1) Let \( p(x) = (x + 2x^2 + 3x^3 + \cdots + nx^n)^2 = a_0 + a_1x + a_2x^2 + \cdots + a_{2n}x^{2n} \). Show that
\[
a_{n+1} + a_{n+2} + \cdots + a_{2n} = \frac{n(n+1)(5n^2 + 5n + 2)}{24}.
\]

2) What is the least number of elements of a set \( A \) such that there exists a function \( f : \{1, 2, 3, \ldots\} \to A \) with the property that \( f(j) \neq f(k) \) whenever \( j - k \) is prime?

3) The sequence \( \{s_n\}_{n \geq 0} \) is defined inductively:
\[
s_0 = s_1 = 1; \quad s_{n+1} = \frac{s_n^2 + 1}{s_{n-1}}.
\]
Show that all the terms of this sequence are integers.