Recent Advances in Operator Related Function Theory II
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Abstracts

Alexandru Aleman, Lund University (aleman@maths.lth.se)
Weak product decompositions for functions in Hardy spaces on the polydisc

ABSTRACT: It is well known that an H^1-function can be written as the product of two functions in H^2. It is a theorem of Sarason that this fact continues to hold true for operator valued functions and very recently, Ferguson and Lacey have found a similar decomposition on the bidisc. Such decompositions are related to boundedness and compactness of Hankel operators and in this talk we shall present an operator-theoretic approach to this problem based on Sarason's theorem mentioned above. (Joint work with Olivia Constantin.)

James Milne Anderson, University College London
Another univalency criterion

ABSTRACT: There are several criteria for univalency in a domain D which imply that D is a quasi-conformal circle. Here we discuss a criterion which does not imply that D is a quasi-conformal circle but has instead, the interior chord-arc property. (Joint work with Jochen Becker.)

Albert Baernstein, Washington University in St Louis (al@dax.wustl.edu)
Hölder continuity of a class of quasiregular maps in the plane

ABSTRACT: Let f be a $K-$quasiregular mapping from a plane domain Ω into the plane. A classical result asserts that f satisfies a local Hölder condition of order $1/K$, and that $1/K$ is the largest possible exponent which works for all such f. We prove, though, that $1/K$ can be improved to a value $\alpha_K > 1/K$ under the additional assumption that the complex derivative $f_{\overline{z}}$ be real valued in Ω. We also offer a conjecture for the best possible value of α_K, and prove it in special cases. An equivalent form of our improvement may be stated as follows: Let a, b, c be real valued measurable functions in Ω, and assume that the eigenvalues of the matrix $\begin{pmatrix} a & b \\ b & c \end{pmatrix}$ lie almost everywhere between $K^{-1/2}$ and $K^{1/2}$. Then each L^2-strong solution u of the elliptic pde $au_{xx} + 2bu_{xy} + cu_{yy} = 0$ belongs locally to C^{1,α_K}, where $\alpha_K > 1/K$.

Roger W. Barnard, Texas Tech University (roger.w.barnard@ttu.edu)
A minimal area problem for nonvanishing functions

ABSTRACT: We find the minimal area covered by the image of the unit disk for nonvanishing functions normalized by conditions $f(0) = 0$, $f'(0) = a$. The method is to intertwine two different approaches and use each to bootstrap on the other, thus contributing to the complete solution of the problem. The first approach reduces the problem, via symmetrization, to the class of typically real functions, where we can employ the well known integral representation to obtain a solution upon a priori knowledge of the extremal function. The second approach requiring smoothness assumptions, leads via some variational formulas to a boundary value problem for analytic functions. We are then able to obtain the complicated but explicit solution to the problem.

Dimitrios Betsakos, Aristotle University of Thessaloniki (betsakos@auth.gr)
Polarization, symmetrization, and harmonic measure

Abstract: We will first review the classical results on the behavior of harmonic measure under polarization and symmetrization. Then we will introduce the harmonic measure associated with symmetric stable processes and we will see how the above results are generalized for this harmonic measure. Special difficulties arise for the equality statements if we do not assume regularity for the Dirichlet problem.

James E. Brennan, University of Kentucky (brennan@ms.uky.edu)
Thomson's Theorem and mean square polynomial approximation

Abstract: Given a positive measure μ of compact support in the complex plane $\mathbb{C}$, are the polynomials dense in $L^2(\mu)$; if not why not? Over the years that question has been the subject of an intense investigation, not only for its own sake, but due also to its connection with the invariant subspace problem for subnormal operators on a Hilbert space.

In 1991 J.E. Thomson determined completely the structure of $H^2(\mu)$, the closed subspace of $L^2(\mu)$ which is spanned by the polynomials. As a consequence he was able to show that if $H^2(\mu) \neq L^2(\mu)$ then every function $f \in H^2(\mu)$ admits an analytic extension to a fixed open set Ω, thereby confirming in full generality a phenomenon noted earlier in various situations by S.N. Bernštein, S.N. Mergeljan, and others. In this talk I shall outline a new proof of Thomson's results which is based on Tolsa's recent work on the semiadditivity of analytic capacity, which gives more information, and is applicable to other problems as well.

Patrick Brown, University at Albany, State University of New York (Patdumpsite_school@hotmail.com)
Uniqueness theorems for inner functions

Abstract: This research comes from seeking out boundary conditions that allow two inner functions to generate a dense subalgebra of H^p (the Hardy Spaces). We will discuss sufficient conditions under which two inner functions are identical or are equal up to an automorphism (and thus generate the same algebra) under certain assumptions of the behavior on the boundary.

Martín Chuaqui, Pontificia Universidad Católica de Chile (mchuaqui@mat.puc.cl)
Injectivity criteria for holomorphic curves in $\mathbb{C}^n$

Abstract: Let $\phi : \mathbb{D} \to \mathbb{C}^n$ be a holomorphic mapping with $\phi'(z) \neq 0$. By appealing to Ahlfors' definition of Schwarzian derivative for parametrized curves $\varphi : (a, b) \to \mathbb{R}^m$, we are able to study the global univalence of ϕ by analyzing its restriction to hyperbolic geodesics in $\mathbb{D}$. In turn, Ahlfors' Schwarzian of the restriction of ϕ to such curves can be expressed in terms of the Osgood-Stowe Schwarzian of ϕ as a conformal map, and the second fundamental form of the image surface $\phi(\mathbb{D})$. The result is a generalization to this context of Nehari's p-criterion for univalence. (Joint work with P. Duren and B. Osgood.)

Olivia Constantin, Lund University (olivia@maths.lth.se)
Atomic decompositions for vector-valued Bergman spaces with Békollé-type weights

Abstract: We prove a general atomic decomposition theorem for vector-valued Bergman spaces $L^p_a(\omega, X)$, $p > 0$, with Békollé-type weights and we present some applications. (Joint work with Alexandru Aleman.)

Manuel D. Contreras, Universidad de Sevilla (contreras@esi.us.es)
Iteration of analytic functions of the unit disk and fixed points

Abstract: Let φ be an analytic self-map of the unit disk. A point b belonging to the boundary of the

unit disk is a (boundary) fixed point of φ if the angular limit of φ at b is equal to b. Our aim in this talk is to study the relationship between iteration and fixed points. In the first part of the talk, we are going to show that, in general, a boundary fixed point of an analytic function is not a boundary fixed points of all of its iterates. However, in the context of fractional iteration, all the iterates have the same fixed points.

In the second part of the talk, we are going to analyze the fixed points in terms of the Koenigs function associated to φ and, again in the context of fractional iteration, in terms of the infinitesimal generator. As an application, we obtain some inequalities related to angular derivatives at boundary contact points for arbitrary analytic self-maps of the unit disk. These and other results will appear in two papers in collaboration with professors S. Díaz-Madrigal and Christian Pommerenke.

Carl Cowen, Indiana University - Purdue University Indianapolis (ccowen@iupui.edu)
Weighted composition operators on functional Hilbert spaces

Abstract: We consider a Hilbert space of functions analytic in a domain in C^N, for example, the Hardy Hilbert space on the unit disk, $H^2(D)$. If ψ is a complex valued function on the domain and φ is an analytic map of the domain into itself, the weighted composition operator $W_{\psi,\varphi}$ is defined by

$$(W_{\psi,\varphi}f)(z) = \psi(z)f(\varphi(z))$$

for z in the domain and f a function from the Hilbert space.

Over the past three decades, it has become increasingly apparent that weighted composition operators arise in important ways in a variety of contexts. For example, in 1964, Frank Forelli showed that every isometry of $H^p(D)$, for $1 < p < \infty$ but $p \neq 2$, is a weighted composition operator. Other examples are being discussed at this conference by other speakers.

This talk will describe recent results from the thesis of Gajath Gunatillake on basic properties of weighted composition operators, specifically on questions of boundedness, compactness, and spectra of compact weighted composition operators. Finally, work of Eung Il Ko and the speaker on a special class of weighted composition operators on $H^2(D)$ will be presented.

Željko Čučković, University of Toledo (zcuckovi@math.utoledo.edu)
Weighted composition operators on Bergman spaces

Abstract: We characterize bounded and compact weighted composition operators acting between different Bergman spaces. Our results use certain integral transforms that generalize the Berezin transform. We also estimate the essential norms of these operators. (Joint work with Ruhan Zhao.)

Eva Gallardo-Gutiérrez, Universidad de Zaragoza (eva@unizar.es)
The adjoint of a composition operator and beyond

Abstract: If $\mathcal{H}$ is a Hilbert space of analytic functions on the unit disk and φ is an analytic function mapping the disk into itself, then for f in $\mathcal{H}$, the equation

$$C_\varphi f = f \circ \varphi$$

defines a composition operator on $\mathcal{H}$. On many common functional Hilbert spaces, conditions for boundedness and compactness of composition operators and results about their spectra and cyclicity are known. For example, the Littlewood Subordination Principle shows that C_φ is bounded on the Hardy space for any analytic function φ that maps the unit disk into itself. In spite of this progress, many interesting and seemingly basic problems remain open. This talk will discuss the problem of the adjoint of a composition operator induced by arbitrary analytic function mapping the unit disk into itself, and how this description, in turn, inspires the definition of a new type of operators on functional Hilbert or Banach spaces that

generalizes standard composition or weighted composition operators. (Joint work with Carl C. Cowen.)

Stephan R. Garcia, University of California, Santa Barbara (garcias@math.ucsb.edu)
Variational principles, the Friedrichs operator, and harmonic conjugation

Abstract: The class of complex symmetric operators includes all normal operators, Hankel operators, compressed Toeplitz (including the compressed shift), and many well-known integral and differential operators (such as the Volterra operator). A recent refinement of the polar decomposition for such operators leads to new "minimax" principles for singular values. We consider an application to the Friedrichs operator of a domain and show how this relates to finding optimal bounds for harmonic conjugation.

Christopher N. Hammond, Connecticut College (cnham@conncoll.edu)
Zeros of hypergeometric functions and the norm of a composition operator

Abstract: Let φ be an analytic self-map of the unit disk; let C_φ denote the corresponding composition operator acting on the Hardy space H^2. The precise value of the norm of C_φ is quite difficult to calculate, even though we have sharp upper and lower bounds in terms of $|\varphi(0)|$. In the last few years, some progress has been made in the situation where φ is a linear fractional map. A new paper by Basor and Retsek demonstrates a connection between the norm of such an operator and the zeros of a particular hypergeometric series. This talk will pursue this line of inquiry further. We will appeal to several results relating to hypergeometric series – most of which date back to the first decade of the 20th century – to deduce more information about the norm of a composition operator. Furthermore, we will use our knowledge of composition operators to establish a (possibly) new result pertaining to the zeros of hypergeometric series.

Haakan Hedenmalm, Royal Institute of Technology in Stockholm (haakanh@kth.se)
To be announced

Rodrigo Hernández, Universidad Adolfo Ibáñez (rodrigo.hernandez@uai.cl)
Schwarzian derivatives and a linearly invariant family in $\mathbb{C}^n$

Abstract: Using the definition given by T. Oda of Schwarzian derivatives of a locally univalent holomorphic mapping F in several complex variables we define a Schwarzian derivative operator $\mathcal{S}F$. We use the Bergman metric to define a norm $\|\mathcal{S}F\|$ for this operator, which in the ball is invariant under composition with automorphisms. We study the linearly invariant family $\mathcal{F}_\alpha = \{F : \mathbb{B}^n \to \mathbb{C}^n \mid F(0) = 0,\, DF(0) = \text{Id},\, \|\mathcal{S}F\| \leq \alpha\}$, and estimate its order and norm order.

Dmitry Khavinson, National Science Foundation (dkhavins@nsf.gov)
Valency of some polynomial and rational harmonic functions and ... gravitational microlensing ?! (thin lenses)

Abstract: The Fundamental Theorem of Algebra first rigorously proved by Gauss states that each complex polynomial of degree n has precisely n complex roots. In recent years various extensions of this celebrated result have been considered. In this talk we discuss the extension of the FT of algebra to harmonic polynomials of degree n. In particular, a recent theorem of D. Khavinson and G. Swiatek proves that the harmonic polynomial $\overline{z} - p(z)$, deg $p = n > 1$ has at most $3n - 2$ roots as was conjectured in the early 90's by T. Sheil-Small and A. Wilmshurst. The case $n = 3$ was settled by B. Crofoot and D. Sarason. Unexpectedly, the proof of the general result involves complex dynamical systems. Another main ingredient is the version of the argument principle for harmonic mappings proved by P. Duren, R. Laugesen and W. Hengartner. Still nothing is known for harmonic polynomials with conjugate degree of $\overline{z}$ larger than 1. Last year G. Neumann and D. Khavinson showed that the maximal number of ze-

ros of rational harmonic functions $\overline{z} - r(z)$, deg $r = n > 1$ is $5n - 5$. It turned out that this result resolved the conjecture by several astrophysists dealing with the estimate on maximal number of images of a star if the light from it is deflected by n co-planar masses. The first nontrivial case of one mass was already studied by A. Einstein in the 30s. Further applications and open problems will be discussed as well.

Yuk J. Leung, University of Delaware (yleung@math.udel.edu)
On an isoperimetric problem in conformal mapping

ABSTRACT: Let Ω be a simply connected domain bounded by a rectifiable Jordan curve of given length $l > 4$. We would like to identify the extremal Ω containing the points $w = -1$ and $w = 1$ in its interior such that the hyperbolic distance between them is a minimum. By Schiffer's variation, such an extremal domain is real symmetric and can be shown to be a quadrature domain with respect to arc-length. If $w = f(z)$ is the conformal map from the unit disk D to Ω with $f'(0) > 0$, and $z = \pm r$ corresponding to $w = \pm 1$ respectively for some r in $(0, 1)$, then this mapping function is related to the first eigenfunction g in the integral equation

$$\int_{-r}^{r} \frac{g(t)}{(1 - xt)}\, dt = \lambda g(x).$$

Using an extremal kernel considered by Macintyre and Rogonsinski (See Chapter 8 in Duren's book on H^p Spaces), the left hand side can be written as

$$\frac{1}{2\pi i} \int_{|z|=1} \frac{g(z)K(z)}{1 - xz}\, dz$$

The function $K(z)$ has the property of mapping the unit disk D slitted along the line segment $[-r, r]$ onto an infinite horizontal strip with a circular disk removed in the middle. The values of r, λ and hence the mapping function $f(z)$ can be computed quite easily by constructing $K(z)$ numerically using Bergman Kernel method when l is much larger than 4. However as l gets closer to 4 or as r gets closer to 1, the integral equation becomes ill conditioned.

Following a method of Alberto Grunbaum in estimating the eigenvalues of a finite Hilbert matrix, we show that the integral operator above commutes with a self-adjoint second order differential operator of Heun type:

$$D(y(x)) = ((1 - r^2x^2)(1 - \frac{x^2}{r^2})y'(x))' + 2x^2y(x)$$

As a result, the infinite Hilbert-like matrix generated by the integral operator commutes with an infinite Jacobi matrix generated by the differential operator. Using a little bit of Schwarz-Christoffel theory, we show that a certain quotient of two linearly independent solutions of the Heun differential equation $D(y(x)) = \mu y(x)$ for the positive accessory parameter μ gives back the kernel function K mentioned above. We should note that the other eigenfunction solutions to this differential equation correspond to negative μ's.

All the results above originated from a harmonic measure problem posed by Glenn Schober to the author. The integral operator appeared above is a special case of the Gabriel's problem posed by Beurling, and has been considered in Shapiro's work on doubly orthogonal functions and serves as a concrete example illustrating the n-width problem considered by Fisher. The appearance of an integral operator together with a commuting differential operator was considered by Pollak and Slepian a while back. Lucky for them, their differential equation has two finite singular points. One of Shapiro's classics "Topics in Approximation" has an elegant discussion of it. If the length constraint of the domain Ω is switched to area, then separately Beurling and Gronwall showed that the solution is simply the image of the unit disk under the mapping $w = \log(\frac{1+sz}{1-sz})$ for some s that can be computed easily in terms of the given area.

Incidentally, this is a famous quadrature domain example with respect to area measure due to Philip Davis.

María J. Martín, Universidad Autónoma de Madrid (mjose.martin@uam.es)
Isometries of the Bloch space among the composition operators

ABSTRACT: The isometries among the composition operators acting on the Bloch space will be characterized in terms of the hyperbolic derivative and cluster set of the symbol of the operator. We will also prove that those thin Blaschke products which fix the origin induce such isometric composition operators. (Joint work with Dragan Vukotić.)

Alec Matheson, Lamar University (matheson@gauss.lamar.edu)
Clark measures, Aleksandrov operators and related questions

ABSTRACT: Various problems related to the action of the Aleksandrov operators on H^1 and composition operators on $BMOA$ are investigated. This will include a compactness criterion and some results on approximation from inside the circle. (Joint work with Joseph A. Cima and Michael Stessin.)

Artur Nicolau, Universidad Autónoma de Barcelona (artur@mat.uab.es)
Approximating inner functions

ABSTRACT: Several open problems concerning approximation of inner functions by interpolating Blaschke products will be presented. It will be proved that the modulus of an inner function can be uniformly approximated in the unit disk by the modulus of an interpolating Blaschke product. (Joint work with Geir Arne Hjelle.)

Maria Nowak, Maria Curie-Skłodowska University (nowakm@golem.umcs.lublin.pl)
Symmetric sets of sampling and interpolation for Bergman spaces

ABSTRACT: Let $\{r_n\}$ be a monotonically increasing sequence of positive numbers converging to 1. We say that the set Γ of points in the unit disk $\mathbb{D}$ is a symmetric set based on $\{r_n\}$ if Γ consists of $\lfloor \frac{\gamma}{1-r_n} \rfloor$, $\gamma > 0$, points symmetrically placed on each circle $|z| = r_n$, $n = 1, 2, \ldots$. If $\sup_n \frac{1-r_{n+1}}{1-r_n} < 1$, then a symmetric set based on $\{r_n\}$ is uniformly discrete and so is an interpolation sequence for some Bergman space A^p. If $\inf_n \frac{1-r_{n+1}}{1-r_n} > 0$, then a symmetric set based on $\{r_n\}$ is an ε net and so is a sampling sequence for some A^p. In the case when $\frac{1-r_{n+1}}{1-r_n} = \beta^{-1}, \beta > 1$, we get the A^p zero sequence considered by Luecking [Complex Variables, 30(1996), 345-362]. For the Luecking sequence Γ we construct the function G (which is essentially a Blaschke product) satisfying the condition

$$|G(z)| \simeq \rho(z, \Gamma)(1 - |z|^2)^{-\alpha}, \quad \alpha = \frac{\gamma}{\log \beta},$$

where ρ denotes the pseudohyperbolic metric in $\mathbb{D}$. This allows us to directly solve interpolation and sampling problems for Bergman spaces in the case of such sequences. (Joint work with A. Kukuryka and P. Sobolewski.)

Joaquim Ortega-Cerdà, Universidad de Barcelona (jortega@ub.edu)
Generalized Marcinkiewicz-Zygmund inequalities

ABSTRACT: I will present a joint work with Jordi Saludes. We study a generalization of the classical Marcinkiewicz-Zygmund inequalities. We relate this problem to the sampling sequences in the Paley-Wiener space and by using this analogy we give sharp necessary and sufficient computable conditions for a family of points to satisfy the Marcinkiewicz-Zygmund inequalities.

John Pfaltzgraff, University of North Carolina, Chapel Hill (jap@math.unc.edu)
The Koebe transform and two-point distortion in several variables

Abstract:

Part 1. Some remarks on Peter Duren's work in univalent functions and variational methods, H^p theory, and capacity.

Part 2. The Koebe Transform $K_\phi F$ of a normalized $(F(0) = 0,\ DF(0) = I)$, locally biholomorphic mapping $F : B \longmapsto \mathbb{C}^n$ is the renormalized composition operator

$$K_\phi F(z) = D\phi(0)^{-1}(DF(\phi(0)))^{-1}\{F(\phi(z)) - F(\phi(0))\},$$

for $\phi \in Aut(B^n)$. The notation means $z = (z_1, \ldots, z_n)$, $B = B^n$ is the open unit ball in $\mathbb{C}^n$, and

$$F(z) = \left(F^1(z), \ldots, F^n(z)\right) = z + A_2(z,z) + \cdots + A_k(z,\ldots,z) + \cdots,$$

$A_k(z,\ldots,z) = (1/k!)\,D^kF(0)(z,\ldots,z)$. The Koebe transform provides the basis for defining a linear invariant family (LIF) $\mathfrak{F}$ of locally biholomorphic mappings of the ball, namely, $F \in \mathfrak{F}$ iff $K_\phi F \in \mathfrak{F}, \forall \phi \in Aut(B)$. The *norm order* of a (LIF) $\mathfrak{F}$ is $||ord||\ \mathfrak{F} = \sup\left\{(1/2)\left|\left|D^2F(0)\right|\right| : F \in \mathfrak{F}\right\}$. If a LIF has finite norm order then it is a normal family [Suff,Pf,2000] J D′ Analyse. The LIF generated by a single mapping F is the family $[F] := \{K_\phi F : \phi \in Aut(B)\}$ and $||ord||\, F$, the *order of the mapping* F is the norm order of the family $[F]$. The concept of LIF's of finite norm order in several variables is introduced and developed in [Suff,Pf,2000] where one can find the distortion theorem that is the starting point for the two-point distortion result below.

Using the LIF, norm order and Caratheodory distance on B we prove the following invariant two-point distortion theorem.

Theorem. $(Graham, G.Kohr, Pf, 2005)$ If F is a normalized locally biholomorphic mapping with finite norm order then F is univalent on B if, and only if,

$$||F(a) - F(b)|| \geq H(C_B(a,b)) \max\{\mathcal{D}F(a), \mathcal{D}F(b)\},\ \forall a, b \in B,$$

where $C_B(a,b)$ is the Caratheodory distance on B, $\mathcal{D}F(z) = \left(1 - ||z||^2\right) / \left|\left|(DF(z))^{-1}\right|\right|$, and H is a function that will be described in this talk.

The first step in this work is a new result giving a lower growth bound that is proved using the lower distortion bound found in [Suff,Pf,2000], Thm. 4.1. The properties of the Caratheodory distance used in this work are its invariance and the formula $C_B(z,0) = (1/2)\log[(1 + ||z||)/(1 - ||z||)]$.

Remarks: Two-point invariant distortion theorems for univalent functions on the disk appear to have their origins in a 1978 paper of C. Blatter and a 1994 paper by Kim and Minda. The speaker has used similar ideas to prove a two-point distortion theorem for plane harmonic mappings of the disk.

Christian Pommerenke, Technical University of Berlin (pommeren@math.TU-Berlin.DE)
The fixed point function and probability

Abstract: Let φ be an analytic self-map of the unit disk $\mathbb{D}$ with $\varphi(0) \neq 0$. Then $w = z/\varphi(z)$ has an inverse $z = f(w)$ mapping $\mathbb{D}$ onto a starlike domain in $\mathbb{D}$, called the "fixed point function". It remains open whether $f(\mathbb{D})$ is always hyperbolically convex. The Bürmann-Lagrange formula allows us to express the coefficients of f in terms of the coefficients of φ. We determine the asymptotic behaviour of the coefficients of f and of related functions. An interesting special case is that φ is the generating function of a random variable X with values $0, 1, 2, \ldots$. Then our results can be interpreted as results about the probabilities of independent combinations of X's. (Joint work with Diego Mejía in Medellín.)

Stefan Richter, University of Tennessee, Knoxville (richter@math.utk.edu)
Two-Isometric d-tuples of commuting operators

Abstract: Let $d \geq 1$ and let $T = (T_1, \ldots, T_d)$ be a d-tuple of commuting operators on a Hilbert space $\mathcal{H}$. T is called a spherical isometry, if $\Delta_1 = \sum_{j=1}^d T_j^* T_j - I = 0$. For $k > 1$ we inductively define $\Delta_k = \left(\sum_{j=1}^d T_j^* \Delta_{k-1} T_j \right) - \Delta_{k-1}$, and we say that T is a k-isometric d-tuple, if $\Delta_k = 0$. Examples of d-isometric d-tuples are given by multiplication by the coordinate functions $M_z = (M_{z_1}, \ldots, M_{z_d})$ on the Arveson space H_d^2 and on all of its invariant subspaces. H_d^2 is defined by the reproducing kernel $k_\lambda(z) = (1 - \langle z, \lambda \rangle)^{-1}$ on the unit ball of $\mathbb{C}^d$. If $d = 1$, then the classical Dirichlet shift is a 2-isometry. In this talk I will present a model theorem for certain 2-isometric d-tuples.

William Ross, University of Richmond (wross@richmond.edu)
Szegö's theorem and common cyclic vectors

Abstract: In this joint work with Warren Wogen, I will apply a generalization of Szegö's classical theorem to show that a certain subclass of multiplication operators on L^2 have a common cyclic vector.

Kristian Seip, Norwegian University of Science and Technology in Trondheim (seip@math.ntnu.no)
Extremal functions

Abstract: Building on Hedenmalm's groundbreaking 1991 paper on factorization, the 1993 paper "Contractive zero-divisors in Bergman spaces" by Duren, Khavinson, Shapiro, and Sundberg has been one of the most influential papers in the development of Bergman spaces. The kind of extremal functions that appear in these works show up in other contexts as well, as I will discuss in this survey talk. We will look at basic properties and some applications reflecting how extremal functions carry essential information about the object at hand.

Harold Shapiro, Royal Institute of Technology in Stockholm (shapiro@math.kth.se)
Some properties of Stefan Bergman's "doubly orthogonal functions"

Abstract: Let D_0 and D_1 be Jordan domains in the complex plane such that D_1 contains the closure of D_0. Moreover, let us denote by $B(D_0)$ and $B(D_1)$ the corresponding Bergman spaces of square integrable analytic functions on these domains. Then, as shown by S. Bergman, there is an orthonormal basis $\{f_n\}$ for $B(D_0)$ whose members each extend analytically into D_1, and moreover the extended functions are in $B(D_1)$, are mutually orthogonal in this space, and span it. This yields, in principle, an interesting test for analytic continuability of a given function f in $B(D_0)$ into the domain D_1, namely: f is so continuable, and its extension is in $B(D_1)$, if and only if the sum of the series $\sum_n L(n)|c(n)|^2$ is finite. Here $c(n)$ denotes the inner product $< f, f_n >$ in the space $B(D_0)$ and $L(n)$ the square of the norm of f_n as an element of the space $B(D_1)$. The drawback to this, in principle very flexible and elegant criterion for analytic continuation, is that there are very few scenarios where these f_n are known explicitly. (From the standpoint of functional analysis the f_n are the eigenfunctions of an integral operator whose kernel is the restriction of the Bergman kernel function for the domain D_1, from $D_1 \times D_1$ to $D_0 \times D_0$.)

In 1980 S.D.Fisher and C.A. Micchelli made a breakthrough concerning the analogous doubly orthogonal systems for the Hardy spaces. They showed, among other things, that the eigenfunction of rank n has precisely n zeros in D_1. (They took for D_1 the unit disk, but their arguments applied to more general situations).

It turns out that such results are not true for the Bergman space eigenfunctions in general, but are true if the domain D_0 is "sufficiently concentrated" within D_1, for example if D_1 is the unit disk and D_0 is contained in the concentric disk of radius $\sqrt{2} - 1$. These are results I obtained in joint work with B.

Gustafsson and M. Putinar, published in J. Functional Analysis 199 (2003). The methods employed are potential-theoretic and involve a non-classical variant of "balayage".

Serguei Shimorin, Royal Institute of Technology in Stockholm (shimorin@math.kth.se)
Branching point area theorems for univalent functions

ABSTRACT: Area methods are a classical tool in the theory of univalent functions. Such topics as Grunsky, Goluzin, or Schiffer-Tammi inequalities are in fact different modifications of the Polynomial Area Theorem which in turn reduces to an appropriate application of the Green formula. In the talk, we discuss a new type of area theorems obtained by considering branching point compositions with univalent functions. As a result, we obtain a new series of sharp integral inequalities. We discuss also branching point versions of Grunsky and Goluzin inequalities. The main point of the study is an analysis in the two-sided Dirichlet space on the unit circle supplied with the natural indefinite inner product.

Michael Stessin, University at Albany, State University of New York (stessin@math.albany.edu)
Composition operators in strongly pseudoconvex domains

ABSTRACT: Let $\Omega_1 \subset \mathbb{C}^m$ and $\Omega_2 \subset \mathbb{C}^n$ be bounded hyperconvex domains. We will consider actions of composition operators induced by holomorphic mappings of Ω_1 into Ω_2 on spaces of holomorphic functions.

Carl Sundberg, University of Tennessee, Knoxville (sundberg@math.utk.edu)
To be announced

Yan-Chun James Tung, Michigan State University (ytung@math.msu.edu)
Analytic functions with lacunary Taylor series

ABSTRACT: For $1 \leq p < \infty$, we give a necessary and sufficient condition for a function with lacunary Taylor series to be a member of the Fock space F^p. We also give a new proof for an analogous result known to hold for Bergman spaces A^p.

Ignacio Uriarte-Tuero, University of Missouri, Columbia (ignacio@math.missouri.edu)
Improved Painlevé removability for planar quasiregular mappings

ABSTRACT: The classical Painlevé problem (characterize geometrically the sets of zero analytic capacity) has been recently solved by Tolsa (with previous partial results by Guy David, etc.). It is natural to try to understand the analogous problem in the quasiconformal world, i.e. understand the removable sets for bounded solutions of the Beltrami equation.
I will present the results of a joint work with Astala, Clop, Mateu and Orobitg. It is known that not all compact sets of sigma-finite length in the plane are removable for bounded analytic functions (1 is the critical dimension for this problem). One of our main results is that, somewhat surprisingly, for the analogous quasiconformal problem (removability for bounded K-quasiregular mappings), all sets of sigma-finite measure at the critical dimension are removable.
The techniques we use come from complex analysis and quasiconformal mappings (conformal welding, integral means estimates, Makarov's compression and expansion for conformal mappings), multifractal analysis, nonlinear potential theory (Riesz and Bessel capacities), harmonic analysis (Calderón-Zygmund theory, Hörmander-Mihlin multiplier theorem), geometric measure theory, etc. I will try to make the talk as self-contained as possible.

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Weighted Bergman spaces in the unit ball

ABSTRACT: For any $\alpha > -1$ the Bergman space A^p_α consists of holomorphic functions that are L^p integrable with respect to the measure $(1-|z|^2)^\alpha dv(z)$. We will extend the theory of Bergman spaces to the case where α is any real number.