Modulated phases in electron-hole bilayers & 2D dipolar Fermi gases

Francesca Maria Marchetti

Workshop NSPM11, Erice, 6 Aug 2011
1. Imbalanced electron-hole bilayers
   ✧ spatially modulated pairing (FFLO)
1. Imbalanced electron-hole bilayers
   - spatially modulated pairing (FFLO)
   - limit of extreme imbalance
   - unusual bosonic limit of FFLO: supersolid
Outline

1. Imbalanced electron-hole bilayers
   ♦ spatially modulated pairing (FFLO)
   ♦ limit of extreme imbalance
   ♦ unusual bosonic limit of FFLO: supersolid

2. 2D dipolar Fermi gases
   ♦ rich many-body physics (even 1 layer)
   ♦ phase diagram (density modulation, superfluidity, collapse, …)
   ♦ why beyond RPA
Background: Two component Fermi gases

✧ Tunable interactions
✧ BEC-BCS crossover

1

\( \frac{1}{k_Fa} \)

Supersolidity in imbalanced electron-hole bilayers
Background: Two component Fermi gases

✧ Tunable interactions
✧ BEC-BCS crossover

[Zwierlein et al. Nature (2005)]
Background: Imbalanced Fermi gases

✧ Tunable interactions
✧ BEC-BCS crossover
✧ Density imbalance frustrates pairing

Can superfluidity persist in presence of a population imbalance?
Background: T=0 phase diagram

Supersolidity in imbalanced electron-hole bilayers

- Polarised SF or breached pairing (space homogeneous)
- Macroscopic phase separation between SF and N phases
-Fully paired SF phase

[Parish, Marchetti et al. Nature Physics (2007)]
[Sheehy et al. PRL (2006)]
Background: $T=0$ phase diagram

- **FFLO phase**
  \[ \Delta(r) = \sum_Q \Delta_Q e^{i r \cdot Q} \]

- **Supersolid phase**
  - Polarised SF or breached pairing (space homogeneous)
  - Macroscopic phase separation between SF and N phases

Supersolidity in imbalanced electron-hole bilayers

- [Parish, Marchetti et al. Nature Physics (2007)]
- [Sheehy et al. PRL (2006)]
- [Shin et al. PRL (2006)]

- Fully paired SF phase
FFLO unlikely in 3D Fermi gases

\[ \Delta(r) = \sum Q \Delta_Q e^{i r \cdot Q} \]

Supersolidity in imbalanced electron-hole bilayers

✧ Displacement of the Fermi surface:
  ...allows Fermi surface nesting
  ...allows the system to polarise

✧ However
  ...nesting is partial
  ...it costs kinetic energy

✧ Phase separation dominates over FFLO

✧ Experiments (Rice & MIT): FFLO elusive

Supersolidity in imbalanced electron-hole bilayers
Electron-hole systems

BEC of bound excitons

exciton insulator

$\text{BEC of bound excitons}\quad \text{exciton insulator}\quad \text{BEC of bound excitons}$

Supersolidity in imbalanced electron-hole bilayers
Supersolidity in imbalanced electron-hole bilayers

Electron-hole systems

BEC of bound excitons

$ne - nh$

exciton insulator

$ne + nh$

BEC

BCS

Supersolidity in imbalanced electron-hole bilayers
Electron-hole bilayers: FFLO more ‘likely’

1. Enhanced Fermi surface nesting

2. No phase separation on macro-scales

BEC of bound excitons

exciton insulator

BEC

BCS

Supersolidity in imbalanced electron-hole bilayers
Electron-hole bilayers: Experimental realisations

1. Optically pumped coupled quantum wells

… and many more…

2. Individually contacted doped layers

Croxall et al. PRL (2008)
Seamons et al. PRL (2009)

⇒ Equilibrium phase diagram
Electron-hole bilayers: Hamiltonian

$$H = \sum \frac{k^2}{2m_\sigma} c_\sigma^\dagger c_\sigma + \frac{1}{\Omega} \sum g_q c_1^\dagger c_2^\dagger c_2 c_1 + \frac{1}{2\Omega} \sum U_q c_\sigma^\dagger c_\sigma^\dagger c_\sigma c_\sigma$$

$$U_q = \frac{2\pi e^2}{\varepsilon q}$$  bare intra-layer Coulomb interaction

$$g_q = -U_q e^{-qd}$$  bare inter-layer

⇒ No spin (spin polarised)
Limit of large imbalance: Variational ground state

* single particle in the 2\textsuperscript{nd} layer + Fermi sea in the 1\textsuperscript{st} layer

\[ \Psi(Q) = \sum_{k > k_F} \varphi_{kQ} c_{Q-k,2}^\dagger c_{k,1}^\dagger |FS\rangle \]
Limit of large imbalance: Variational ground state

- single particle in the 2\textsuperscript{nd} layer + Fermi see in the 1\textsuperscript{st} layer

\[ |\psi(Q)\rangle = \sum_{k>k_F} \varphi_{kQ} c_{Q-k,2}^{\dagger} c_{k,1}^{\dagger} |FS\rangle \]

Relative $k$ \hspace{1cm} CoM $Q$

$|\psi(Q = 0)\rangle$ (SF=superfluid) \hspace{1cm} $|\psi(Q \neq 0)\rangle$ (FFLO)
Limit of large imbalance: Variational ground state

- single particle in the 2nd layer + Fermi sea in the 1st layer

\[ |\psi(Q)\rangle = \sum_{k>k_F} \varphi_{kQ} c_{Q-k,2}^{\dagger} c_{k,1}^{\dagger} |FS\rangle \]

\[ \varphi_{kQ} = \delta_{k,k_F} \delta_{Q,q_F} \]

\[ |\psi_0\rangle = c_{0,2}^{\dagger} c_{kF,1}^{\dagger} |FS\rangle \]

Interpolate by screening the interactions (within RPA) = dress with density fluctuations, including particle-hole excitations

|\psi(Q = 0)\rangle (SF=superfluid)

|\psi(Q \neq 0)\rangle (FFLO)

Good description of both low and high density limit

Supersolidity in imbalanced electron-hole bilayers
Phase diagram

1. Eigenvalue equation $\langle \psi(Q) | H | \psi(Q) \rangle$

2. Unbinding transition $\langle \psi_0 | H | \psi_0 \rangle |\psi_0\rangle$

✧ Read also as mean-field (linearised) gap equation

⇒ Current theories
[D] some neglect screening
[D] some neglect finite Q FFLO

SF=superfluid

$|\psi(Q = 0)\rangle$

FFLO

$|\psi(Q \neq 0)\rangle$

✧ Parameters:

- exciton Bohr radius $a_0 = \frac{\varepsilon}{me^2}$
- mass ratio $\alpha = \frac{m_2}{m_1}$
- interlayer distance $d/a_0$
- dimensionless density $r_s = \frac{2}{k_F a_0}$

Supersolidity in imbalanced electron-hole bilayers
Phase diagram

1. Eigenvalue equation
\[ \langle \psi(Q) | H | \psi(Q) \rangle \]

2. Unbinding transition
\[ \langle \psi_0 | H | \psi_0 \rangle \]

- Read also as mean-field (linearised) gap equation

- Current theories
  - ... some neglect screening
  - ... some neglect finite Q FFLO

- Parameters:
  - exciton Bohr radius \( a_0 = \frac{\varepsilon}{me^2} \)
  - mass ratio \( \alpha = \frac{m_2}{m_1} \)
  - interlayer distance \( \frac{d}{a_0} \)
  - dimensionless density \( r_s = \frac{2}{k_Fa_0} \)

Supersolidity in imbalanced electron-hole bilayers
Phase diagram of fully imbalanced EH bilayer

Supersolidity in imbalanced electron-hole bilayers

GaAs:
\[ \alpha = 0.25 \]
\[ \alpha = 4 \]

FFLO region enhanced if minority particle lighter

\[ \Rightarrow \text{mass ratio } \alpha = \frac{m_2}{m_1} \]
\[ \Rightarrow \text{interlayer distance } \frac{d}{a_0} \]
\[ \Rightarrow \text{dimensionless density } r_s = \frac{2}{k_F a_0} \]
Effect of screening on the LOFF phase

✧ Unscreened case

Numerically
Analytically

[Parish, Marchetti & Littlewood EPL (2011)]
Effect of screening on the LOFF phase

 únscreened case

[Numerically]

[Analytically]

[Pieri et al. PRB (2007)]
[Subasi et al. PRB (2010)]

⇒ Numerically
⇒ Analytically

[Parish, Marchetti&Littlewood EPL (2011)]
Which kind of FFLO phase?
Dilute gas of minority particles: Interactions

ività Normal phase:

effective interaction between two unbound minority particles (RPA)

⇒ repulsive and dipolar $V^{22}(r) \rightarrow \frac{1}{r^3}$:
  the dilute gas is a Fermi liquid

ità Excitonic phase:

well separated excitons (dipoles)

⇒ also repulsive and dipolar $V^{ex}(r) \rightarrow \frac{1}{r^3}$:
  no phase separation
  (biexciton formation suppressed for single spin species & $\frac{d}{a_0} \gtrsim 0.25$)
Phenomenology of ‘bosonic’ FFLO

- Landau theory for \( |\psi|^2 \) = exciton density

\[
F[\psi] = \int dr \left[ -\mu |\psi|^2 + \frac{\lambda}{2} |\psi|^4 \right]
\]

chemical potential \hspace{1cm} \text{minimum energy at } |q| = Q_{\text{min}}
Phenomenology of ‘bosonic’ FFLO

✧ Landau theory for $|\psi|^2 = \text{exciton density}$ (weak crystalisation theory)

$$F[\psi] = \int dr \left[ -\mu |\psi|^2 + \gamma (\nabla^2 + Q_{min}^2) |\psi|^2 + \frac{\lambda}{2} |\psi|^4 \right]$$

chemical potential \hspace{2cm} minimum energy at $|q| = Q_{min}$

✧ Complex order parameter

$$\psi(r) = \sum_n a_n e^{i q_n \cdot r} \bigg|_{|q_n| = Q_{min}}$$

✧ Minimal energy solution

$$|\psi(r)|^2 \propto |\cos(q_1 \cdot r) - i \cos(q_2 \cdot r + \varphi)|^2$$

⇒ supersolid: exciton condensate with 2D spatial modulation (diagonal and off-diagonal order)
Observing FFLO

✧ FFLO enjoys a sizeable region of existence away from the Wigner crystal
✧ Trions only for $d/a_0 << 1$

✧ Finite temperature
  Exciton binding energy in GaAs
  = upper bound for the FFLO critical temperature

\[
\frac{E_B}{k_B} = 5 \text{ K}
\]

\[
T_{BKT} \propto \frac{E_0}{k_B r_s^2} \frac{n_2}{n_1} \sim 100 \text{ mK}
\]

GaAs
\[
\alpha = 0.25 \\
a_0 = 7 \text{ nm} \\
E_0 = 17 \text{ meV} \\
d/a_0 = 0.5 \\
r_s = 10 \\
n_2/n_1 \sim 0.2
\]

1. Light scattering off the spatial modulations
2. Photon angular emission
   (electron hole recombining): finite momentum pairing ($\pm q_1, \pm q_2$)
Conclusions part 1

- Electron-hole bilayers: promising for observing exotic pairing phenomena

- Evidence of FFLO phase at large imbalance: finite-Q exciton in presence of a fermi sea

- Dilute gas of finite-Q excitons: condensation with 2D spatial modulation (a supersolid)

  $\Rightarrow$ bosonic limit of FFLO

- Prospects for experimental observation
2D dipolar Fermi gases

✧ Quantum gas of ultracold polar fermions
   ⇒ $^{40}\text{K}^{87}\text{Rb}$ tightly bound heteronuclear molecules
   ⇒ quantum degeneracy
   ⇒ 2D confinement

[Ospelkaus et al. Science (2010)]
de Miranda et al. Nature Physics (2011)]

✧ Dipole-dipole interaction: long range and anisotropic

✧ Rich many-body physics (even for single component and single layer)
   ⇒ $\theta = 0$ isotropic & repulsive
2D dipolar Fermi gases

✧ Quantum gas of ultracold polar fermions
  ⇒ $^{40}\text{K}_{-}{^\text{87}}\text{Rb}$ tightly bound heteronuclear molecules
  ⇒ quantum degeneracy
  ⇒ 2D confinement

[Ospelkaus et al. Science (2010)]
[de Miranda et al. Nature Physics (2011)]

✧ Dipole-dipole interaction: long range and anisotropic
✧ Rich many-body physics (even for single component and single layer)
  ⇒ $\theta = 0$ isotropic & repulsive

spontaneous density modulation & crystalline phase

Modulated density phase in 2D dipolar Fermi gases
2D dipolar Fermi gases

✧ Quantum gas of ultracold polar fermions
  ⇒ \(^{40}\text{K}\)-\(^{87}\text{Rb}\) tightly bound heteronuclear molecules
  ⇒ quantum degeneracy
  ⇒ 2D confinement

[Ospelkaus et al. Science (2010)]
[de Miranda et al. Nature Physics (2011)]

✧ Dipole-dipole interaction: long range and anisotropic

✧ Rich many-body physics (even for single component and single layer)
  ⇒ \(\theta = 0\) isotropic & repulsive

⇒ \(\theta > \arcsin(1/\sqrt{3})\) attractive sliver

spontaneous density modulation & crystalline phase

superfluidity & collapse

Modulated density phase in 2D dipolar Fermi gases
Phase diagram

\[ V_\theta(q, \phi) = 2\pi D^2 \left[ v_0 - q \left( \cos^2 \theta - \sin^2 \theta \cos^2 \phi \right) \right] \]

Marchetti & Parish (in prep.)

Bruun & Taylor PRL (2008)

Wigner crystal

\[ U \gtrsim 60 \]

Astrakharchik PRL (2007)
Why not RPA (e.g., for $\theta=0$)

\[ V_0(q) = 2\pi D^2 [v_0 - q] \]

✧ Hartree-Fock ground state energy per particle ($U = mD^2k_F \rightarrow 0$):

\[ \varepsilon(n) = \frac{4\pi n}{m} \left( \frac{1}{4} + \frac{32U}{45\pi} \right) \]

✧ Compressibility sum rule

\[ -m \frac{\partial^2 [n\varepsilon(n)]}{\partial n^2} = \lim_{q \rightarrow 0} \chi^{-1}(q, \omega = 0) = \lim_{q \rightarrow 0} \frac{1}{\Pi(q, \omega = 0)} - V_0(q) [1 - G(q)] \]

✧ The local field factor allows to include exchange correlations neglected by RPA ($G(q) = 0$)
Why not RPA

Modulated density phase in 2D dipolar Fermi gases

![Graph showing the relationship between G(q) and q/k_F for Coulomb and dipolar forces.](image-url)
Phase diagram

Modulated density phase in 2D dipolar Fermi gases

\[ U = mD^2 k_F \]

- Collapse
- Stripe phase
- Superfluid
- RPA

[Marchetti & Parish (in prep.)]
[Bruun & Taylor PRL (2008)]
[Sun, Wu & Das Sarma PRB (2010)]
Phase diagram

- Approaching the transition to the stripe phase

\[ S(q, \phi = \pi/2) \]

\[ \chi^{-1}(q, \phi = \pi/2) \times \Pi(q) \]

Static structure factor

Inverse density-density response function

\[ q_{\text{inst}} < 2k_F \]

\[ \phi = \frac{\pi}{2} \]
Phase diagram

No collapse within RPA [Sun, Wu & Das Sarma PRB (2010)]

[RPA]

[Marchetti & Parish (in prep.)]

[Bruun & Taylor PRL (2008)]

$U = mD^2 k_F$

Modulated density phase in 2D dipolar Fermi gases
Thanks to

Meera Parish  
Cavendish Laboratory,  
University of Cambridge, UK

Peter Littlewood  
Argonne National Laboratory