Confining and slowing airborne sound with a corrugated metawire

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In this letter we present a theoretical study on the acoustic wave propagation along a periodically corrugated perfect rigid wire surrounded by air. It is shown how acoustic surface waves (ASWs) can be engineered with their propagation properties controlled by geometrical means. These highly localized ASWs give rise to strong acoustical field confinement along the wire, whereas the slowing down of sound decelerate the group velocity down to zero. What is believed to be a promising feature of these low-loss propagation properties is the ability to tune sensing and screening applications with good transducer coupling.
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Manipulating the natural properties of acoustic waves remains a subject of growing interest. With the emergence of the concept of *acoustic metamaterial*,[1–3] several different phenomena have been reported, such as acoustic focusing and beaming[4–9] or cloaking.[10,11] In this letter we present a theoretical work devoted to the analysis of the propagation of acoustic surface waves (ASWs) along cylindrical wires. A perfectly rigid cylindrical wire does not support the propagation of ASWs. However, it is known that when a flat interface between two fluids is periodically corrugated, ASWs are supported,[12] which is also the case for an equivalent solid-solid interface with shear-horizontal waves.[13] Our aim is to study these geometrically induced ASWs in a cylindrical geometry. Sound wave propagation along corrugated wires has been studied before.[14–17] However, our motivation is to demonstrate the capabilities of these ASWs for acoustic wave focusing and slowing. Scanning and probing for acoustical nondestructive testing and high-intensity focused ultrasound[18,19] would take advantage of the ASWs described in this letter. In what follows, we assume that the longitudinal sound wave propagation takes place in inviscid stationary air, which is governed by the linear continuity and momentum equations

$$\nabla \mathbf{u}' - \frac{i\omega}{c_0^2\rho_0}p' = 0, \quad \nabla p' - i\omega\rho_0\mathbf{u}' = 0, \tag{1}$$

where ω is the frequency and c_0 is the sound velocity. For the pressure p, velocity $\mathbf{u}$, and density ρ, one can write $p=p_0+p'$, $\mathbf{u}=\mathbf{u}'$, and $\rho=\rho_0+\rho'$, where the terms p_0 and ρ_0 denote the background pressure and density in an undisturbed medium, respectively. The primed quantities p', $\mathbf{u}'$, and ρ' describe the variation in the corresponding magnitudes due to the presence of a low-amplitude acoustic field in the medium. In deriving Eq. (1), only linear terms in the primed quantities are taken into account while all higher-order terms are neglected. Consider an acoustically perfect rigid ($\partial_{\mathbf{n}}p'=0$) cylinder of radius R_o into which periodically rings are grooved [see Fig. 1(a)]. The rings that are separated with constant Λ have depth $h=R_o-R_i$ and width a. Since the structure is considered to be perfectly rigid, Λ is chosen to be the unit length of the structure. Initially, we are interested in calculating the dispersion relation $[k_z(\omega)]$ of the geometrically induced ASWs propagating along the corrugated wire. To simplify the problem, we will assume that the pressure field does not have azimuthal (ϕ) dependence. As a possible solution for the Helmholtz equation in region I [see Fig. 1(a)], a Sommerfeld-type[20] wave is sought, composed of a discrete set of Bloch waves:

$$p_I'(r,z) = \sum_{n=-\infty}^{\infty} C_n K_0(q_n r)\sigma_n(z), \tag{2}$$

where $\sigma_n(z)=e^{ik_{z,n}z}/\sqrt{\Lambda}$ and the expansion coefficients are C_n. The radial dependence is governed by the zero-order modified Neumann function K_0. The wave vector component in the r direction is $q_n=\sqrt{k_{z,n}^2-k_0^2}$ with $k_{z,n}=k_z+n(2\pi/\Lambda)$ and $k_0=2\pi/\lambda$. Our main interest is devoted to regimes where $k_z>k_0$, in which p_I' decays exponentially with r as $r\to\infty$. In this case, the geometrically induced ASWs are truly bound. As no pressure field can penetrate into the sound-hard wire, the only nonzero field distribution in region II wire occurs within the radial grooves:

$$p_{\mathrm{II}}'(r,z) = \sum_{m=0}^{\infty} A_m[J_0(\beta_m r) - \alpha(m)N_0(\beta_m r)]\psi_m(z), \tag{3}$$

where $\alpha(m)=J_1(\beta_m R_i)/N_1(\beta_m R_i)$ and $\beta_m=\sqrt{k_0^2-(m\pi/a)^2}$. The pressure field inside the grooves is expanded in terms of the ring waveguide modes, in which the z dependence is controlled by the functions $\psi_m(z)=\sqrt{(2-\delta_{m,0})/a}\cos m\pi/a(z+a/2)$ and the radial dependence by the zero-order/first-order Bessel and Neumann functions J_0, J_1, N_0, and N_1, respectively. By applying the appropriate boundary conditions (radial component of the velocity being continuous everywhere along the I-II interface but pressure being continuous only at the openings), a set of linear equations for the expansion coefficients, $\{A_m\}$, can be built up. Then, the dispersion relation for the ASWs can be extracted by just looking at the zeroes of the determinant of the corresponding matrix. Figure 1(b) displays $k_z(\omega)$ for acoustic

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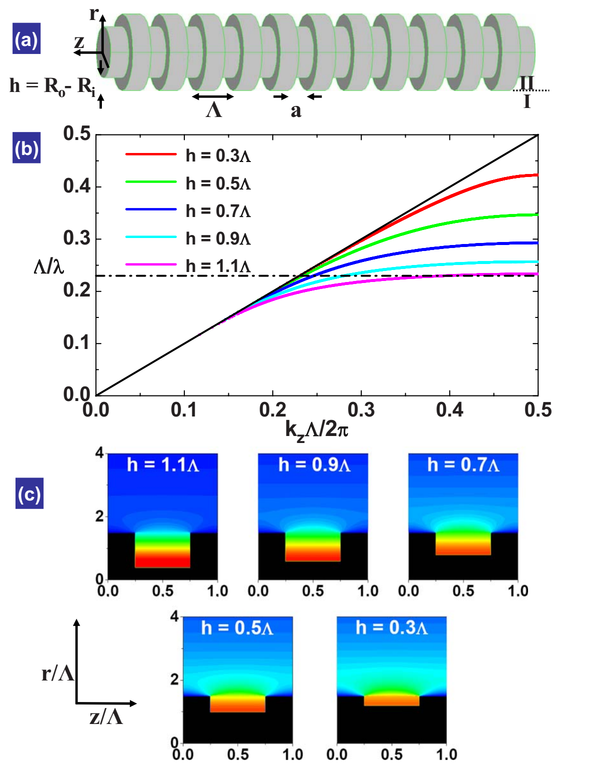

FIG. 1. (Color online) (a) Schematic view of the acoustic metawire analyzed. (b) Dispersion relation for infinite structures with $a=0.5\Lambda$, $R_o=1.5\Lambda$, and several values of h. The bands are obtained by means of the modal expansion technique. (c) Pressure field ($|p'|$) within one unit cell (rz plane) for metawires of various depths h at fixed frequency ($\Lambda/\lambda=0.23$), which corresponds to the intersections of the dash-dotted line with the ASW bands.

metawires with fixed $a=0.5\Lambda$ and $R_o=1.5\Lambda$ but for different values of h. For very shallow grooves ($h=0.3\Lambda$), a corrugated wire has weak sound guiding properties as $k_z(\omega)$ runs very close to the sound line ($k_z=\omega/c_0$). However, as the ring depth h becomes more and more pronounced, the tailored ASWs are getting more localized as the increase in the propagation constant ($k_z \gg k_0$) gives rise to a large value for $q_n \approx \sqrt{k_z^2-k_0^2}$. Note that this increasing confinement is accompanied by a strong reduction in the group velocity $c=\partial\omega/\partial k_z$ toward a *flat dispersion relation* $k_z(\omega)$. The increase in confinement is visualized by virtue of the pressure field plots shown in Fig. 1(c), which show the pressure field amplitudes (evaluated at $\Lambda/\lambda=0.23$) for different ring depths. In the subwavelength regime ($\lambda \gg a$), we have checked that the fundamental ring mode ($m=0$) suffices the expansion within the grooves provided $a \le \Lambda/2$. In this case, $k_z(\omega)$ can be extracted via the transcendental equation

$$\sum_{n=-\infty}^{\infty} \frac{\beta_0}{q_n}\frac{K_s(q_nR_o)}{K_t(q_nR_o)}|S_{0n}|^2 = \frac{J_s(\beta_0R_o)N_t(\beta_0R_i)-J_t(\beta_0R_i)N_s(\beta_0R_o)}{J_t(\beta_0R_o)N_t(\beta_0R_i)-J_t(\beta_0R_i)N_t(\beta_0R_o)}, \quad (4)$$

where $s=0$, $t=1$, and $S_{0n}=\sqrt{a/\Lambda}\,\mathrm{sinc}(k_{z,n}a/2)$. From Ref. 21, it is known that the dispersion relation for the geometrically induced surface plasmon polaritons (SPPs) propagating along a perfectly conducting wire takes the same form as Eq. (4) but with $s=1$ and $t=0$. This is a very interesting result as, due to the difference in the boundary conditions (either perfectly rigid or conducting for geometry-induced ASWs or SPPs, respectively), the dispersion relations for the surface waves in the electromagnetic and acoustic cases are quite similar but not identical. Remarkably, this is not the case in one-dimensional (1D) structures (periodic array of 1D grooves in a flat interface) where the dispersion relation of the surface waves is given by[12,13,22]

$$k_z = k_0\sqrt{1+\frac{a^2}{\Lambda^2}\tan^2 k_0h} \quad (5)$$

independently of the type of wave. This last result [Eq. (5)] can be recovered from Eq. (4) by taking the limit $R_o, R_i \gg \Lambda$. In order to illustrate the confining properties that are connected to the excitation of ASWs, finite element (FE) (COMSOL MULTIPHYSICS) simulations have been employed for a metawire of finite length, $L=40\Lambda$, with the parameters corresponding to Fig. 1(b) for $h=0.5\Lambda$. Depending on the wavelength of the impinging acoustic wave, this can be guided along the corrugated wire or be radiated away (similar to phononic crystals[23]). This is exposed in Fig. 2 for three different wavelengths. For $k_0=0.25(2\pi/\Lambda)$, which is in the nearest vicinity of the sound line, only poor field confinement to the wire is expected, but as one tends to higher frequencies ($k_0=0.30(2\pi/\Lambda)$) a strong acoustic wave localization can be observed. Note that when the ASW reaches the end of the metawire, this surface wave is scattered and yields a strong sound radiation at the wire tip. For the last case with $k_0=0.35(2\pi/\Lambda)$, the gap of the ASW band is reached and the incident pressure field is being radiated away at the entrance of the wire as no ASWs are supported at that wavelength. Apart from the possibility of subwavelength field confinement of sound by taking advantage of the strong localization associated with the ASWs, in this letter we propose two schemes for focusing sound at the end of a corrugated wire and/or stopping sound of different frequencies at different places along the rod. The basic structure able to support these two phenomena is a corrugated wire in which the depth of the grooves is adiabatically increased along the wire [see Fig. 3(a)]. If the gradual increase in h is chosen such that the depth of the grooves at the final end leads to an asymptote frequency ω_4, then an incident acoustic wave of that particular frequency will be focused at the tip of the corrugated wire. Moreover, if now the incident acoustic wave is not monochromatic but contains several frequencies above ω_4, each of these frequencies will be stopped at different places along the wire. This is due to the univocal relation between h and the frequency of the ASW band edge, as demonstrated in Fig. 1(b). The slow sound phenomenon is

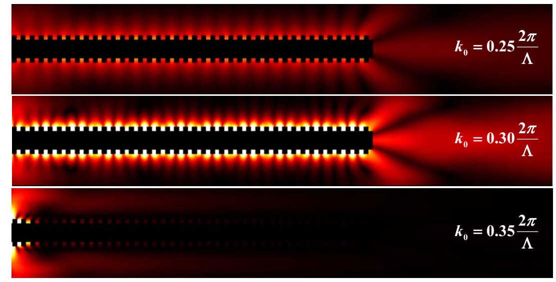

FIG. 2. (Color online) Numerical (FE) pressure field-mapping $|p'|$ of a truncated metawire (40Λ) for three different wavenumbers k_0. Geometries are as in Fig. 1 with ring depths $h=0.5\Lambda$.

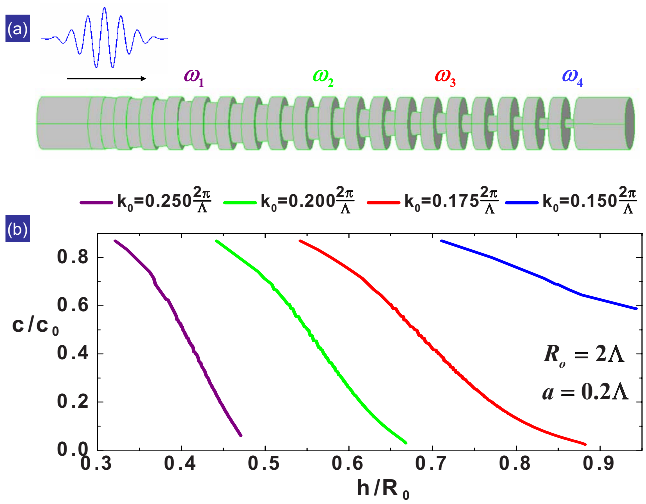

FIG. 3. (Color online) (a) Schematic of the metawire with adiabatic reduction in R_i and incident wave packet. (b) Normalized c vs h calculated with the mode matching technique for $k_0=0.15(2\pi/\Lambda)$, $0.175(2\pi/\Lambda)$, $0.2(2\pi/\Lambda)$, and $0.25(2\pi/\Lambda)$ with $a/\Lambda=0.2$ and $R_o=2\Lambda$.

illustrated in Fig. 3(b) in which the evolution of the group velocity, c, as a function of h for four different frequencies is displayed. For calculating these four curves we have considered infinitely periodic corrugated wires with uniform h. It is then envisaged that in a finite wire presenting a gradual and adiabatic increase in h, the wave component associated with each frequency will be stopped at the spatial location (h) in which $c\rightarrow 0$ for that particular frequency. The prospect of engineering an acoustic surface wave along a corrugated wire opens up a possibility to confine and slow down sound. Moreover, by gradual energy concentration, superfocusing on the submillimeter range could be achieved and the possibility to create an axial guide with tunable frequency passbands is facilitated. With minor technical extensions, acoustical scanning, spectroscopy, medical ultrasound instrumentation, and imaging could obtain good field resolution.

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