1) Let $a, b, c$ be the sides of a triangle and let $T$ be its area. Show that
\[ a^2 + b^2 + c^2 \geq 4\sqrt{3}T. \]
When do we have the equality?

2) Show that $\cos(\sin x) > \sin(\cos x)$ for every $x \in \mathbb{R}$.

3) Prove that there are infinitely many natural numbers not containing the digit 0 that are divisible by the sum of their digits.

4) Let $A, B, C$ be pairwise commuting matrices. Prove that there exist real numbers $\alpha, \beta, \gamma$ not all equal to 0 such that
\[ \det (\alpha A + \beta B + \gamma C) = 0. \]