1) A real sequence \( \{a_n\} \) is defined by an inductive rule \( a_n = a_{n-1}(a_{n-1} - 1) \). For which choices of the initial value \( a_1 \) does this sequence converge?

2) Let \( n \geq 1 \) be a fixed integer. Prove that one can colour the plane using \( n \) colours in such a way that each nondegenerate circle (i.e., each circle which is not a single point) contains points of all the \( n \) colours.

3) Let \( a, b, c, d \) be real numbers such that \( a + b = c + d \) and \( a^3 + b^3 = c^3 + d^3 \). Prove that \( a^{2013} + b^{2013} = c^{2013} + d^{2013} \).

4) Find all polynomials \( Q \) satisfying \( Q(x^2 + 1) = Q(x)^2 + 1 \).

5) Given two positive rational numbers \( a, b \) such that \( \sqrt[3]{a} + \sqrt[3]{b} \) is rational, prove that \( \sqrt[3]{a} \) and \( \sqrt[3]{b} \) are rational.

6) (a) Solve in integers (positive or negative): \( \frac{1}{x} + \frac{1}{y} = \frac{1}{6} \).

(b) Solve the diophantine equation \( \frac{1}{x} + \frac{1}{y} = \frac{1}{z} \) (write down a formula which gives all integer solutions).