Skill-Biased Technical Change and the Decline in Low-Skill Wages in a Signaling Model.

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May 21, 2013

Abstract
Skill-biased technical change is the most frequently-cited explanation for the rise in the college education premium observed in the US over the last 30 years. However, skill-biased technical change does not necessarily explain the decline in non-college wages, the main driver of the rise in the college education premium.

In this paper we introduce wealth heterogeneity into Spence’s (1973) education signaling model. We find that under some general conditions of wealth distribution, the decline in non-college wages can be explained as a consequence of ability composition effects induced by skill-biased technical change.

JEL-Classification: D82, J31, I20
Keywords: Information asymmetry, College Enrollment, College Premium, Education Signaling, Skill-Biased Technical Change

1 Introduction.
The coincident increase in the returns to higher education and in the supply of college graduates in the US over the last thirty years is well documented.\footnote{\footnote{I gratefully acknowledge the comments from Antonio Cabrales, Caterina Calsamiglia, Guillermo Caruana, David Perez-Castrillo and Pedro Rey-Biel as well as to the participants in the seminars in UAB, UC3M, URV, SAEE, ECORE and in the ASSET 2011 meeting. All errors are my own responsibility. This research has been supported by the Ministerio de Educaci´on y Competitividad, through grant ECO2012-34581.}} Returns to higher education are measured by the college wage premium, that is, by differences in earnings between college graduates and individuals with lower levels of education. A remarkable fact that has received much less attention is that the rise in the the college premium is mainly driven by a reduction in low-skill wages, see Figure 1.\footnote{See Katz and Autor (1999) for a general survey of the literature and Figure 1 to observe changes in returns to education and the increase of college graduates.} We use a signaling approach to show that the reduction in low-skill wages can be understood as an indirect consequence of skill-biased technical change, the most comprehensive explanation of changes in the college premium.

1 See Katz and Autor (1999) for a general survey of the literature and Figure 1 to observe changes in returns to education and the increase of college graduates.
2 We use the terms “college” and “high-skill” interchangeably, as well as “non-college” and “low-skill” in this article.
Skill-biased technical change consists in changes of technology that favor high-skilled labor productivity. Consequently it must have a positive impact on high-skill wages. Nevertheless, skill-biased technical change does not explain a reduction in low-skill wages, a point emphasized in recent work by Acemoglu and Autor (2011) “despite its notable success, the canonical model is largely silent on a number of central empirical developments of the last three decades, including, significant declines in real wages of low-skill workers, particularly low-skill males”.3

We show that a simple version of the Spence’s (1973) signaling model, with wealth heterogeneity, similarly to Hendel et al. (2005), provides a theoretical explanation for the decline in low-skill wages. In this model, some general conditions on the joint distribution of wealth and ability are sufficient to show the reduction of low-skill wages to be an indirect consequence of skill-biased technical change. Our explanation is compatible with the three observations noted above: the increase in the supply of college graduates, the rise in returns to higher education and the decline in wages of low-skill workers.

We use a stylized model, with two types of ability: high and low. The intuition behind this specification is as follows. In a signaling model, wages equal average productivity for a given education level. College education reveals (or partially reveals) the unobservable productivity or ability of individuals. Nevertheless, if there are imperfect credit markets and wealth heterogeneity, higher education is not only a signal of ability, but also of individuals’ (parents’) wealth. Consider an extreme case in which individuals can only be rich or poor and of high or low-ability. If higher education becomes more attractive, then both poor individuals with high-ability and rich with low-ability might move from the non-college-educated to the college-educated share of the population. Therefore, the ability composition effect on college wages can be either positive or negative. However, only a minority of poor individuals of low-ability do not enroll in college. This pushes non-college wages down toward low productivity level. We show that a monotone likelihood ratio and/or log-concavity of the cumulative distribution of wealth are sufficient conditions to flesh out this intuition in more general cases. These two conditions guarantee that the fraction of high-ability types that choose higher education after the change in technology is relatively large, with respect to their current presence at non-college education levels, pushing low-skill wages down. Under these conditions, an increase in the college premium induced by skill-biased technical change always occurs concurrently with a reduction in low-skill wages, while it does not necessarily entail an increase in high-skill wages.

There is a substantial amount of evidence regarding the importance of households’ resources

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3Inspired by Tinbergen’s (1974, 1975) pioneering works, the authors refer to the canonical model as the demand-and-supply model of high-skill and low-skill workers under skill-biased technical change.
as a determinant of college attendance. See Bailey and Dynarski (2011), Corazzini et al. (1972) or Heller (2011). Therefore, accounting for wealth heterogeneity is important in understanding the effects of ability composition on the college premium. Moreover, wealth is relevant to the formation of skills before college attendance. We assume that the wealth distributions of different ability types satisfy the monotone likelihood ratio property. This ensures a positive correlation between wealth and ability. However, our results also hold in the specific case of the independence of wealth and ability. The assumption of a log-concave cumulative distribution of wealth also accords with the existing literature.

Another notable fact is the substantial increase in tuition fees in higher education over the last 30 years. As Heller (2005, 2011) points out, the share of family income required to pay for four years of tuition -at public institutions of higher learning- has more than doubled between 1971 and 1997 for the poorest quintiles and nearly doubled for the richest one. Despite these observations, previous signaling literature on the college premium needs from improved access to higher education to explain the increase in the college premium. Here we show, that, on the one hand, that the signaling hypothesis is compatible with an increase in the college premium, even with increased tuition costs, though interestingly, it cannot be reconciled with the decline in low-skill wages. On the other hand, we also show that if access to higher education is improved though a reduction in the interest rate on college loans, this does not necessarily induce an increase in the college premium. Therefore, our conclusions are more favorable to the skill-biased technology change than to changes in the costs of education as an explanation of the rise in the college premium.

2 Literature Review.

Economic explanations for changes in the college premium can be distinguished according to the two main economic interpretations of education: human capital and signaling.

The human capital literature is mainly represented by the canonical model, also known as Tinbergen’s framework. This model considers the relative demand and supply of high-skill and

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4 Although not the main determinant Blau (1999) documents a positive relationship between parental’ income and child development. Cunha and Heckman (2010) finds a positive (but indirect) effect of wealth on ability, driven by the importance of the early years in the development of non-cognitive skills.

5 The two main hypotheses are that wealth distribution is Pareto and that is it log-normal. See Atkinson and Harrison (1978). Although the density functions of these two distributions are not log-concave, their cumulative distribution functions are; see Bagnoli and Bergstrom (2005). The latter is sufficient to guarantee our results.

6 Archibald and Feldman (2008, 2010) is one of the few works that supports improved affordability of post-secondary education. Their assessment arises because they measure affordability based on total income available after paying for higher education.

In the canonical model, the rise in the returns to higher education is compatible with the increase in the relative supply of college graduates because of a contemporaneous rise in the relative demand for college-educated workers. The most prevalent hypothesis to explain the increase in the relative demand for college graduates is the skill-biased technical change. Despite its success in modeling changes in the college premium, the skill-biased technical change argument is not compatible with the observed decline in non-college wages in these models. Notable exceptions are presented in Acemoglu (1999) and Acemoglu and Autor (2011). Acemoglu (1999) characterizes the decline in non-college wages as a consequence of structural changes in firms’ production models, changes made to adapt to a more skilled workforce. Acemoglu and Autor (2011) endogenize the assignment of skills to different tasks. In their model, the necessary condition for a reduction in non-college wages following a skill-biased technical change is the substitution of low-skill workers for better prepared workers, in tasks where low-skill workers previously had a competitive advantage.

In contrast to these two works, we consider that technological change does not have a direct negative effect on low-skilled workers. In our model, the decline in low-skill wages is a consequence of individuals’ educational decisions, that occurs through the effects of these decisions on the ability composition of education groups. Thus, we argue that the decline in low-skill wages may occur because low-skills have become a clearer signal of low productivity. Although our framework is not as rich as that of the canonical model, it is in line with the empirical works of Carneiro and Lee (2011) and Taber (2001), which suggest the relevance of ability composition for changes in the college premium. Despite, our theoretical framework is far from the empirical work of Carneiro and Lee (2011), it may help to complement their results at an individual decision level.

The role of ability composition or signaling in generating changes in the college education premium has received less attention than has human capital. Riley (2001) provides a discussion of the plausibility of the signaling hypothesis. Lang and Kropp (1986) is one of the first works that seeks to explain the college education premium using an ability composition argument. Bedard (2001) does not directly address changes in the college education premium, but suggests

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that ability composition plays an important role in the college enrollment decision because of its effects on the returns to college.

Our work is especially closely related to Hendel et al. (2005) and Zheng (2010), with whom we share a signaling framework that incorporates wealth heterogeneity and imperfect credit markets. However, substantial differences between our work and theirs exist. First, we extend the model in Hendel et al. (2005), to allow both high and low wages to be sensitive to changes in ability composition. Hendel et al. (2005) consider a particular separating equilibrium in which only high-ability types enroll in college. Zheng (2010) examines a pooling equilibrium that restricts the ability composition effect to low-skill wages. In their models, improved access to higher education always induces an increase in the college premium. In our framework, by contrast, an increase in the college premium is also compatible with decreased college affordability, as documented in Heller (2005, 2011). A second important difference with these works is that we analyze the effects of skill-biased technical change. To our knowledge, this is the first paper to address skill-biased technical change from a signaling perspective. According to our results, skill-biased technical change results in an increase in the college premium and a decline in non-college wages, while this is not the case when we account for changes in affordability.

We do not present the ability composition effect as an alternative to skill-biased technical change, but as a complement to it. Other recent studies also explicitly take account of the non-exclusiveness of these hypotheses and examine their relative contributions to the dynamics of the college premium. Fang (2006) and Cunha et al. (2011) distinguish between the signaling and human capital hypotheses, finding that although both can play a role, human capital and skill-biased technical change have played a larger role in the increase in the college premium. According to our results, skill-biased technical change always results in an increase in the college education premium, independently of the direction of the composition effect. However, this is not the case when we consider only the composition effect without changes in productivity (i.e., changes in affordability). This conforms to the view that human capital plays a major role in the rise of the college premium.

3 The Model.

To model wage formation, we use a version of Spence’s signaling model with wealth heterogeneity similar to Hendel et al. (2005). Individuals are endowed with an initial wealth level and with a particular level of ability, where the latter is perfectly correlated with productivity and academic performance. Then, with imperfect credit markets, their decision regarding acquisition of college
education depends not only on ability, but also on the affordability of education.

3.1 Individuals.

The population is of size one. A fraction $\pi$ is high-ability, $H$, with productivity $q_H \in \mathbb{R}_{++}$. The remaining fraction $1 - \pi$ is of low-ability, $L$, with productivity $q_L \in \mathbb{R}_{++}$. As in Spence’s original framework, the productivity of high-ability types is greater than that of low-ability types, that is, $q_H > q_L$. We denote by $\bar{q}$ the average productivity of the population, i.e., $\bar{q} = \pi q_H + (1 - \pi) q_L$.

To obtain education, individuals must pay the tuition, which we denote by $T > 0$. However, a decision to study does not immediately lead to a credential. As in Weiss (1983), there is some risk of failing. Thus, individuals have probability $P_i, i = H, L$, of successfully completing their education. High-ability individuals have a greater probability than low-ability individuals of obtaining a credential, and without loss of generality, we assume that $1 = P_H > P_L > 0$, which captures Spence’s (1973) original idea that education is more challenging for low-ability individuals than for high-ability individuals. Once paid, tuition is never reimbursed, independently of whether the student completes education.

An individual who obtains a degree works in the qualified sector and receives a wage of $w^e$, greater, in equilibrium, than the wage the individual would receive working in the non-qualified sector, $w^n$, i.e., not enrolling or failing. Workers are risk neutral and have incentives to study only if their expected gain, i.e., the college education premium, denoted $cp = w^e - w^n$, is sufficiently large relative to the cost of education. At the same time, given credit market imperfections, the education decision is also subject to individuals’ initial wealth.

The initial wealth of individuals is denoted by $b \in \mathbb{R}_+$. Endowments follow a continuous density function, $f_i(b)$, with full support over $[0, \bar{b}]$. We use $F_i(b)$ to denote the cumulative distribution of wealth of type $i$ individuals. Wealth and ability are potentially correlated. Although not the only factor, wealth is an important determinant of individuals’ ability formation. The following assumption guarantees a positive correlation between wealth and ability:

- **Assumption (MLRP):** The likelihood ratio $f_H(b) / f_L(b)$ is monotone increasing.

This assumption implies the stochastic dominance of high types wealth distribution, i.e., $F_H(b) \leq F_L(b)$. This accounts for the positive correlation between wealth and ability. We also note, as a final remark, that our results hold in the specific case of the independence of wealth and ability.
3.2 Firms.

Because ability affects productivity, it is valued by firms, but unobservable. However, education can be used to signal ability. Firms can only observe workers’ credentials and set wages according to workers education levels, but cannot observe individuals’ ability or wealth. Firms know the proportion of high-ability workers in the population, as well as the wealth distributions of high-ability and low-ability types. Firms compete a la Bertrand, which implies that they fix wages according to expected productivity, conditional on a workers education level. The expected productivity in each sector can be represented as $g^s$, $s = e, n$:

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\begin{align*}
    g^e &= \frac{\pi(1-\rho_H)q_H+(1-\pi)P_L(1-\rho_L)q_L}{\pi(1-\rho_H)+(1-\pi)P_L(1-\rho_L)} \\
    g^n &= \frac{\rho_H q_H+(1-\pi)(1-P_L(1-\rho_L))q_L}{\pi(1-\rho_H)+(1-\pi)(1-P_L(1-\rho_L))}
\end{align*}
$$

where $\rho_i$, with $i = H, L$, is the proportion of each type that decides not to enroll in higher education. In contrast to pure signaling models, we add a productive role for higher education, indicated by the parameter $\alpha > 1$. $\alpha > 1$ represents the productivity enhancement role of higher education.

3.3 Educational Decision.

Following Hendel et al. (2005), individuals can borrow in order to study. We denote by $x$ the borrowing interest rate. We also assume that there is an interest rate for lending, normalized to 1. Credit markets are imperfect in that the borrowing interest rate exceeds the lending interest rate, i.e., $x > 1$.

As borrowing is costly, poorer individuals require higher college education premiums to compel them to enroll in higher education. We denote by $b^*_i$ the lowest wealth level of type $i$ individuals who choose to study in equilibrium. $b^*_i$ is a function of the college premium, i.e., the endogenous part of the model, as well as of the models exogenous variables. Given the continuity of wealth distributions and the monotonicity of borrowing costs with respect to $b$, $F_i(b^*_i)$ represents the proportion of type $i$ individuals who decide not to enroll in higher education. That is, we can use $\rho_i = F_i(b^*_i)$ of the average productivity equation (1), so that average productivity in each sector is a function of each ability types wealth cut-off level, i.e., $g^s(b^*_H, b^*_L)$, for $s = e, n$.

Hendel et al. (2005) assume that low-ability individuals have no incentives to study. Consequently, in their model, average productivity in the college-educated sector is always $q_H$. We relax this assumption, to allow low-ability individuals to receive higher education if they are sufficiently affluent. At the same time, we allow that ability differences are still relevant to the
decision to enroll in higher education. The following assumptions ensure that this is the case:

- **Assumption 0 (A0):** \[0 < T < \bar{b}.\]
- **Assumption 1 (A1):** \[T < \alpha q_H - \bar{q} < T/P_L < \alpha q_H - q_L < T(1 + x)\]
- **Assumption 2 (A2):** \[F_H(T) < \frac{(1-\pi)(\alpha q_H - q_L - \bar{b})}{\pi (\bar{P}_L - (\alpha-1)q_H)}\]

These assumptions guarantee that in equilibrium, both wealth and ability are relevant to the attainment of higher education. This is consistent with the empirical evidence showing that resources and ability are the two main determinants of college attendance, see Corazzini et al., (1972); Hearn (1984 and 1988); Hossler et al. 1999 and Sewell (1971). Technically, this would be represented as a pooling equilibrium in which both ability types obtain both education levels.

A0 simply states that the most affluent individual of each ability type can always afford the cost of education without borrowing, while the poorest cannot.

A1 establishes that both wealth and ability are relevant for the education decision. It is a weaker version of the assumptions in the model of Hendel et al. (2005).

As firms pay according to average productivity, the upper bound of the college education premium corresponds to the greatest possible difference in productivity between individuals, that is, \(\alpha q_H - q_L\). According to the third inequality in A1, both types have incentives to study when the college education premium takes a value arbitrarily close to its upper bound. However, this is not true when the college premium is sufficiently low, in particular when it equals the difference between high productivity and population average productivity. In this case, only high-ability types have incentives to study, according to the first and the second inequalities in A1. Therefore, differences in ability are relevant, but much less so than in Hendel et al. (2005), where only high ability individuals can study.

The last inequality in A1 indicates that paying for education entirely by borrowing is too expensive, independently of ability level. Therefore, very poor individuals, i.e., \(b = 0\), never enroll in higher education. This assures that family wealth is also relevant in the education decision, at least for the poorest individuals.

Assumption 2 guarantees that the proportion of high-ability types who can afford the tuition cost is sufficiently large to sustain a college premium that incentivizes low-ability types to study, i.e., \(q_H - q^*(T, \bar{b}) > \frac{T}{P_L}\). Therefore, at least the most affluent individuals of both types study in equilibrium. This avoids the existence of equilibria in which only ability is relevant to the attainment of higher education.

\[8 A1 \text{ guarantees the non-emptiness of the Assumption 2 condition.}\]
For our equilibrium concept, we use perfect Bayesian equilibrium. The first inequality in A1 guarantees that only high-ability types will deviate from a pooling situation in which no one undertakes higher education. Thus, firms’ beliefs, in accordance with the Intuitive Criterion of Cho and Kreps (1987), must be such that if no one studies, any deviator would be seen to be of high-ability. We incorporate this refinement into the construction of the equilibrium. Thus the expected productivity of a college graduate is $\alpha q_H$, in the hypothetical case in which no one receives higher education.

We use $\widetilde{w}^s$, with $s = e, n$, to denote equilibrium wages, with $cp^* = \tilde{w}^e - \tilde{w}^n$, and with $CP(b_H^*, b_L^*) = g^e(b_H^*(cp), b_L^*(cp)) - g^n(b_H^*(cp), b_L^*(cp))$ denoting the equilibrium difference in productivity between those who obtain education and those who do not. As in Hendel et al. (2005), we refine our equilibrium concept with the notion of tâtonnement stability.\(^9\)

**Definition 1. Equilibrium**

A perfect signaling Bayesian equilibrium is a set of education choices, $s^*(i, b) \in \{e, n\}$, based on ability and wealth level, firms’ beliefs $\beta(L) | s$ about ability for a given education choice, and wage formation according to $g^e(b_H^*, b_L^*)$ and $g^n(b_H^*, b_L^*)$, such that:

$$g^n(b_H^*(\tilde{w}^e - \tilde{w}^n), b_L^*(\tilde{w}^e - \tilde{w}^n)) = \tilde{w}^n$$
$$g^e(b_H^*(\tilde{w}^e - \tilde{w}^n), b_L^*(\tilde{w}^e - \tilde{w}^n)) = \tilde{w}^e$$

An equilibrium is stable if and only if:

$$\frac{\partial CP(b_H^*, b_L^*)}{\partial cp^*} < 1$$

In the above definition, we have written productivity as $g^s(b_H^*(\tilde{w}^e - \tilde{w}^n), b_H^*(\tilde{w}^e - \tilde{w}^n))$, to emphasize the fixed point nature of this problem. However, we will generally use the simplified notation $g^s(b_H^*, b_L^*)$.

**Proposition 1.** If $A0$, $A1$ and $A2$ are satisfied, then there is at least one stable equilibrium.

All proofs are left to the appendix. Our combined assumptions exclude separating equilibria. As noted above, this is in line with the evidence suggesting that both wealth and ability are relevant to the attainment of higher education. According to the last inequality in A1, at least the poorest individual of both ability types does not enroll in higher education. At the same time, $A2$ and the Cho-Kreps Intuitive Criterion guarantee the presence of both ability types in higher education. Then, the following proposition arises:

\(^9\)An equilibrium is tâtonnement stable when the college premium returns to the equilibrium level under slight perturbations of $cp^*$.  

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Proposition 2. If \( A_0, A_1 \) and \( A_2 \) are satisfied, then the equilibrium must be pooling, with \( \frac{T_{PL}}{F_L} \leq cp^* \).

We can distinguish two different types of equilibrium outcome, given these propositions: one in which the college premium strictly exceeds the low-ability type’s education cost (i.e., \( cp^* > \frac{T_{PL}}{F_L} \)), and another in which the two coincide (i.e., \( cp^* = \frac{T_{PL}}{F_L} \)). The latter situation is a very unlikely knife-edge case. Hence, we focus on the case where \( cp^* > \frac{T_{PL}}{F_L} \), so that both types borrow in order to study. We call this a borrowing equilibrium, since some individuals of both ability borrow to study when \( cp^* > \frac{T_{PL}}{F_L} \).

We require continuous differentiability of \( F_i(b) \), where \( i = H, L \), to guarantee the stability of a borrowing equilibrium.

Proposition 3. \( A_0, A_1 \) and \( A_2 \), together with the continuous differentiability of \( F_i(b), i = H, L \), for any \( b > T/P_L \), guarantees the stability of a borrowing equilibrium.

4 Skill-Biased Technical Change.

To analyze the effects of skill-biased technical change, we conduct comparative statics with respect to \( \alpha \). The parameter \( \alpha \) represents the productivity enhancement role of higher education. An increase in \( \alpha \) represents a relative increase in the productivity of high-skill workers. Therefore, by conducting comparative statics on \( \alpha \), we model skill-biased technical change.

Proposition 4. Given skill-biased technical change, in a borrowing equilibrium:

i) The college education premium rises;

ii) The non-college wage falls if the cumulative distribution of wealth is log-concave and the likelihood ratio, \( \frac{f_H(b^*H)}{f_L(b^*L)} \), is monotone increasing;

iii) The college wage increases if \( \frac{1-F_H(b^*H)}{1-F_L(b^*L)} < \frac{f_H(b^*H)}{f_L(b^*L)} \).

As seen in this proposition, skill-biased technical change explains the rise in the college education premium. Moreover, because of the ability composition effect, it also accounts for the fall of low-skill wages.

We can divide the overall consequences of skill-biased technical change into two effects: the direct effect of technology and the ability composition effect.

First, skill-biased technical change enhances productivity of higher education, which positively affects college wages and consequently the college premium. This makes higher education more attractive, which in turn triggers the ability composition effect.
The ability composition effect can also be divided into two parts. First, there is the sensitivity of the cut-off level of wealth for each ability type to a change in technology, i.e., $\frac{\partial b^*}{\partial \alpha}$. Second, there is the relative size of the mass of agents concentrated around the wealth cut-off level for each ability type, as these are the individuals who, by changing their education decision, drive the composition effect.

The higher probability of successful college completion of high-ability types is equivalent to obtaining higher returns from higher education. Therefore, high-ability types respond more positively than low-ability types to technological change. In terms of our model, this means that the reduction in the wealth cut-off level is larger for high-ability individuals than for low-ability individuals, i.e., $\frac{\partial b^*}{\partial \alpha} > 0$.

Because equilibrium wages equal average productivity, an increase in the number of high-ability individuals obtaining college education induces a rise in the equilibrium college wage, $\frac{\partial g_e(b^*_H, b^*_L)}{\partial b^*_H} < 0$, and a decline in the non-college wage, $\frac{\partial g_n(b^*_H, b^*_L)}{\partial b^*_L} > 0$. At the same time, an increase in the number of low-ability individuals obtaining higher education has the opposite effect, i.e., $\frac{\partial g_e(b^*_H, b^*_L)}{\partial b^*_L} > 0$ and $\frac{\partial g_n(b^*_H, b^*_L)}{\partial b^*_H} < 0$. Which of these effects dominates depends on the relative sizes of the masses of agents concentrated around the wealth cut-off levels for each ability type, which in turn depends on the properties of the joint distribution of wealth and ability. The following lemma provides insight into these effects and is useful in proving many of our results.

**Lemma 1.** If wealth follows log-concave distribution functions with a monotone increasing likelihood ratio, $f_H(b), \frac{f_H'(b)}{f_L(b)}$, then $|\frac{\partial g^*(b^*_H, b^*_L)}{\partial b^*_H}| \leq |\frac{\partial g^*(b^*_H, b^*_L)}{\partial b^*_L}|$ and $|\frac{\partial g^*(b^*_H, b^*_L)}{\partial b^*_L}| \geq |\frac{\partial g^*(b^*_H, b^*_L)}{\partial b^*_H}|$.

Non-college wages are only affected by the ability composition effect. Combining both parts of the composition effect, we can explain the reduction in low-skill wages. First, the monotonicity of the likelihood ratio, together with log-concavity of the cumulative function, guarantees that the non-college wage is more sensitive to a change in the high-ability wealth threshold than in the low-ability wealth threshold $|\frac{\partial g^*(b^*_H, b^*_L)}{\partial b^*_H}| > |\frac{\partial g^*(b^*_H, b^*_L)}{\partial b^*_L}|$. Combining this with the larger change in the wealth cut-off level of high-ability types seen above, $\frac{\partial b^*_H}{\partial \alpha} > 0$, guarantees the fall in low-skill wages, i.e., $\frac{\partial g^*(b^*_H, b^*_L)}{\partial \alpha} = \frac{\partial g^*(b^*_H, b^*_L)}{\partial b^*_H} \frac{\partial b^*_H}{\partial \alpha} + \frac{\partial g^*(b^*_H, b^*_L)}{\partial b^*_L} \frac{\partial b^*_L}{\partial \alpha} < 0$.

The above conditions are not sufficient to guarantee a particular sign for the change in college wages. Skill-biased technical change has a direct positive impact, but the composition effect is ambiguous. Log-concavity of the surviving function and monotonicity of the likelihood ratio cause the composition effect on college wages to be negative, in accordance with Carneiro and
A positive composition effect guarantees an increase in college wages. In other words, the college wage can fall only if the composition effect is negative and exceeds, in absolute value, the technology effect.

Although the composition effect on college wages can be negative, the overall effect of technical change (i.e., of the two effects combined) on the college premium is always positive. Interestingly, an increase in the college premium may occur even with a reduction in college wages, as long as non-college wages fall further.

5 Changes in Affordability of Higher Education.

The signaling literature has traditionally focused on changes in the affordability of higher education to explain changes in the college premium. Here, we analyze the effects of positive and negative changes in affordability from two sources: an increase in tuition fees and a reduction in the college loan interest rate. Although these two measures have opposite effects on access to higher education, both can potentially generate an increase in the college premium. However, in contrast to skill-biased technical change, this is not necessarily always the case.

**Proposition 5.** Given a change in tuition, then in a borrowing equilibrium:

\[
\frac{\partial \text{CP}(b^*_H, b^*_L)}{\partial T} > 0 \iff |\frac{\partial \text{CP}(b^*_H, b^*_L)}{\partial b^*_L}| > |\frac{\partial \text{CP}(b^*_H, b^*_L)}{\partial b^*_H}| \iff \frac{\partial b^*_L}{\partial T} > \frac{\partial b^*_H}{\partial T}
\]

This proposition states conditions required for an increase in the college premium following a change in tuition. An increase in tuition fees can induce an increase in the college premium. This is the case if and only if low-ability types react strongly to an increase in the price of education.

**Proposition 6.** When an increase in tuition fees leads to an increase in the college premium, both college and non-college wages increase if wealth follows a log-concave distribution and the likelihood ratio is monotone increasing.

Given an increase in tuition fees, an increase in the college premium is not compatible with a reduction in non-college wages.

Because there is now no technological change, all effects come through the composition effect. The composition effect works as in the previous case. The only difference is that, given an increase in \( T \), an increase in the college premium only occurs if \( \frac{\partial b^*_H}{\partial T} > \frac{\partial b^*_L}{\partial T} \). Then, the change in the wealth cut-off level is larger for low-ability types. At the same time, log-concavity

\(^{10}\)The usual assumptions of a log-normal or Pareto distribution of wealth ensure the log-concavity of \( F_i(b) \), but not of \( 1 - F_i(b) \). See Bagnoli and Bergstrom (2005)
and monotone likelihood guarantee that the mass of low-ability types around the wealth cut-off level is larger than for high-ability types. The combination of these assumptions guarantees an increase in non-college wages. Therefore, the increase in tuition can not account for the simultaneous increase in the college premium and decline in low-skill wages.

Given that the increase in the college education premium only occurs simultaneously with an increase in non-college wages, college wages must also increase.

Many programs designed to facilitate access to education involve attaching advantageous conditions to college loans. We can analyze the effects of such policies on the college premium through a comparative statics analysis of interest rate reductions.

**Proposition 7.** If access to higher education is expanded through a reduction in the college loan interest rate, then in a borrowing equilibrium:

i) The college education premium can either increase or decrease; ii) The non-college wage decreases if wealth follows a log-concave distribution function and the likelihood ratio is monotone increasing; iii) The college wage can either increase or decrease.

The reduction in the college loan interest rate does not necessarily cause an increase in the college premium. However, it is always compatible with a reduction in low-skill wages. The intuition for this is very similar to the previous cases. The main difference from the case of tuition changes is that with a reduction in the college loan interest rate, changes in the wealth cut-off of high-ability is larger than in the wealth cut-off of low-ability. i.e., $0 < \frac{\partial b^*_L}{\partial x} < \frac{\partial b^*_H}{\partial x}$.

Hence, a rise in the college education premium is more likely to occur. The higher net returns of college for high-ability types makes them more willing than low-ability types to borrow to pay for higher education. Thus, a reduction in the college loan interest rate represents a larger savings for them. This implies that the indifferent high-ability type’s wealth cut-off level is more sensitive than that of low-ability types to changes in the college loan interest rate.

From this section can find out that differently from skill-biased technical change, changes in affordability do not necessarily lead to an increase in the college premium. However if that is the case, the increase in tuition is not-compatible with the decline in low-skill wages while the reduction of college loan interest rate is.

Finally, the following remark covers the specific case of the independence of wealth and ability, i.e. $F_H(b) = F_L(b) = F(b) \forall b$.

**Remark 1.** If wealth and ability are independent, logarithmic concavity of the cumulative distribution of wealth is sufficient to guarantee the results in Propositions 4, 6 and 7.
If the distribution of wealth is the same for both ability types, we no longer require an assumption regarding the relationship between the wealth distributions of the different ability types. In such cases, there is no need for monotonicity of the likelihood ratio.

6 Conclusions.

Signaling explanations for changes in the college premium have been largely seen as unrelated to skill-biased technical change. Here, we show that these arguments are not only compatible, but also that signaling provides a rationale for the observed reduction in low-skill wages. At the same time, affordability arguments, previously used in the signaling literature, perform more poorly in explaining these changes.

With imperfect credit markets, not only do high-ability types obtain higher education, but high wealth low-ability types do so as well. Thus, the joint distribution of wealth and ability plays an important role in the composition effect. Assuming log-concavity of the cumulative wealth distribution and a monotone likelihood ratio, skill-biased technical change attracts both low-wealth, high-ability individuals and high-wealth, low-ability individuals to higher education, in such a way that average ability among the non-college-educated is reduced. This entails a reduction in low-skill wages.

References


7 Appendix A

7.1 Proof of Proposition 1.

Define the college premium as \( cp = w^c - w^n \). In equilibrium, necessarily \( w^n < w^c \). Then, the set \( CP = [0, \alpha q_H - q_L] \), containing all possible equilibrium values of \( cp = w^c - w^n \), is convex and closed. Consider a mapping from \( CP \) into itself, \( G : CP \to CP \), with \( G(cp) = g^n(b_H^*(cp), b_L^*(cp)) - g^n(b_H^*(cp), b_L^*(cp)) \), where \( G(cp) \) is the reduced form of \( CP(b_H^*, b_L^*) \) in the main text. Given assumptions A0-A2, the Cho-Kreps Intuitive Criterion and individuals’ decisions, \( G(cp) \) is equal to:

\[
G(cp) = \begin{cases} 
\alpha q_H - \bar{q} & \text{if } cp < T \\
\alpha q_H - \frac{\pi_F(h^*_H)(1 - \pi_H) q_L}{\pi_F(h^*_H)(1 - \pi_H) q_L + (1 - \pi_H) q_L} & \text{if } T \leq cp < \frac{T}{P_L} \\
\Lambda - \Gamma & \text{if } \frac{T}{P_L} \leq cp 
\end{cases}
\]

where \( \Lambda = \alpha H \frac{\pi_F(h^*_H)(1 - \pi_H) q_L}{\pi_F(h^*_H)(1 - \pi_H) q_L + (1 - \pi_H) q_L} \) and \( \Gamma = \frac{\pi_F(h^*_H)(1 - \pi_H) q_L}{\pi_F(h^*_H)(1 - \pi_H) q_L + (1 - \pi_H) q_L} \). Notice that \( b_H^* \) and \( b_L^* \) can take any value in the interval \([T, \bar{b}]\), when \( cp = T \) and when \( cp = T/P_L \), respectively. Consequently, \( G(cp) \) is not single valued at these two points.

Because \( G(cp) \) has a closed graph and its image set is bounded, it is upper-hemicontinuous, with the property that the set \( G(cp) \in CP \) is nonempty and convex for every \( cp \). Kakutani’s theorem guarantees the existence of a fixed point.
Call \( W^e \) and \( W^n \) the sets of feasible wages for studying and not studying, respectively. Let \( g : W^e \times W^n \rightarrow W^e \times W^n \) with \( g(w^e, w^n) = (g^e(b^*_H, b^*_L), g^n(b^*_H, b^*_L)) \). Then for any fixed point \( G(cp^*) = cp^* \), there is also a fixed point of \( g(w^e, w^n) \), i.e., \( w^e = g^e(b^*_H(cp^*), b^*_L(cp^*)) \), \( w^n = g^n(b^*_H(cp^*), b^*_L(cp^*)) \).

To show stability, we know by A1 that \( b^*_H(\alpha q_H - q_L) < \bar{b} \). Then, \( g^e(b^*_H(\alpha q_H - q_L), b^*_L(\alpha q_H - q_L)) < \alpha q_H \), while by construction \( g^e(\alpha q_H - q_L) \) can never be lower than \( q_L \). So \( G(\alpha q_H - q_L) < \alpha q_H - q_L \). Because of the non-emptiness, upper-hemicontinuity and convexity of \( G(cp) \), the previous inequality holds for any \( \hat{c}p \) such that \( cp^* \leq \hat{c}p \leq \alpha q_H - q_L \), where \( cp^* \) is a fixed point. We also know that for any college premium \( \hat{c}p \), such that \( \hat{c}p < T \), no one undertakes higher education, i.e., \( b^*_H(\hat{c}p) = \bar{b} \) and \( b^*_L(\hat{c}p) = \bar{b} \). Then, by the Intuitive Criterion refinement, \( g^o(\bar{b}, \bar{b}) = \alpha q_H \), while \( g^o(\bar{b}, \bar{b}) = \bar{q} \). Therefore, \( G(\hat{c}p) = \alpha q_H - \bar{q} > T > \hat{c}p \), where the first inequality arises from A1.

Given that \( G(\hat{c}p) < \hat{c}p \) for all \( \hat{c}p \geq cp^* \) and that \( G(\hat{c}p) > \hat{c}p \), the function \( G(cp) \) must cross the 45° line from above at least at one fix point, which guarantees stability.

### 7.2 Proof of Proposition 2.

From Proposition 1, we know that there is at least one equilibrium. The first inequality in A1 and the Intuitive Criterion imply that \( G(cp) > cp \) for any \( cp \in [0, T) \). A2 implies that \( G(cp) > cp \) for any \( cp \in \{T, T/P_L\} \). Therefore, all equilibria must satisfy \( cp^* \geq T/P_L \).

According to the last inequality in A1, paying the whole cost of college by credit is too costly, independently of ability level. Therefore, there is no separating equilibrium, with only one ability-type not attending college.

Given that \( cp^* \geq T/P_L \), some individuals of both types have incentives to study in equilibrium. This eliminates the possibility of a pooling equilibrium in which only high-ability types obtain higher education.

Therefore, the equilibrium must be a pooling equilibrium.

### 7.3 Proof of Proposition 3.

We know from the proof of Proposition 2 that \( G(cp) > cp \) for all \( cp \in [0, T/P_L) \). From the proof of Proposition 1, we know that \( G(\alpha q_H - q_L) < \alpha q_H - q_L \). Thus, the upper-hemicontinuity, non-emptiness and convexity of \( G(cp) \) guarantee that \( G(cp) \) crosses the 45° line from above at least once, for \( cp \in \{T/P_L, \alpha q_H - q_L\} \), which is sufficient for stability.

Notice that there is a continuum of images for \( cp = T/P_L \). We call \( \min\{G(T/P_L)\} \) to the lower value of these images.

If \( \min\{G(T/P_L)\} \geq T/P_L \), then \( G(cp) \) does not cross the 45° line at \( cp = T/P_L \). This
guarantees that \( G(cp) \) crosses the 45° line from above at the borrowing equilibrium closest to \( T/PL \).

If \( \min G(T/PL) < T/PL \), the top left candidate to a borrowing equilibrium, \( \hat{c}p \), cannot be stable. Since we have assumed \( F_i(b), i = H, L \) to be continuously differentiable, we can apply the transversality theorem to \( G(\hat{c}p) - \hat{c}p \). It guarantees that \( G(cp) \) crosses the 45° line at \( \hat{c}p \) for almost any choice of \( T \) and \( x \). But then, \( G(cp) > cp \) for some \( cp > \hat{c}p \), which together with the fact that \( G(qH - qL) < qH - qL \), implies that \( G(cp) \) must cross again (and from above) the 45° line, which brings stability.

### 7.4 Proof of Proposition 4.

i) College premium.

In equilibrium \( b^*_i = T(1 + x) - PL(H_i, b^*_L) \), \( i = H, L \). Then \( \partial b^*_i/\partial x = -P_i \partial CP(b^*_H, b^*_L)/\partial x \). Using these expressions in \( \partial CP(b^*_H, b^*_L)/\partial x = p^*(cp)/\alpha + \partial CP(b^*_H, b^*_L)/\partial b^*_H + \partial CP(b^*_H, b^*_L)/\partial b^*_L \) we obtain:

\[
\frac{\partial CP(b^*_H, b^*_L)}{\partial x} = \frac{p^*(cp)/\alpha}{1 + \frac{\partial CP(b^*_H, b^*_L)}{\partial b^*_H} \frac{P_i}{x} + \frac{\partial CP(b^*_H, b^*_L)}{\partial b^*_L} \frac{P_i}{x}}
\]

Stability (i.e., \( \partial CP(b^*_H, b^*_L)/\partial cp < 1 \)) guarantees that \( 1 + \frac{\partial CP(b^*_H, b^*_L)}{\partial b^*_H} \frac{P_i}{x} + \frac{\partial CP(b^*_H, b^*_L)}{\partial b^*_L} \frac{P_i}{x} > 1 \). Consequently, the college premium increases.

ii) Changes in Wages.

Using the chain rule to obtain \( \partial g^*(b^*_H, b^*_L)/\partial x, s = e, n, \) and \( \partial b^*_i/\partial x = -P_i \partial CP(b^*_H, b^*_L)/\partial x \), \( i = H, L \), and solving the system of equations, we obtain:

\[
\frac{\partial b^*_i}{\partial x} = \left( \frac{\partial g^*(b^*_H, b^*_L)}{\partial b^*_H} + \frac{\partial g^*(b^*_H, b^*_L)}{\partial b^*_L} P_i \right) \frac{p^*(b^*_H, b^*_L)}{\alpha}
\]

\[
\frac{\partial p^*(b^*_H, b^*_L)}{\partial x} = \left( \frac{\partial g^*(b^*_H, b^*_L)}{\partial b^*_H} + \frac{\partial g^*(b^*_H, b^*_L)}{\partial b^*_L} P_i \right) \frac{p^*(b^*_H, b^*_L)}{\alpha}
\]

The non-college wage is affected only by the composition effect. Lemma 1 guarantees that the numerator is positive, i.e., \( \frac{\partial g^*(b^*_H, b^*_L)}{\partial b^*_H} + \frac{\partial g^*(b^*_H, b^*_L)}{\partial b^*_L} P_i > 0 \). The stability condition guarantees that the denominator is also positive. Hence, we conclude that \( \frac{\partial g^*(b^*_H, b^*_L)}{\partial b^*_i} < 0 \).

These conditions are not sufficient to determine the sign of the change in the college wage. From the above, we can see that \( \frac{\partial g^*(b^*_H, b^*_L)}{\partial b^*_H} > 0 \) if and only if \( \frac{\partial g^*(b^*_H, b^*_L)}{\partial b^*_H} + \frac{\partial g^*(b^*_H, b^*_L)}{\partial b^*_L} P_i - x < 0 \). This can be written as \( \frac{\partial g^*(b^*_H, b^*_L)}{\partial cp} > -1 \). Stability requires that \( \frac{\partial g^*(b^*_H, b^*_L)}{\partial cp} - \frac{\partial g^*(b^*_H, b^*_L)}{\partial cp} < 1 \), which means \( \frac{\partial g^*(b^*_H, b^*_L)}{\partial cp} > -1 \) is a sufficient condition to guarantee that \( \frac{\partial g^*(b^*_H, b^*_L)}{\partial cp} > -1 \) is satisfied. Using

\[11\] To check that \( D(G(cp) - 1) \) is full rank we can use the expression in the proof of proposition 5 for \( \frac{\partial CP(b^*_H, b^*_L)}{\partial cp} \) and

\[
\frac{\partial CP(b^*_H, b^*_L)}{\partial x} = \frac{\partial CP(b^*_H, b^*_L)}{\partial b^*_H} \left( \frac{x(H - q_i)}{x - \frac{P_i}{x}} \right) + \frac{\partial CP(b^*_H, b^*_L)}{\partial b^*_L} \left( \frac{x(H - q_i)}{x - \frac{P_i}{x}} \right) + \frac{\partial CP(b^*_H, b^*_L)}{\partial b^*_H} \left( \frac{x(H - q_i)}{x - \frac{P_i}{x}} \right) + \frac{\partial CP(b^*_H, b^*_L)}{\partial b^*_L} \left( \frac{x(H - q_i)}{x - \frac{P_i}{x}} \right).
\]
the expressions in the proof of lemma 1 for \( \frac{\partial g(b_H, b_L)}{\partial b_i} \), for \( i = H, L \), this can be rewritten as:

\[
\frac{1-F_H(b_H)}{1-F_L(b_L)} < f_H(b_H) - f_L(b_L)
\]

### 7.5 Proof of Lemma 1.

Computing the derivative of each productivity with respect to \( b_H^* \) and \( b_L^* \):

\[
\frac{\partial g(b_H^*, b_L^*)}{\partial b_H} = \frac{\alpha(1-F_L(b_L^*))f_H(b_H^*)P_L(q_H-q_L)\pi(1-\pi)}{(P_L(1-P_L)(1-\pi)+(1-F_H(b_H^*))\pi)^2} < 0
\]

\[
\frac{\partial g(b_H^*, b_L^*)}{\partial b_L} = \frac{(1-P_L(1-P_L)(1-\pi)+(1-F_H(b_H^*))\pi)^2}{(1-P_L(1-P_L)(1-\pi)+F_H(b_H^*))^2} > 0
\]

Using these expressions we can see that \( \frac{\partial g(b_H^*, b_L^*)}{\partial b_H} \leq |\frac{\partial g(b_H^*, b_L^*)}{\partial b_L}| \) and \( |\frac{\partial g(b_H^*, b_L^*)}{\partial b_H}| \geq |\frac{\partial g(b_H^*, b_L^*)}{\partial b_L}| \)

if the following holds,

\[
P_L \frac{F_H(b_H^*)}{1-F_L(b_L^*)} < \frac{f_H(b_H^*)}{f_L(b_L^*)} \frac{1-F_H(b_H^*)}{1-F_L(b_L^*)} \quad (a)
\]

The monotone-likelihood ratio property implies that \( F_{b_L}(b_H) < f_{b_L}(b_H) < \frac{F_H(b_H)}{f_H(b_H)} \) for all \( b \). While log-concavity guarantees that \( F_{b_L}(b_H) < f_{b_L}(b_H) < \frac{F_H(b_H)}{f_H(b_H)} \) for \( i = H, L \). The combination of these two conditions guarantees that \( (a) \) is satisfied.

### 7.6 Proof of Proposition 5.

The derivative of the college premium with respect to \( T \) is

\[
\frac{\partial CP(b_H^*, b_L^*)}{\partial T} = \frac{\partial CP(b_H^*, b_L^*)}{\partial b_H} \frac{\partial b_H^*}{\partial T} + \frac{\partial CP(b_H^*, b_L^*)}{\partial b_L} \frac{\partial b_L^*}{\partial T}
\]

Given that \( cp^* > \frac{T}{F_L} \), we know that the wealth threshold level for each type is \( b_i^* = \frac{T(1+x)-CP(b_H^*, b_L^*)}{x} \). Writing the derivative with respect to tuition, \( \frac{\partial b_i^*}{\partial T} = \frac{1+x}{x} - \frac{P_i}{x} \frac{\partial CP(b_H^*, b_L^*)}{\partial T} \).

Using this expression in the derivative of the college premium we can see that

\[
\frac{\partial CP(b_H^*, b_L^*)}{\partial T} = \frac{1+x}{x} \left[ \frac{\partial CP(b_H^*, b_L^*)}{\partial b_H} + \frac{\partial CP(b_H^*, b_L^*)}{\partial b_L} \right]
\]

Stability guarantees that the denominator of the previous expression is positive. Given that \( \frac{\partial CP(b_H^*, b_L^*)}{\partial b_H} < 0 \) and \( \frac{\partial CP(b_H^*, b_L^*)}{\partial b_L} > 0 \), the following condition is necessary and sufficient for \( \frac{\partial CP(b_H^*, b_L^*)}{\partial T} \geq 0 \),

\[
|\frac{\partial CP(b_H^*, b_L^*)}{\partial b_H}| \leq |\frac{\partial CP(b_H^*, b_L^*)}{\partial b_L}|.
\]

Looking at the derivatives of the indifferent individuals’ wealth (\( \frac{\partial b_i^*}{\partial T} \)) and given that \( P_H > P_L \) it is immediate that \( \frac{\partial b_H^*}{\partial T} \geq \frac{\partial b_L^*}{\partial T} \iff \frac{\partial CP(b_H^*, b_L^*)}{\partial T} \geq 0 \).

Finally, if \( \frac{\partial b_i^*}{\partial T} \) is negative for both \( i = H, L \), it must be the case that \( \frac{\partial CP(b_H^*, b_L^*)}{\partial T} \geq 0 \) which is equivalent to \( \frac{\partial b_H^*}{\partial T} \geq \frac{\partial b_L^*}{\partial T} \).
Finally the combination of $\frac{\partial g}{\partial b_0}$. Using the chain rule and previous inequalities, we see that
\[
\frac{\partial g}{\partial b_0} + \frac{\partial g}{\partial b_1} > 0.
\]

Proceeding exactly as in the proof of Lemma 1, it is easy to see that the sufficient and necessary

\[ 7.7 \text{ Proof of Proposition 6.} \]

We know from Proposition 5 that when the college premium increases after an increase in tuition: $\frac{\partial g}{\partial T} < \frac{\partial g}{\partial T}$. At the same time Lemma 1 guarantees that if monotone likelihood and log-concavity are satisfied then $\left| \frac{\partial g}{\partial b_0} \right| > 0$.

Using the chain rule and previous inequalities, we see that
\[
\frac{\partial g}{\partial b_0} + \frac{\partial g}{\partial b_1} + \frac{\partial g}{\partial b_2} > 0.
\]

Finally the combination of $\frac{\partial g}{\partial b_1} > 0$ and $\frac{\partial g}{\partial b_2} > 0$ automatically guarantee that $\frac{\partial g}{\partial T} > 0$.

\[ 7.8 \text{ Proof of Proposition 7.} \]

The derivative of $b_i$ with respect to the interest rate can be written: $\frac{\partial b_i}{\partial x} = \frac{T}{x^2} + P_i \left( \frac{CP(b_i^*, b_j^*)}{x^2} - \frac{1}{x} \frac{\partial CP(b_i^*, b_j^*)}{\partial x} \right)$. Using it in the derivative of the equilibrium college premium we obtain,
\[
\frac{\partial CP}{\partial x} = \frac{\partial CP(b_i^*, b_j^*)}{\partial b_i} \frac{\partial b_i}{\partial x} = \frac{\partial CP(b_i^*, b_j^*)}{\partial b_i} \left( \frac{1}{x^2} \left( \frac{\partial CP}{\partial b_i} \right) - \frac{T}{x^2} \right) + \frac{\partial CP(b_i^*, b_j^*)}{\partial b_j} \left( \frac{P_i}{x^2} \frac{\partial CP}{\partial b_i} \right) \frac{T}{x^2}.
\]

Since we know by stability that the denominator is always positive, the previous derivative is positive if and only if $\left( \frac{\partial CP(b_i^*, b_j^*)}{\partial b_i} \right) - \frac{T}{x^2} \frac{\partial CP}{\partial b_i} (P_i - P_j) \frac{T}{x^2} > 0$.

To analyze the change in each wage, first we can see that:
\[
\frac{\partial b_i}{\partial x} = \frac{1}{x^2} \left( P_i CP(b_i^*, b_j^*) - T \right) + \frac{\partial CP(b_i^*, b_j^*)}{\partial b_i} \left( P_i - P_j \right) \frac{T}{x^2} > 0
\]

where $i = H, L$ and $j \neq i$.

The positive value of these expressions arise from the fact that $\frac{\partial CP(b_i^*, b_j^*)}{\partial b_i} < 0$ and $\frac{\partial CP(b_i^*, b_j^*)}{\partial b_j} > 0, P_L < 1$ and that we are in an equilibrium with $CP(b_i^*, b_j^*) > \frac{T}{x^2}$, which guarantee the positive value of the numerator. On the other hand stability guarantees that the denominator is also positive.

Given that $\frac{\partial b_i}{\partial x} > 0$ for both $i = H, L$, we can use $\frac{\partial b_i}{\partial x} = \frac{T}{x^2} + P_i \left( \frac{CP(b_i^*, b_j^*)}{x^2} - \frac{1}{x} \frac{\partial CP(b_i^*, b_j^*)}{\partial x} \right)$ to conclude that $\frac{\partial b_i}{\partial x} > \frac{\partial b_i}{\partial x}$. Then, using Lemma 1 we can proceed as in the proof of Proposition 4 to show that log-concavity and monotone likelihood are sufficient to guarantee the reduction of low-skill wages. At the same time, this is not sufficient to obtain a particular sign for the derivative of college wages.

\[ 7.9 \text{ Proof of Remark 1.} \]

Proceeding exactly as in the proof of Lemma 1, it is easy to see that the sufficient and necessary condition to guarantee the same change on each productivity $\frac{\partial g}{\partial e_i} s = e, n, i = H, L$
\[
\frac{F(b_L^*)}{F(b_H^*)} < \frac{f(b_L^*)}{f(b_H^*)} < \frac{1-F(b_L^*)}{1-F(b_H^*)}
\]
which is directly guaranteed by log-concavity. Changes on wealth cut-off levels are exactly as in the previous proofs.

8 Appendix B.

Figure 1: College Enrollment, College Premium and Wages.

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<td><img src="image1.png" alt="Graph" /></td>
<td><img src="image2.png" alt="Graph" /></td>
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</table>

Source:

College Enrollment (as a percentage of all 18 to 24 year-old): National Center for Education and Statistics (NCES). Digest of Education Statistics, 2009. Table 204.