The following proof of the irrationality of $\sqrt{2}$ is credited to Thomas Apostol. It relies on geometry and the principal of mathematical induction.

Suppose that there are natural numbers $m$ and $n$ such that $m^2 = 2n^2$ and that $n$ is the smallest natural number for which $m^2 = 2n^2$. Draw a circle centered at $A$ so that $AB = BC = n$ and $AC = m$.

Let $DE$ be tangent to the circle at $D$ and $BC$ be tangent to the circle at $B$. Then $DE = EB = DC = m - n$. Then $\triangle DCE$ is a right triangle with sides $m - n < n$ and hypotenuse $n - (m - n) = 2n - m$ and $n$ is not the smallest natural number satisfying $m^2 = 2n^2$.