1. Orbits of A

2. Preservación del volumen en campos conservativos (dimensión 1). Las partículas con más momento se desplazan más.

3. Orbits del punto 0 por $T_{\sqrt{17}}$.

4. $\mathbb{Z} \setminus \mathbb{Q}$ y $\mathbb{Z}^2 \setminus \mathbb{R}^2$.
Teorema ergódico

densidad de \( m \sqrt{7} \).

Puntos de coordenadas enteras en la corona.

\( \text{PSL}_2(\mathbb{R}) \)
$\mu(gu) = \mu(u)$

$d(gx,gy) = d(x,y)$

**10.**

distancia $t$

$(z,0)$

$(z,0) u_t$

**flujo horocíclico**

**11.**

distancia $t$

$(z,0)$

$(z,0) a e^t$

**flujo geodésico**

**12.**

distancia $t$

$(z,0)$

$(z,\pi)$

**flujo horocíclico exterior**
Los flujos generan $PSL_2(\mathbb{R})$.

$g=(z,0)$

$(0,1)=g \cdot (z,1)$

$\equiv \mathbb{Z}^2 \setminus \mathbb{R}^2$

$(4 \mathbb{Z} \times 2 \mathbb{Z}) \setminus \mathbb{R}^2 \equiv \bigcirc$

Tesselaciones de $\mathbb{R}^2$
The geodesic flow expands the closed horocycles, pushing them away from the cusp; indeed, we have

$$g_t(H_y) = H_{e^{-t}y}.$$  

Figure 3. A long closed horocycle $H_y$ passes randomly through many fundamental domains for $SL_2(\mathbb{Z})$.

Random elliptic curves

Using mixing of the geodesic flow, it is not hard to show that $H_y$ is uniformly distributed on $B = T_1(\mathfrak{M}_1)$ as $y \to 0$ (see, e.g., [EM, §7]). That is, uniform measure along $H_y$ converges to the invariant measure $\mu_B$ on $B$ as the length of $H_y$ tends to infinity. See Figure 3.

Because of this uniform distribution, one can construct a “nearly random” Riemann surface $X$ of genus one as follows: Pick $y > 0$ very small, pick $x \in [0, 1]$ at random, let $\tau = x + iy$, and set $X = \mathbb{C}/\mathbb{Z} \oplus \mathbb{Z}\tau$. As $y \to 0$, the distribution of $X$ on $\mathfrak{M}_1$ converges to hyperbolic area measure.

$E$ as a torus bundle

We now return to the space of lattice translates

$$E = \overline{ASL_2(\mathbb{R})/ASL_2(\mathbb{Z})}.$$

The projection $ASL_2(\mathbb{R}) \to SL_2(\mathbb{R})$ (sending $Ax + b$ to $A$) makes the space of lattice translates into a torus bundle

$$E \xrightarrow{D} B.$$
GAPS IN $\sqrt{n} \bmod 1$ AND ERGODIC THEORY

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$$g_t(H_y) = H_y$$

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The geodesic flow expands the closed horocycles, pushing them away from the cusp; indeed, we have

$$x_t(H_y) = H_{e^{i
u}y}.$$  

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**$E$ as a torus bundle**

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$$E \xrightarrow{D} B.$$
The geodesic flow expands the closed horocycles, pushing them away from the cusp; indeed, we have
\[ g_s(H_y) = e^{-s} H_y. \]

Equidistribución de \( x_0, T x_0, T^2 x_0, \ldots, T^{N-1} x_0 \) en \( X \).

Figure 3. A long closed horocycle \( H_y \) passes randomly through many fundamental domains for \( \text{SL}_2(\mathbb{Z}) \).

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Using mixing of the geodesic flow, it is not hard to show that \( H_y \) is uniformly distributed on \( B = T_1(M_1) \) as \( y \to 0 \) (see, e.g., [EM, §7]). That is, uniform measure along \( H_y \) converges to the invariant measure \( \mu_B \) on \( B \) as the length of \( H_y \) tends to infinity. See Figure 3.

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**E as a torus bundle**

We now return to the space of lattice translates
\[ E = \text{ASL}_2(\mathbb{R}) / \text{ASL}_2(\mathbb{Z}). \]

The projection \( \text{ASL}_2(\mathbb{R}) \to \text{SL}_2(\mathbb{R}) \) (sending \( Ax + b \) to \( A \)) makes the space of lattice translates into a torus bundle
\[ E \xrightarrow{D} B. \]
19 \quad t \quad 0 < t < 1

Porque el flujo horocíclico \( T_t \) avanza un poco más al estar más arriba.

extension de invariancia de \( g \)
\[ u_t \rightarrow u \]

20

extension de invariancia de \( g \)
\[ u_t \rightarrow u \]

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El flujo horacídico se pare puntos (con ángulo similar) muy eles pacio
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