1) Prove that if in a triangle of area $A$ the product of two medians is $3A/2$ then these two medians are orthogonal.

2) Let $A = A_n$ be the square matrix of size $n$ with entries $A(j, k) = 1/\min(j, k)$ $(1 \leq j, k \leq n)$. Find a formula for $\det A_n$.

3) Consider the Taylor expansion

$$(1 - t + t^2) e^t = \sum_{n=0}^{\infty} a_n t^n$$

at the origin. Prove that all coefficients $a_n$ are rational numbers of the form $p_n/q_n$, where $p_n, q_n \in \mathbb{Z}$ and each $p_n$ is equal to 0, 1 or a prime number.

4) Let $f$ be a real-valued function on $[0, 1]$ such that $|f(x)| \leq 1$. Prove that

$$\int_0^1 \sqrt{1 - f(x)^2} dx \leq \sqrt{1 - \left( \int_0^1 f(x) \right)^2}.$$