Exercises week 1

September 28, 2016

Please submit your work before the following class (name the script file as your_name_problems-week1.m) at the following e-mail address francesca.marchetti@uam.es.

1 Exercise:

Consider the following matrices

\[ A = \begin{pmatrix} 4 & 2 & 1 \\ 5 & 9 & 12 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 2 & -7 \\ 9 & 2 & 0 \end{pmatrix}, \quad C = \begin{pmatrix} 2 & 5 \\ -3 & 2 \\ 5 & -9 \end{pmatrix}. \]

1. Consider the following operations \( A \ast B, A \ast C, A \ast C', B \ast C, B \ast C', A \ast C, C \ast A, A \ast B, A \ast B' \), and determine which of these operations is valid and explain the result;

2. Explain what is the difference between the operation \( \ast \) and \( \ast \). When can you use one and when the other?

3. Evaluate \( A \wedge 2 \) and explain the result; why you cannot consider \( A \wedge 2' \)?

2 Exercise:

Define two row vectors \( a \) and \( b \) of 4 elements each: \( a \) has the first even numbers (2,4,6,8) and \( b \) the first odd numbers in reverse order (7,5,3,1) — use a different definition than the trivial one!

1. Find two equivalent ways to define the vector dot product between the two vectors \( \sum_{i=1}^{4} a(i)b(i) \);

2. Find two equivalent ways to define the modulus of each vector;

3. Evaluate the angle between \( a \) and \( b \) in radians and degrees;

4. Describe which kind of matrix/vector one gets by considering \( a \ast b', a' \ast b, a \ast b, (b \ast a)' \).

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3 Exercise:

Plot the following functions in the required intervals:

\[ f_1(x) = x^2 + x - 4 \quad x \in [-3, 2] \]
\[ f_2(x) = e^{x^2} - 3x^2 \quad x \in [-1.5, 1.5] \]
\[ f_3(x) = \ln(x + 3) - x^2 \quad x \in [-1, 3] \]

For each function and interval find the local extrema (minima and maxima), as well as the global ones, and compare the results you get with the ones you obtain analytically.

4 Exercise:

It is in general convenient to plot power-laws \((f(x) = x^n)\) in logarithmic scale, as in the following example

```python
x=logspace(log10(1), log10(1000), 100)
plot(log10(x), log10(x.^3), 'o')
loglog(x,x.^3,'+')
```

1. What is the difference between `plot()` and `loglog()`?
2. Can you determine the power-law exponent from the plots in logarithmic scale?
3. What happens if you use `linspace()` rather than `logspace()`?

5 Exercise:

The number \(e\) can be equivalently defined as

\[ e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n = \sum_{n=0}^{\infty} \frac{1}{n!} \]

Find an estimate of \(e\) by using both the definitions given above and compare them with the built-in value of Matlab (or `exp(1)`) — for summing the vector components you can use the command `sum` or you can find an equivalent way of doing it by using the multiplication operation between vectors.

6 Exercise:

Write a script that evaluates the factorial \(n!\) of a given natural number \(n\) and compare the results with the built-in function `factorial(n)` — remember that the factorial is defined as \(n! = n(n-1)(n-2)\ldots2\times1\); Hint: store the result in a variable \(f\) that needs to be initialised to \(f=1\) prior to the loop.