Polarised Fermi Superfluids
Phase Diagram & Dynamics

F.M. Marchetti

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Why atomic gases?

Search for novel phases of quantum coherent matter

- Tuning the interaction strength
- Mixtures of different statistics
- Optical lattices
- 1D, 2D
- ...
Fermi superfluids
Polarised Fermi superfluids

Can superfluidity persist in presence of a population imbalance?
Why interesting?

- Magnetised superconductors (Zeeman)
- Quantum Chromodynamics (and neutron stars)
- Electron-hole bilayers
BEC-BCS crossover in electron-hole systems!


- Microcavity polaritons
  - (1/2-exciton 1/2-photon quasi-particles)

[See M. Szymanska’s talk]

[Discussing microcavity polaritons, referring to J. Kasprzak et al., Nature 443, 409 (2006).]
Outline

- BEC-BCS crossover

1. Unbalanced populations
   - Homogeneous phase diagram: $T=0$ & finite $T$
   - Trap
   - Experiments

2. Unequal masses

3. Dynamics of phase separation

- Conclusions & prospectives
3. [A. Lamacraft & F.M. Marchetti, preprint cond-mat/0701692]
4. [F.M. Marchetti, C. Mathy, & M.M. Parish, (related work on BF mixtures!)]
Feshbach resonances

[C. A. Regal et al., Nature 424, 47 (2003)]
Feshbach resonances

[C. A. Regal et al., Nature 424, 47 (2003)]
Feshbach resonances

\[ C. A. \text{Regal et al., Nature 424, 47 (2003)} \]
At $T=0$, described by the same ground state

$$e^{\lambda \sum_k \varphi_k c_{k,\uparrow}^\dagger c_{-k,\downarrow}^\dagger} |0\rangle = \prod_k \left( u_k + v_k c_{k,\uparrow}^\dagger c_{-k,\downarrow}^\dagger \right) |0\rangle$$

- weakly repulsive diatomic molecules
- weakly attractive fermionic atoms
Finite T BEC-BCS crossover

[BEC: condensate forms out of preformed molecules]


BCS: pairing instability

\[ \Delta_{\text{BCS}} \sim k_B T_{\text{BCS}} \ll \varepsilon_F \]

\[ T_{\text{BEC}} \sim T_F \ll T_{\text{diss}} \]
Finite T BEC-BCS crossover


BCS: pairing instability

\[ \Delta_{\text{BCS}} \sim k_B T_{\text{BCS}} \ll \varepsilon_F \]

BEC: condensate forms out of preformed molecules

\[ T_{\text{BEC}} \sim T_F \ll T_{\text{diss}} \]
What if not every fermion can pair up?
**Single-channel model**

\[
\hat{H} = \sum_{\sigma=\uparrow,\downarrow} \mu_{\sigma} \hat{N}_{\sigma} = \sum_{\mathbf{k},\sigma} (\epsilon_{\mathbf{k}} - \mu_{\sigma}) \hat{c}_{\mathbf{k}\sigma}^\dagger \hat{c}_{\mathbf{k}\sigma} + \frac{U}{V} \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} \hat{c}_{\mathbf{k}\uparrow}^\dagger \hat{c}_{\mathbf{k}'\downarrow}^\dagger \hat{c}_{\mathbf{k}\downarrow} \hat{c}_{\mathbf{k}\uparrow}
\]

- **Contact interaction**
  \[
  \frac{1}{U} = \frac{m}{4\pi a} - \frac{1}{V} \sum_{\mathbf{k}} \frac{1}{2\epsilon_{\mathbf{k}}}
  \]

- **Allow for different populations**
  \[
  \hat{n}_{\uparrow} = \frac{1}{V} \sum_{\mathbf{k}} \hat{c}_{\mathbf{k}\uparrow}^\dagger \hat{c}_{\mathbf{k}\uparrow}
  \]
  \[
  \hat{n}_{\downarrow} = \frac{1}{V} \sum_{\mathbf{k}} \hat{c}_{\mathbf{k}\downarrow}^\dagger \hat{c}_{\mathbf{k}\downarrow}
  \]

- **Averaged chemical potential & ‘Zeeman’ term**
  [total density & population imbalance (or ‘magnetisation’)]
  \[
  \mu = (\mu_{\uparrow} + \mu_{\downarrow})/2
  \]
  \[
  h = (\mu_{\uparrow} - \mu_{\downarrow})/2
  \]
  \[
  \hat{n} = \hat{n}_{\uparrow} + \hat{n}_{\downarrow}
  \]
  \[
  \hat{m} = \hat{n}_{\uparrow} - \hat{n}_{\downarrow}
  \]
Analogy with magnetised superconductors

- A population imbalance like a Zeeman term in a superconductor

\[ H_{\text{BdG}} = \begin{pmatrix} \epsilon_k - \mu - h & -\Delta \\ -\Delta & -(\epsilon_k - \mu) - h \end{pmatrix} \]

- Neglect the orbital effect?
T=0 magnetised superconductors

BCS side of the resonance

$\frac{1}{\sqrt{2}} \frac{h}{\Delta_{\text{BCS}}}$

$0^\text{th}$

SF (BCS)

T ≠ 0 magnetised superconductor

BCS side of the resonance

\[ T_c/\Delta_{BCS} \]

\[ \frac{1}{\sqrt{2}} \]

\[ h/\Delta_{BCS} \]

0.57

2\text{nd}

N

1\text{st}

tricritical point

SF

PS

population imbalance

\[ m = n_\uparrow - n_\downarrow \]

Analogy with $^3\text{He}-^4\text{He}$ mixtures

$^3\text{He}-^4\text{He} = \text{Bose-Fermi mixture}$…
… and the polarised Fermi gas is a Bose-Fermi mixture on the BEC side of the resonance

Expect the same structure on the BEC side?
Paired states start to be depleted when:

1. BCS side

\[ h = \min_k E_k = \Delta \ll \varepsilon_F \]

2. BEC side

\[ h = \min_k E_k = \sqrt{\Delta^2 + \mu^2} \sim \mu \gg \varepsilon_F \]

\[ E_{k,\sigma=\uparrow,\downarrow} = \sqrt{\left(\epsilon_k - \mu\right)^2 + \Delta^2} \pm h \]
Mean-field grand-canonical free energy

\[ \Omega^{(0)}(\mu, h) = \min_{\Delta} f^{(0)}(\Delta; \mu, h) \]

\[ f^{(0)}(\frac{\Delta}{|\mu|}, \frac{h}{|\mu|}) \]

\[
\begin{align*}
    n &= -\frac{\partial \Omega^{(0)}}{\partial \mu} \\
    m &= -\frac{\partial \Omega^{(0)}}{\partial h}
\end{align*}
\]

\[ \text{polarisation} \]

\[ \frac{m}{n} \]
1\textsuperscript{st} order phase transition

\[ \frac{1}{k_Fa} < \left( \frac{1}{k_Fa} \right)_{\text{tricrit}} \]

and

\[ \frac{T}{\varepsilon_F} < \left( \frac{T}{\varepsilon_F} \right)_{\text{tricrit}} \]

(oil&water)

\[ f(0) \left( \frac{\eta}{|\mu|} \right) \]

Increasing $h/|\mu|$

Phase separation

$\Delta / |\mu|$
$1^{st}$ order phase transition

$$\frac{1}{k_F a} < \left( \frac{1}{k_F a} \right)_{\text{tricrit}}$$

and

$$\frac{T}{\varepsilon_F} < \left( \frac{T}{\varepsilon_F} \right)_{\text{tricrit}}$$

and

$$\frac{\Omega(0)}{\Omega} < -1$$

$\frac{h}{|\mu|} \rightarrow SF$

$\frac{\Delta}{|\mu|} \rightarrow \text{phase separation}$
2\textsuperscript{nd} order phase transition

\[ \frac{1}{k_F a} > \left( \frac{1}{k_F a} \right)_{\text{tricrit}} \]

or

\[ \frac{T}{\varepsilon_F} > \left( \frac{T}{\varepsilon_F} \right)_{\text{tricrit}} \]

(towards BEC or high temperature)
T=0 phase diagram

[M. Parish, F.M. Marchetti et al., Nature Physics 3, 124 (2007)]

\[ n = n_\uparrow + n_\downarrow \]
\[ m = n_\uparrow - n_\downarrow \]
**$\text{SF}_M$ phase: Breached Pairing**

Always unstable even for unequal masses.

[W. V. Liu & F. Wilczek, PRL 90 047002 (2003)]

[M. Parish, F.M. Marchetti et al., Nature Physics 3, 124 (2007)]

[M. Parish, F.M. Marchetti et al., PRL 98, 160402 (2007)]
Finite T phase diagram

[M. Parish, F.M. Marchetti et al., *Nature Physics* 3, 124 (2007)]

\[ n = n_{\uparrow} + n_{\downarrow} \]
\[ m = n_{\uparrow} - n_{\downarrow} \]
Adding pair fluctuations (finite $T$)

One loop correction to mean-field $T_c$ ($\Delta = 0$)

$$\Omega(\mu, h) = \Omega^{(0)}(\mu, h) + \Omega^{(1)}(\mu, h)$$

$$n = -\frac{\partial \Omega}{\partial \mu} = n^{(0)} + n^{(1)}$$

$$m = -\frac{\partial \Omega}{\partial h} = m^{(0)} + m^{(1)}$$

condensed pairs + qp’s
thermal pairs

Finite T phase diagram

[M. Parish, F.M. Marchetti et al., Nature Physics 3, 124 (2007)]

\( n = n_\uparrow + n_\downarrow \)

\( m = n_\uparrow - n_\downarrow \)
Finite T phase diagram

\[ \frac{T}{\varepsilon_F} \]

\[ \frac{m}{n} \]

\[ n = n_{\uparrow} + n_{\downarrow} \]

\[ m = n_{\uparrow} - n_{\downarrow} \]

[1st, 2nd, N, SF, PS, FFLO]

[M. Parish, F.M. Marchetti et al., Nature Physics 3, 124 (2007)]
Single- vs. two-channel model

[A. Andreev et al., PRL 93, 130402 (2004)]

\[ \hat{H}_{1C} = \sum_{k,\sigma=\uparrow,\downarrow} \epsilon_k c_{\sigma}^\dagger c_{\sigma} + \frac{U}{V} \sum_{k,k',q} c_{\uparrow}^\dagger c_{\uparrow}^\dagger c_{\downarrow} c_{\downarrow} \]

\[ \hat{H}_{2C} = \sum_{k,\sigma} \epsilon_k c_{\sigma}^\dagger c_{\sigma} + \sum_{k} \left( \frac{\epsilon_k}{2} + \delta_0 \right) b^\dagger b + \frac{g}{\sqrt{V}} \sum_{k,k'} (b c_{\uparrow}^\dagger c_{\downarrow}^\dagger + \text{h.c.}) \]

- The single-channel model is recovered in the limit

\[ \frac{4\pi a}{m} = \frac{-g^2}{\delta_0 - \frac{g^2}{V} \sum_k \frac{1}{2\epsilon_k}} \equiv \frac{-g^2}{\delta} \rightarrow \infty = \text{const} \]

- \( g \) can be a small parameter (narrow resonances) and controls the fluctuations corrections above mean-field
Single- vs. two-channel model

[A. Andreev et al., PRL 93, 130402 (2004)]
[M. Parish, F.M. Marchetti et al., Nature Physics 3, 124 (2007)]
Trapped Fermi Gases

- LDA
  \[ \mu_{\uparrow,\downarrow}(r) = \mu_{\uparrow,\downarrow} - V(r) \]

\[ \mu \text{ decreases} \]

\[ \mu_{\uparrow} = \mu + h \]

\[ \mu_{\downarrow} = \mu - h \]
Phase Diagram for Trapped Gases

\[ \frac{1}{k_F a} = 0 \]

\[ \frac{T_1}{h_1} \]

\[ \frac{T_2}{h_2} < \frac{T_1}{h_1} \]

[M. Parish, F.M. Marchetti et al., Nature Physics 3, 124 (2007)]
Experiments on Imbalanced Fermi Clouds

- In-situ imaging of phase separation (3D density distribution $n_{\uparrow,\downarrow}(\mathbf{r})$)

[Y. Shin et al., PRL 97, 030401 (2006)]
Experiments on Imbalanced Fermi Clouds

- Sharp phase boundary at low temperatures (1\textsuperscript{st} order transition)

\[ T < 0.05 \, T_F \]
\[ m/n = 0.35 \]

[G. B. Partridge \textit{et al.}, PRL 97, 190407 (2006)]
Temperature Dependence of Phase Separation

\[ T/T_F < 0.05 \]

\[ T/T_F = 0.2 \]

[G. B. Partridge et al., PRL 97, 190407 (2006)]
Unequal Masses

\[
\frac{m_{\downarrow}}{m_{\uparrow}} = 0.5 \\
\frac{m_{\downarrow}}{m_{\uparrow}} = 2 \\
\frac{m_{\downarrow}}{m_{\uparrow}} = 10
\]

[M. Parish, F.M. Marchetti et al., PRL 98, 160402 (2007)]
Unequal Masses

[M. Parish, F.M. Marchetti et al., PRL 98, 160402 (2007)]
Dynamics of Phase Separation

[A. Lamacraft & F.M. Marchetti, cond-mat/0701692]
Dynamics of Phase Separation

- Spinodal: phase separation starts via a linear instability

[A. Lamacraft & F.M. Marchetti, cond-mat/0701692]
Spinodal Decomposition

- Early time dynamics

\[ \ell_{\text{unst}} = \frac{1}{q_{\text{unst}}} \]

\[ t_{\text{unst}} = \frac{1}{\omega_{\text{unst}}} \]

- E.g. Temperature quenches in polymers in solutions,…

[Courtesy of Nigel Clarke, Polymer IRC]
Spinodal Region

\[ \tilde{\Omega}^{(0)} \left( \frac{m}{n} \right) \]

PS

metastable

unstable

metastable

PS
Unstable Modes

- Matrix response function (to changes of the density)

\[
\frac{m}{n} = 0.5
\]

[A. Lamacraft & F.M. Marchetti, cond-mat/0701692]
Most Unstable Modes

- Characteristic length and time scales (for $T_F = 1 \mu K$)
  
  $\ell_{\text{unst}} \approx 0.1 \mu m$  
  $t_{\text{unst}} \approx 400 \mu s$

$\frac{m}{n} = 0.5$

[A. Lamacraft & F.M. Marchetti, cond-mat/0701692]
Conclusions

1. Phase diagram of polarised Fermi superfluids

2. Probing the order of the transition in experiments

3. Dynamics of phase separation
Some future work

- Bose-Fermi mixtures (with a Feshbach resonance)

(with C. Mathy and M. Parish)