1. \( \text{a) (1.5 points)} \) Consider the differential equation

\[
\frac{dy}{dt} = y^3 - y
\]

Plot the corresponding phase line, find the equilibrium solutions and determine their type. Sketch the solution that passes through the point \((t, y) = (0, 1/2)\).

\( \text{b) (1.5 points)} \) Sketch the bifurcation diagram for the differential equation

\[
\frac{dy}{dt} = y^3 - ay
\]

with \(a\) a parameter. Show the corresponding phase lines.
2. Consider the initial value problem

\[
(*) \quad \begin{cases}
    \frac{dy}{dt} = y/2 + t/2 \\
y(0) = 1.
\end{cases}
\]

a) (1.5 points) Calculate the approximation to \(y(1)\) given by Euler method with \(\Delta t = 0.5\).

b) (1.5 points) Knowing that \(y = -t - 2\) is a particular solution to \(\frac{dy}{dt} = y/2 + t/2\), calculate the exact value of \(y(1)\) for the initial value problem \((*)\).

c) (1 point) Taking into account (a) and (b), what would be your guess for the error in the approximation to \(y(1)\) given by Euler method with \(\Delta t = 0.005\)?
3. A tank contains 40 gallons of pure water. Brine with 3 pounds of salt per gallon flows in at a rate of 2 gallons/minute, and the stirred mixture flows out at 3 gallons/minute.

   a) (1.5 points) Find the amount of salt in the tank when the brine in it has been reduced to 20 gallons.

   b) (1.5 points) When is the amount of salt in the tank largest?