information for learning is described as a system of differential equations, the learning paths, defined as paths that follow the information vectors at each point, are simply the solutions of the system of differential equations represented by the field. This means that the concept of information space connects our theory to the theory of ordinary differential equations, and thereby allows us to take advantage of the theory of ordinary differential equations, which is of course one of the most natural and powerful theories for studying systems that change over time (e.g., Strogatz, 1994).

Let us now reconsider the colliding balls experiment of Jacobs et al. (2001). Given (1) the conjectured specificity between information for learning and change due to learning, (2) a point in an information space representing the variable that a participant uses initially, and (3) a particular information field, we ought to be able to predict the learning path of that participant. Because the information fields illustrated in Figure 8 are different, the predicted paths for observers starting at the same locus are expected to be different for observers in the speed-correlation zero condition and observers in the angle-correlation zero condition. We illustrate this with two simulations, one for each condition. Remember that several participants in this experiment initially exploited the exit-speed difference.
Hence, the simulated participants were also assumed to use the exit-speed difference in the first block of trials. They then moved to a second point in the space, representing a second variable, according to the information vector at that point, as defined in the section on information for learning. The simulated participants were assumed to use that second variable in the second block and move to a third point in the space according to the information vector at the second point. We likewise calculated the variables used by the simulated participants—or their positions in the information space—in the fourth and fifth block.

Figure 11 presents the simulated paths. The positions in the different blocks are indicated with numerals in the original information space, which also shows the information fields. In the speed-correlation zero condition (left panel), the information vectors around the point (1, 0), the point representing the exit-speed difference, are large, because the usefulness of the variable, as measured by its correlation with relative mass, is zero. Consequently, the simulated movement through the space of the participant is fast. On the other hand, in the angle-correlation zero condition (right panel), the information vectors are small, and the simulated movement is slow.  

To compare the performance of these simulated participants to the performance of actual participants, we computed the judgments of the simulated participants, per block of
trials, that would result from the use of the variables represented by their locations in the information space. As we did for the actual participants, we then correlated these simulated judgments with the original candidate variables: exit-speed difference, scatter-angle difference, and an invariant specifying relative mass. The squares of the correlations are presented in Figure 12. Note the similarity of the figures obtained from the simulated participants and figures for the actual participants (look back to Figure 1). Although it is too early to draw conclusions concerning particular information fields and learning paths, these results illustrate how the direct-learning approach can be used to analyze empirical observations.

Conclusions

A first purpose of the present article was to present a view of perceptual and perceptual-motor learning that does not entail loans of intelligence. This led us to explore a theory of direct learning. Remember in this regard that the direct-perception approach portrays perception as specific to properties of ambient energy arrays, and thereby obviates loans of intelligence (Shaw, Turvey, & Mace, 1982). Likewise, the direct learning approach portrays change in learning devices as specific to properties of ambient energy arrays; this in sharp contrast with the view that learning devices somehow compute how to change, for instance on the basis of summary statistics (probabilities) or representation-like entities. Hence, the direct-perception and direct-learning approaches are based on the same principles, which means that if direct perception does not entail loans of intelligence, then neither does direct learning entail such loans.
To come back to our more general purpose—to identify regularities that scientists might want to search for, we suggest that research be aimed at discovering the properties of the environment-actor system, the change in set-up, and the single-valued functions that are involved in learning. In other words, we suggest that research be based on the conjecture of direct learning as a methodological doctrine. Note that this contains an implicit warning for researchers studying the informational bases of perception or action. Namely, attempts to prove that perceivers use some variable, or that they do not use that variable, may have limited generality. The differences and changes in variable use that we have seen indicate that the lawfulness of perception and action does not always reside at that level. In our view, after having carefully scrutinized the level of perception and action, the level of learning is simply the next best place to search for lawfulness.

Note also, as an aside, that the present article has been concerned with two time-scales: the short scale of perceiving and acting and the longer scale of learning. We did not consider yet longer scales. However, if one assumes perceptual and perceptual-motor systems to approximate the use of specifying information in the long run, which is to say, if one assumes that information fields have an attractor at a point representing specifying information, then the theory can be extended with a third time scale, the scale of ecological realism (e.g., Shaw et al., 1982; Turvey et al., 1981). We refer the reader to Jacobs and Michaels (2002) for more detail on the relation of the scales of learning and realism.

Finally, let us mention that we like to think about the information to which perceptual and perceptual-motor devices “resonate” as being a single entity, without
necessarily disentangling the respective contributions of information for perception, calibration, and learning. Such a holistic perspective emphasizes the close relation of perceiving and learning—perceiving and learning being, in fact, a single process. In any case, our view has in common with the traditional direct perception approach that we seek informational couplings, or dualities, between animal and environment. Our view differs from the direct-perception approach in that we do not search for duality that is a consequence of learning or evolution, but for duality that encompasses, so far, perceptual and perceptual-motor learning.
References

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Author Note

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Footnotes

1 Note that the expected primacy of affordances does not imply that animals perceive only affordances; it is our belief that they also perceive other properties of objects and events—as well as object and events themselves (cf. Bingham’s, 2000, commentary on Stoffregen, 2000).

2 We refer the reader to Appendix A of Jacobs et al. (2001) for details about how local constraints were manipulated to create the different sets of collisions. Note, though, that the usefulness of different nonspecifying variables can to a large extent be manipulated independently. Also note that the specificity of information granted by natural laws or other global constraints, which has been shown to exist in the case of the relative mass of colliding balls (e.g., Runeson et al., 2000), is not affected by adding local constraints.

3 Note that there are many (sometimes equivalent) ways to define information spaces. One could also define candidate informational variables for the Michaels and de Vries (1998) experiment as the maximum of the fractional time derivative of (possibly noninteger) order $\alpha$ of the displacement of the center of mass (Podlubny, 1999). Then $\alpha$ would be the coordinate variable of a space in which the variable maximal displacement would be represented by $\alpha=0$, maximal velocity by $\alpha=1$, and maximal acceleration (i.e., force) by $\alpha=2$. Using this space would lead to results that are to a large extent similar to the results that we present later in this article.
Note that several somewhat arbitrary choices have to be made in the second step of this method, concerning whether to use a hyperplane or a hypersphere, or some other hypersurface. Or, if a plane, which one? In the following, therefore, we have to be careful to use concepts that do not depend on such choices. Relevant to our earlier work (e.g., Jacobs, 2001; Jacobs & Michaels, 2002), *convergence* is such a concept.

In the following we used, in fact, information fields that are proportional to the one defined in Equation 2. In Figure 8 we used a factor such that the largest arrows in the figures just did not overlap, and in the simulations presented in Figure 8 we divided the vector field as indicated by Equation 2 by 4.

Note that learning paths are assumed to be continuous and that the apparent discreteness of these results is a mere consequence of the block-by-block analysis.
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Figure 1. Squares of correlations between judgments of mass ratio and the various kinematic variables for two participants in Experiment 3a of Jacobs et al. (2001). *Invariant* refers to a variable specifying relative mass; *speed* refers to the exit-speed difference; and *angle* refers to the scatter-angle difference.

Figure 2. Schematic representation of thermometers as analogies for perceptual systems. A. Thermometer that does not calibrate. B. Thermometer that calibrates the height of the scale.

Figure 3. One-dimensional information space for Experiment 4 of Michaels and de Vries (1998). A point on the horizontal axis, $q$, represents the ambient energy variable $E_q = \cos(q) \cdot \text{velocity} + \sin(q) \cdot \text{displacement}$. The vertical axis represents the correlations between $E_q$ and the to-be-judged property, force.

Figure 4. A schematic representation of the construction of an information space for Experiment 3a of Jacobs et al. (2001). A point in the bottom plane, $(a, b)$, represents the variable $E_{(a,b)} = a \cdot \text{angle} + b \cdot \text{speed} + (1 - a - b) \cdot \text{invariant}$. Thus, $E_{(a,b)}$ is the variable associated to the point $(a, b, 1 - a - b)$ in the slanted plane in the upper part of the picture, which is the point that corresponds to $(a, b)$ via the projection.

Figure 5. Usefulness of variables in the sets of collisions used in the speed-correlation zero condition (left graph) and angle-correlation zero condition (right graph) of
Experiment 3a of Jacobs et al. (2001). A point in the horizontal plane, \((a, b)\), represents the variable \(E_{(a, b)} = a \cdot \text{angle} + b \cdot \text{speed} + (1 - a - b) \cdot \text{invariant}\), and the vertical axis presents the squares of the correlations between these variables and the to-be-judged property, relative mass.

*Figure 6.* Information field for calibrating thermometer (arbitrary units).

*Figure 7.* Information field for Experiment 4 of Michaels and de Vries (1998).

*Figure 8.* The information field defined by Equation 2, represented as vector field in an information space, for the sets of collisions used in the speed-correlation zero condition (left panel) and the angle-correlation zero condition (right panel) of Experiment 3a of Jacobs et al. (2001). The locations of the invariant, the exit-speed difference, and the scatter-angle difference are represented, respectively, by the diamond, dot, and triangle.

*Figure 9.* Calibration space for Experiment 2 of Jacobs and Michaels (2006). The locations of the numerals represent the values of calibration parameter \(c_1\), per block of trials, averaged over all monocular participants. The arrows indicate the corresponding changes, over blocks of trials, in the location in the calibration space.

*Figure 10.* Information spaces and experimental results for three participants in Experiment 4 of Michaels and de Vries (1998). The horizontal locations of the numerals
represent the variable that the participant appeared to use on the associated blocks of trials. The heights of the numerals represent the correlation between force and the judgments of the participant in the associated blocks of trials. The solid lines indicate the usefulness of the variables represented by points on the horizontal axes.

*Figure 11.* Information spaces for Experiment 3a of Jacobs et al. (2001), with the information fields as defined by Equation 2 and the predictions following from them. The locations of the numerals represent the variables used by simulated participants in the corresponding blocks of trials in the speed-correlation zero condition (left panel) and angle-correlation zero condition (right panel).

*Figure 12.* Squares of correlations between judgments of mass ratio and the various kinematic variables for two simulated participants, based on Experiment 3a of Jacobs et al. (2001). The simulated judgments were computed assuming that participants used the variables as represented by the locations of the numerals in Figure 11.
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\[ c_2 \]

\[ \rightarrow \rightarrow \rightarrow \leftarrow \leftarrow \leftarrow \]

-3 -2 -1 0 1 2 3