Introducción a la Superconductividad y Superfluidez

Curso de Mecánica Estadística Avanzada

• asignatura optativa (segundo semestre)
• 6 créditos
Introduction to BEC & superfluidity
Macroscopic occupation of the ground state

Introduction to BEC & superfluidity

High temperatures: billiard balls

Quantum-statistical phase transition (it occurs even in absence of interactions)

$\lambda_T = \left( \frac{2\pi\hbar^2}{mk_BT} \right)^{1/2}$

$n\lambda_T^3 \geq 2.612$

Giant matter-wave
BEC: from 1925 to 1995

• 1924/1925
Following the work of Bose on the statistical description of light quanta, Einstein predicted that a gas of non-interacting massive bosons, below a critical temperature, undergoes a phase transition associated with the condensation of the atoms in the lowest energy state: Bose-Einstein condensation (BEC)

“...A separation is affected; some part condenses, the rest remains a ‘saturated ideal gas’...”

“...condensation without attractive forces...”

Satyendranath Bose  Albert Einstein
BEC

NON INTERACTING IDEAL BOSE GAS

⇒ paradigm of quantum statistical mechanics
  • indistinguishable particles
  • wave nature of particles
  • thermal equilibrium

⇒ macroscopic quantum phenomena
  • macroscopic wavefunction
    (many-body ground state wavefunction is the product of N identical single-particle ground-state wavefunctions)
• **1938** Discovery of superfluidity in liquid helium $^4$He (Allen & Misener; Kapitza).
**BEC: from 1925 to 1995**

- **1938** Discovery of superfluidity in liquid helium $^4$He (Allen & Misener; Kapitza).

What’s the relation between BEC & superfluidity?

- Immediately after, London suggested the connection between the superfluidity of $^4$He and BEC

  ⇒ first to bring out the idea of BEC displaying quantum behaviour on a macroscopic scale

- It was a source of debate for decades
- e.g., the Landau’s criterion for superfluidity does not explicitly mention the notion of BEC
- it is now recognised that superfluidity in $^4$He is related to BEC (though, because of strong interactions in $^4$He, there is a strong reduction of the lowest energy state occupancy)
BEC: from 1925 to 1995

• **1947** microscopic theory of interacting Bose gases (Bogoliubov)
  
  …provides the microscopic picture behind Landau’s theory

• **1951** off-diagonal long range order (Landau & Lifshitz; Penrose)

• and much more theoretical work… (which we are going to see in class)
<table>
<thead>
<tr>
<th>BEC vs. Superfluidity</th>
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<tbody>
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<td><strong>INTERACTING BOSE GAS</strong></td>
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<td>⇒ mainly related to transport phenomena (flow without friction)</td>
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<td>• is essential the form of the dispersion of the elementary excitations (Landau criterion)</td>
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<td>• interactions are essential (change the dispersion from quadratic to phonon-like)</td>
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<td>• superfluidity is possible even with few% of atoms in the ground state (see $^{4}$He)</td>
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**IDEAL BECs ARE NOT SUPERFLUID**

**LINK BETWEEN BEC & SUPERFLUIDITY: ORDER PARAMETER and ODLRO**
BEC in other systems

- BEC is involved in several macroscopic quantum phenomena (even if some systems are not ideal Bose gases):
  - $^4$He (but is a strongly interacting system)
  - superconductors (BEC of Cooper pairs)
    - (we will see the BEC-BCS crossover)
  - $^3$He (also fermions)
  - lasers (but out of equilibrium: requires inversion of the population)
  - ...

**QUEST TO REALISE A BEC:**
Search of weakly interacting Bose gases
Searching for weakly interacting Bose gases

Why so hard?

⇒ At very low T most substances are in the solid (or liquid) phase & interaction becomes strong

• BEC in its ideal form can be realised only in conditions of metastability

• Thermal equilibrium but the gas has a finite lifetime

70 years to realise a BEC in dilute atomic gases

very sophisticated cooling and trapping techniques
Dilute ultracold atomic gases: Experiments

• **1959** spin-polarized (by a magnetic field) hydrogen proposed as a good candidate for a weakly interacting Bose gas

• **'80** Developments in magnetic trapping, laser and evaporative cooling of alkali atoms

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**Diagram:**
- **cooled atom cloud**
- **vacuum chamber**
- **mirror**
- **laser beam** (circularly polarised)
- **magnet coils in anti-Helmholtz configuration**

**Periodic Table:**
- **Group 1**
  - **1 H**
- **Group 2**
  - **3 Li**
  - **11 Na**
- **Group 3**
  - **19 K**
  - **37 Rb**
  - **55 Cs**
- **Group 4**
  - **87 Fr**
First realisation of a BEC in ultracold gases

- **1995** BEC in alkali atoms ($^{87}$Rb, $^{23}$Na, $^{7}$Li, …)

\[ T \sim 500\text{nK} - \mu\text{K} \]
\[ n \sim 10^{11} - 10^{13}\text{cm}^{-3} \]

Coolest system in the universe!

Nobel prize (2001)

Carl Wieman & Eric Cornell  Wolfgang Ketterle
BEC & superfluidity

- Landau criterion
- Macroscopic phase coherence

[Andrews et al. Science (1997)]

[From R. Grimm’s group]
BEC & superfluidity

- Landau criterion
- Macroscopic phase coherence
- Quantised vortices
  (rotating condensates)

[Abo-Shaeer et al. Science (2001)]

[Andrews et al. Science (1997)]

Introduction to BEC & superfluidity
BEC & superfluidity

• Landau criterion
• Macroscopic phase coherence
• Quantised vortices
• Metastable persistent flow

[Abo-Shaeer et al. Science (2001)]

[Andrews et al. Science (1997)]

[Ryu et al. PRL (2007)]
Ultracold atoms today

• Tune the interaction strength (Feshbach resonances)

• Bosons, Fermions, mixtures

• Optical lattices

• Reduced dimensions (2D, 1D, 0D)

• Disorder

Simulate solid state systems (with advantage of external control)

[Bloch Nature Physics (2005)]

Introduction to BEC & superfluidity
Synopsis for the first half of the course

1. BEC: ideal Bose gas & weak interactions
2. BEC & superfluidity (Landau criterion and response to a moving defect)
3. Gross-Pitaevskii equation (non uniform condensates)
4. experiments in ultracold atoms (elements)

5. Applications: interference between two BECs
   ⇒ Josephson coupling & oscillations: analogy with superconductors
6. Applications: rotating BEC & vortices
   ⇒ vortex lattices
7. ...

Introduction to BEC & superfluidity
List of possible topics for the presentation

1. How to measure the speed of sound in a superfluid (ultracold atomic BEC)
2. Defect moving through a superfluid and Cherenkov radiation
3. Interference between two expanding condensates and the Josephson effect
4. Rotating superfluids and vortices
5. ...
6. BEC-BCS crossover
7. Imbalanced Fermi mixtures and analogy with a BCS superconductor in a Zeeman magnetic field
8. Your proposals!
Moving defect in a BEC & Cherenkov radiation

\[ v < v_c \]

moving defect
Moving defect in a BEC & Cherenkov radiation

\[ u > u_c \]

-moving defect-

[from E. Cornell’s group]
Moving defect in a BEC & Cherenkov radiation

$v_c = \min \frac{E_k}{k} = c_s$

sound velocity $c_s = \sqrt{\frac{gn}{m}}$
Moving defect in a BEC & Cherenkov radiation

\[ c_s - v > 0 \]

\[ v_c = \min \frac{E_k}{\kappa} = c_s \]

- \[ E_k \]
- \[ c_s \]
- \[ \kappa \]

- \[ (v^2 - c_s^2)/v \]
- \[ F_d \text{ [arb. units]} \]

- \[ [0, 0.5, 1, 1.5, 2] \]
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[G. E. Astrakharchik and L. P. Pitaevskii, PRA (2004)]
[I. Carusotto et al., PRL (2006)]
Moving defect in a BEC & Cherenkov radiation

$E_k$

$c_s - \nu < 0$

$v_c = \min \frac{E_k}{k} = c_s$

$F_d \text{ [arb. units]}$

$\left( \frac{v^2 - c_s^2}{v} \right)$

[G. E. Astrakharchik and L. P. Pitaevskii, PRA (2004)]
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Introduction to BEC & superfluidity
BEC-BCS crossover

• Tune the interaction strength (Feshbach resonances)
BEC-BCS crossover

- Tune the interaction strength (Feshbach resonances)

\[ \frac{1}{k_F a} \]

- \( a(B) \)

- \( \sqrt{2m\Delta} \)

- \( k_X \)

- \( k_Y \)

BEC

BCS
Imbalanced Fermi mixtures

• Tune the interaction strength (Feshbach resonances)

Can superfluidity persist in presence of a population imbalance?

Analogy with a superconductor in a magnetic Zeeman field

\[
\frac{1}{k_F a}
\]

\[a(B)\]

\[n_\uparrow - n_\downarrow\]

BEC

BCS