LOCAL SMOOTHING FOR THE BACKSCATTERING TRANSFORM

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3. Special fundamental solutions of the ultrahyperbolic operator.
4. Local smoothing estimates for the backscattering transform.
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The course presents results obtained by Anders Melin (Lund University) and the lecturer. We consider the inverse backscattering problem for Schrödinger operators $H_v = -\Delta + v$, $v \in L^\infty_{\text{comp}}(\mathbb{R}^n; \mathbb{R})$, in odd dimensions $n \geq 3$. The backscattering transform $Bv$ of the potential $v$ is, up to a smooth function, the real part of the inverse Fourier transform of the backscattering part of the scattering matrix. The mapping $L^\infty_{\text{comp}}(\mathbb{R}^n) \ni v \mapsto Bv \in \mathcal{D}'(\mathbb{R}^n)$ is entire analytic, and we write

$$Bv = \sum_1^\infty B_N v,$$

where $B_N v$ is the $N$th order term in the power series expansion of $B$ at $v = 0$. Here $B_1 v = v$.

We shall give estimates for $B_N v$ in Sobolev spaces $H^{(s)}(\mathbb{R}^n)$ and show that $v \mapsto Bv$ extends to an entire analytic mapping on $H^{(s)}(\mathbb{R}^n) \cap \mathcal{E}'(\mathbb{R}^n)$ with values in $H^{(s+\alpha)}_{\text{comp}}(\mathbb{R}^n)$, when $s \geq (n - 3)/2$. We show moreover that, when $s > (n - 3)/2$ and $v \in H^{(s)}(\mathbb{R}^n) \cap \mathcal{E}'(\mathbb{R}^n)$, the regularity of $B_N v$ increases with $N$ and $v - Bv$ is locally of class $H^{(s+\alpha)}(\mathbb{R}^n)$, where $0 \leq \alpha < 1$, $a < s - (n - 3)/2$. Finally, further results on global estimates will be briefly presented.

We conclude this abstract with a few references the course is based upon.

REFERENCES