Resonant Transmission of Cold Atoms through Subwavelength Apertures

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(Received 16 June 2005; published 20 October 2005)

DOI: 10.1103/PhysRevLett.95.170406

Classical particles can traverse a hole whenever they are smaller than the hole size. Quantum particles, for instance atoms, must additionally obey the constraints imposed by their own wave nature. One basic principle in wave physics states that transmission through an aperture much smaller than the wavelength is negligible. Therefore, when cold atoms are considered (with temperature in the order of μK, and associated de Broglie wavelength of about 1 μm), their transmittance through an aperture of size in the order of a fraction of a micron is almost zero, even though the atoms would classically fit inside the aperture. However, it has been recently found that this undulatory limitation can be overcome for electromagnetic waves impinging on a perforated metallic film, if the holes are arranged in a periodic fashion [1]. In this Letter we show that matter waves can be harnessed in a similar way, thus traversing narrow slits otherwise opaque.

The phenomenon of extraordinary optical transmission through arrays of subwavelength holes relies on the resonant excitation of surface plasmon (SP) modes decorating the surface of a metallic film [2]. The first question to address is the possible existence of the SP analog for the case of matter waves. It is easily shown that the simplest potential function that supports surface matter waves (SMWs) is the one depicted in Fig. 1: a square potential in the z direction (the direction orthogonal to the surface) and translationally invariant in the xy plane. When the potential well has a bound state (with energy $E_0 < 0$), the dispersion relation of the associated SMW propagating in the x direction is given by

$$E(q) = E_0 + \frac{(h q)^2}{2m},$$

where $E$ and $h q$ are the surface wave energy and momentum, respectively, and $m$ is the atom mass. This mode is confined in the z direction (decaying exponentially with z outside the potential well), and propagates in the x direction. Such SMWs at a flat interface cannot be excited by an impinging atom, due to energy and momentum conservation. However, scattering by an array of grooves in the interface can provide the crystal momentum needed for the coupling. When perforating grooves are considered, it seems natural to test whether the phenomenon of extraordinary transmission through an array of narrow apertures also appears for matter waves.

Let us consider cold atoms incident on a film (thickness $t = 0.16$ μm) perforated with a periodic array of slits (period $\lambda = 0.80$ μm, width $w = 0.16$ μm). In this first simple model the potential inside the film is infinite, and its crosscut along the z direction displays a square potential well of width $h = 0.30$ μm and depth $V_0 = 1.060 \times 10^{-11}$ eV (see inset of Fig. 2 and Fig. 1). The potential inside the slits is also assumed to be $V_0$ (Fig. 2, inset). In

![FIG. 1 (color online). Potential function supporting surface matter waves. Here, the crosscut along the direction z orthogonal to the flat surface is shown (black thick line), the potential being independent on x and y. The infinite potential barrier models the interface impenetrability. The potential well has one bound state of energy $E_0$ (blue line). The corresponding wave function (red dashed line) is confined to a region close to the interface and propagates parallel to it. The indicated transition (dashed arrow) from a collision state of energy $E_{in}$ to the SMW is only possible once the surface is modulated (see Fig. 2).](https://doi.org/10.1103/PhysRevLett.95.170406)
the following, the incident atom will be rubidium. Figure 2 depicts the transmittance spectrum (red curve) for a plane matter wave orthogonally incident on the structure described above. The spectrum displays two main features: (i) two close peaks that reach 100% transmittance, and (ii) a zero at a slightly higher energy. In the photonic analog, these two features are the well-known fingerprints of extraordinary optical transmission through subwavelength holes [1,2] and their presence in Fig. 2 is indicative that this phenomenon also occurs for matter waves. In order to further confirm this conclusion, it is worth comparing the transmittance curve with the approximate predictions obtained from the dispersion relation [Eq. (1)] of an uncorrugated surface. For normal incidence, and considering only first order processes, the momentum acquired by the SMW (transferred by the periodic grating) is \( hK = \pm h2\pi/\lambda \). As dictated by energy conservation, the associated energy transfer \( (hK)^2/2m = 1.486 \times 10^{-11} \text{ eV} \) plus the energy of the bound state \( E_0 = -0.095 \times 10^{-11} \text{ eV} \) fit well with the energy of the transmission zero. Notice that, for the corrugated film, the SMW is distorted and its dispersion relation is shifted down, thus explaining why the maxima occur for slightly lower energy than predicted by Eq. (1). The surface modes at both interfaces couple through the slits, and so their energy degeneracy is broken, splitting into even and odd modes that are responsible for the two maxima seen in the calculated transmittance. All these facts have a very close parallel with the photonic analog [2,3]. Let us stress that, at the transmission maxima, the de Broglie wavelength (\( \lambda = 0.844 \mu\text{m} \)) is more than 5 times larger than the slit width. This is the reason why the slits are opaque to atoms when no surface mode is available (Fig. 2, black dashed line).

Now that we have demonstrated that the phenomenon of extraordinary transmission is possible for matter waves, let us next discuss a possible implementation of the basic structure analyzed above. The fundamental ingredient is the existence of a potential well close to the material interface. In principle, one could take advantage of the atom-surface potential present in front of a dielectric surface as a result of the long-range van der Waals (vdW) attraction and the very short-range repulsion between the electronic cloud of the atom and that of the surface. Although this potential supports surface states [4], it suffers from two important drawbacks. First, the energies involved are very different from the ones we are interested in (cold atoms at temperatures of \( \mu\text{K} \) have energies of the order of \( 10^{-11} \text{ eV} \) whereas typical values in vdW potentials are of the order of \( 10^{-3} \text{ eV} \)). And second, the centers of mass of these bound states are very close to the surface so that strong electronic interaction between the incident atom and the surface atoms is expected. In principle, this should be avoided in order to maintain the coherence of the atom wave and to minimize atom heating during the transmission process. In addition, if it were possible to externally tune the potential, one could implement an atom switch by opening or closing the transmission channel.

For these reasons external forces have to be exerted on the cold atom to obtain a potential well with a depth of about \( 10^{-10} \text{ eV} \) at a distance away from the surface of the order of 1 \( \mu\text{m} \). Such a potential can be built by adding to the inherent attractive vdW force a repulsive dipolar interaction due to a laser field [5,6]. The frequency of the laser field is blue detuned slightly above the frequency of a given optical transition of the atom so that its polarizability becomes negative, resulting in an effective repulsive potential. The sum of the optical repulsive potential plus the vdW attraction gives rise to a potential barrier keeping the atoms sufficiently far away from the surface (a so-called evanescent atomic mirror [7,8]), and to a potential well that may support bound states at an appropriate distance away from the surface. Similar ideas have been used to guide cold atoms along optical fibers [9,10]. In addition, periodicity is also needed to couple the incident plane matter wave to the SMW. A structure fulfilling all these requirements is an array of parallel cylindrical optical fibers carrying a blue-detuned optical mode propagating along them. This is the structure we propose to experimentally test the phenomenon of extraordinary transmission of matter waves through very narrow slits.

In what follows, we present calculations for a realistic structure in order to show that the proposed effect is within

![FIG. 2 (color online). Calculated transmittance of cold \(^{87}\text{Rb} \) atoms through a film perforated by a periodic array of very narrow slits, as a function of the incident energy. Atoms incide along the direction \( z \) normal to the film. Red line: transmittance reaches 100% when the structure supports surface states. The corresponding 2D potential is displayed in the inset (colors code the potential as follows: red (dark gray) \( \rightarrow V = +\infty \), orange (medium gray) \( \rightarrow V = 0 \), blue (light gray) \( \rightarrow V = V_0 = \text{const} < 0 \); see crosscut along direction \( z \) in Fig. 1). Black dashed line: transmittance is negligible when no potential well is available, i.e., when \( V_0 = 0 \). In the computer simulations, the corners of the rectangular areas have been rounded with radius \( r = 64 \text{ nm} \).](170406-2)
reach. Although some design was needed, no attempt at optimization was done, so it is expected that the phenomenon will also appear for other parameter values and/or other geometries (for instance, it is possible that the potential landscape could be created without a dielectric by fully optical means). In our numerical simulations we have chosen the following parameter values: the optical fibers have radius $R = 0.20 \, \mu m$, dielectric constant $\varepsilon = 13$, and the array period is $\Lambda = 0.80 \, \mu m$. We take again $^{87}$Rb cold atoms and will be considering the atomic transition $D_2$ ($S_{1/2} \rightarrow S_{1/2}$) at $\omega_0 = 2\pi \times 384 \, \text{THz}$. The laser frequency is chosen in such a way that the detuning is $\delta = \omega - \omega_0 = 2\pi \times 6 \, \text{THz}$. The repulsive optical potential at the position $r$ outside the fiber is given by [11]

$$V_{\text{opt}}(r) = \frac{\hbar \delta}{2} \frac{\Gamma^2}{\Gamma^2 + 4\delta^2} \frac{I(r)}{I_{\text{sat}}},$$

where $\Gamma = 2\pi \times 6 \, \text{MHz}$ is the linewidth of the considered transition, $I_{\text{sat}} = 2.5 \, \text{mW/cm}^2$, and $I(r)$ is the intensity profile of the laser guided mode. The chosen detuning is sufficiently small so that a two-level atom approximation is valid, but large enough so that recoil heating is low. The vdW attractive potential is obtained by means of a simple calculation assuming pairwise interaction between the atom and the points inside the fiber [12]. The total potential is then $V(r) = V_{\text{opt}}(r) + V_{\text{vdW}}(r)$.

Figure 3 plots the total potential for the case when the optical power carried by the laser mode running along one fiber is $P_0 = 20.7 \, \text{mW}$. As desired, the overall picture of this potential resembles our basic model [compare inset of Figs. 2 and 3(a)]. White regions in Fig. 3(a) correspond to the locations of the fibers. In the immediate vicinity of the fibers’ surfaces vdW attraction dominates (black narrow ring adjacent to the fiber), but for distances to the fiber center between 0.21 $\mu m$ and 0.32 $\mu m$ the total potential is repulsive (red doughnut-shaped region around the fiber). This potential barrier is much higher than the incident energy so that tunneling to the inner attractive ring adjacent to the surface fiber is negligible. Because of the evanescent character of the optical field outside the fiber, vdW attraction again dominates for distances larger than 0.40 $\mu m$. In the intermediate region, a potential well develops (blue and green colored areas). Importantly, the potential between the fibers is strongly repulsive, leaving an extremely narrow effective slit (width $\approx 0.05 \, \mu m$) for the transmission of atoms through the structure. Figure 3(b) displays a one-dimensional (1D) crosscut of the potential along the $z$ direction through the center of one fiber. This potential well has four bound states, found by solving the 1D Schrödinger equation. The energy of the uppermost level is $E_4 = -0.155 \times 10^{-11} \, \text{eV}$ (blue line). This bound state is the candidate to assist in the transmission of Rb atoms with de Broglie wavelengths of the order of the array periodicity.

Figure 4(a) renders the transmittance spectra of orthogonally incident Rb atoms for different values of the laser field power $P$, taking $P_0$ as a reference. Let us first analyze the transmission corresponding to $P = P_0$ (black line). The spectrum shows two very close maxima reaching 100% transmission and zero for slightly larger energy. We can safely conclude that the phenomenon of resonant extraordinary transmission of matter waves is present in the designed structure. This conclusion is reinforced by looking at the pattern of the wave function modulus $|\psi(r)|$ [Fig. 4(b)], associated to the left peak at about $1.364 \times 10^{-11} \, \text{eV}$. Here, it can be distinctly seen that surface matter waves above and below the structure are excited, building up an even mode. These SMWs have maxima at a distance away from the fiber axis of 0.90 $\mu m$, in good agreement with the maximum’s location of the fourth eigenmode in

![Figure 3](color). (a) 2D total potential for $^{87}$Rb cold atoms impinging on an array of cylindrical dielectric fibers carrying a blue-detuned optical guided mode. The white circles represent the fibers’ cross section. Color scale codes potential energies in units of $10^{-11} \, \text{eV}$. Blue and green regions correspond to an attractive potential well. The effective slit width is of the order of 0.05 $\mu m$. (b) Black line: crosscut along the vertical $z$ line of potential shown in left panel. Green and magenta lines: van der Waals and dipolar contribution to the total potential, respectively. The energy of the fourth level of this 1D potential is represented by the blue line, and the red dashed line shows the corresponding wave function.
the 1D potential [Fig. 3(b), dashed curve]. If the power $P$ of the laser field is now changed, the total potential and the corresponding transmission spectrum are modified. We have found that transmittance is extremely sensitive to the laser power. This feature opens up the possibility of using the structure as an atomic switch. When the power $P$ is reduced by just 10%, the transmission peak almost vanishes [Fig. 4(a), orange curve]. Notice that, as $P$ is being lowered, two physical magnitudes change: first, the energy of the eigenmode involved in the transmission process sinks to lower energies, explaining the shift to lower energies of the transmission peak found in Fig. 4(a). Second, the channels between the fibers broaden so that the coupling between the SMWs at both interfaces becomes less resonant provoking a reduction in the height of the peaks. On the other hand, when $P$ is increased the channel between the fibers is effectively shut. The two peaks become narrower and closer to each other and, in the limit, they merge together with the transmission zero and disappear.

In conclusion, we have shown that resonant coupling to surface matter waves can lead to 100% transmission of cold atoms through narrow slits that would be otherwise opaque at those energies. In a carefully designed device, the effective slit width and surface wave properties can be externally tuned, so that the structure behaves as an atom switch. Moreover, in this Letter we have focused on transmittance enhancement through periodic arrays, but in the photonic analog it is known that surface corrugation leads to photon collimation, lensing, and enhanced transmission through single apertures [13]. Apart from its fundamental interest, this new phenomenon could help to control the flow of neutral atoms, with possible applications in atom lithography [14] and interferometry [15]. At lower temperatures, for which the Bose-Einstein condensate realm is entered, the ideas presented here could be extended to diluted condensates [16].