Second-harmonic generation using 4-quasi-phasematching in a GaAs whispering-gallery-mode microcavity

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The 4 crystal symmetry in materials such as GaAs can enable quasi-phasematching for efficient optical frequency conversion without poling, twinning or other engineered domain inversions. 4 symmetry means that a 90° rotation is equivalent to a crystallographic inversion. Therefore, when light circulates about the 4 axis, as in GaAs whispering-gallery-mode microdisks, it encounters effective domain inversions that can produce quasi-phasematching. Microdisk resonators also offer resonant field enhancement, resulting in highly efficient frequency conversion in micrometre-scale volumes. These devices can be integrated in photonic circuits as compact frequency convertors, sources of radiation or entangled photons. Here we present the first experimental observation of second-harmonic generation in a whispering-gallery-mode microcavity utilizing 4-quasi-phasematching. We use a tapered fibre to couple into the 5-μm diameter microdisk resonator, resulting in a normalized conversion efficiency η ≈ 5 × 10⁻⁵ mW⁻¹. Simulations indicate that when accounting for fibre-cavity scattering, the normalized conversion efficiency is η ≈ 3 × 10⁻³ mW⁻¹.
Frequency conversion can result from a nonlinear interaction of light with a material, where the material's dielectric polarization responds nonlinearly with the incident electric field. Second-harmonic (SH) generation was observed in quartz more than 50 years ago, when the newly developed laser was essential to observe the nonlinearity. Efficient optical frequency conversion using these nonlinear processes requires the phases of the driving polarization and generated electric field be matched or properly compensated. Consider sum-frequency generation where \( \omega_2 = \omega_1 + \omega_2 \) (SH generation (SHG) is obtained if \( \omega_2 = \omega_3 \)). For the general case of sum-frequency generation, the generated electric field at frequency \( \omega_3 \) propagates as \( E_3 \propto \exp(-ik_2z) \), while the nonlinear driving polarization propagates as \( P_{\text{in}}(\omega_1) \propto \gamma^{(2)} E_1 E_2 \propto \exp[-i(k_1 + k_2)z] \), where \( k_i \) is the wavevector at frequency \( \omega_i \) and \( \gamma^{(2)} \) is the second-order nonlinear coefficient. Perfect phasematching is achieved when \( k_1 = k_2 + k_3 \) and can be obtained by, for instance, mixing orthogonally polarized waves in a birefringent crystal. Efficient conversion can also be attained using quasi-phasematching (QPM), where \( \gamma^{(2)} \) is modulated at period \( A = 2\pi|k_1 - k_2 + k_3| \) (\( j \) is an integer). This modulation can be produced by periodic domain inversions where \( \gamma^{(2)} \) changes sign, by on-off modulation, or by modulation in the magnitude of \( \gamma^{(2)} \) (ref. 7). Materials such as GaAs that possess 4 crystal symmetry can exhibit an effective \( \gamma^{(2)} \) modulation when the fields propagate in curved geometries (such as in microrings or microdisks). A 90° rotation about the 4 axis is the same as a crystallographic inversion, and hence fields propagating along the 4 axis (\( \langle 001 \rangle \)) in an uniform GaAs microdisk effectively encounter four domain inversions per round trip. Thus, the 4 crystal symmetry allows QPM to be achieved without externally produced domain inversions. GaAs is particularly attractive for nonlinear frequency conversion, as it features a high nonlinear susceptibility, higher than that of LiNbO₃ and GaP.

Second-order nonlinear mixing has been achieved in other whispering galaxy-mode resonators using other forms of phasematching. Efficient SHG has been observed in millimetre-scale disk resonators made of periodically poled LiNbO₃ (ref. 13) where QPM was achieved using engineered domain inversions. Birefringent phasematching was utilized to demonstrate efficient conversion in uniform LiNbO₃ disk resonators (ref. 14, 15). SHG was observed in a GaP photonic crystal cavity (ref. 16). Finally, frequency conversion using four-wave mixing (\( \chi^{(3)} \)) has been shown in GaAs microring resonators (ref. 17).

We present here the first experimental observation of SHG in a whispering-gallery-mode resonator utilizing 4 quasi phasematching. The GaAs-based 4 QPM whispering galaxy-mode resonator allows for efficient SHG in micrometre-scale structures that are compatible with integrated photonic architecture. These resonators could be used in all-optical circuits (ref. 18, 19) and as integrated photonic chip sources of classical and non-classical light (ref. 20, 21).

**Results**

**Basic concepts.** SHG in a \( \langle 001 \rangle \)-surface-normal GaAs microdisk (Fig. 1) occurs between a TE-polarized (electric field in the plane of the disk) fundamental (\( f \)) wave and a TM-polarized (electric field orthogonal to the disk plane) SH wave. These polarizations are dictated by the \( \chi^{(2)} \) tensor for GaAs and other 43\( m \) point-group materials, which has only one non-zero susceptibility tensor element, \( d_{13} = d_{42} \). Writing the electric fields at \( \omega_i \) as \( E_i = A_i(\theta)e^{i(\omega_i t + m_i \phi)} \), where \( i = f \) or SH, \( A_i(\theta) \) is the slowly varying amplitude and \( m_i \) is the azimuthal number; the change in the SH amplitude with propagation angle \( \theta \) is (refs 8, 11)

\[
\frac{dA_{\text{SH}}}{d\theta} = A^2_{f} \left( K_f e^{i(\Delta m + 2)\theta} + K_{-f} e^{i(\Delta m - 2)\theta} \right). \tag{1}
\]

Figure 1 | Schematic of second-harmonic generation in a fibre-taper-coupled GaAs microdisk. Input light at the fundamental wavelength (represented by red light) is converted inside the microdisk to second-harmonic light (represented by blue light). The inset shows a scanning electron micrograph of a fabricated device. The scale bar is 1 µm.

\( K_{+} \) and \( K_{-} \) are the SHG coefficients calculated from mode-overlap integrals (see Supplementary Discussion) and \( \Delta m = m_{\text{SH}} - 2m_f \).

\[
A_{\text{SH}}(2\pi) - A_{\text{SH}}(0) = 2\pi A^2_{f} \left( K_{+} e^{i(\Delta m + 2)\pi} \text{sinc}[(\Delta m + 2)\pi] + K_{-} e^{i(\Delta m - 2)\pi} \text{sinc}[(\Delta m - 2)\pi] \right) = A^2_{f} K 
\tag{2}
\]

Optimizing conversion with off-resonance waves. While the fundamental wavelength must equal twice the generated second-harmonic wavelength by energy conservation (\( \lambda_{f} = 2\lambda_{\text{SH}} \)), the microdisk resonances are not guaranteed to be properly spaced to satisfy \( \lambda_{\text{f NS}} = 2\lambda_{\text{SH NS}} \), where \( \lambda_{\text{f NS}} \) are resonance wavelengths. The most efficient SHG is obtained when both the fundamental and SH waves are resonant with the microdisk (\( \lambda_{f} = \lambda_{\text{f NS}} \) and \( \lambda_{\text{SH NS}} = \lambda_{\text{f NS}} \)) and the 4-QPM condition is satisfied (\( \Delta m = \pm 2 \)). However, frequency conversion can still be obtained if one of the waves is off-resonance with the microdisk (\( \lambda_{f} = 2\lambda_{\text{SH NS}} \) but \( \lambda_{\text{SH NS}} \not= \lambda_{\text{f NS}} \)), which leads to partially doubly resonant or singly resonant SHG. The generated second-harmonic spectrum, as a function of the fundamental wavelength, is proportional (see Supplementary Discussion)

\[
P_{\text{SH NS}}^{\text{f NS}}(\lambda_{f}) \propto \left( P_{\text{f NS}}^{\text{circ}}(\lambda_{f}) \right)^{2} \left( P_{\text{circ}}^{\text{f NS}}(\lambda_{f}/2) \right)^{2} (K(\lambda_{f}))^{2}, \tag{3}
\]

where \( P_{\text{f NS}}^{\text{f NS}}(\lambda_{f}) \) and \( P_{\text{circ}}^{\text{f NS}}(\lambda_{f}/2) \) are the circulating-power spectra of the microdisk cavity in the fundamental and SH wavelength ranges, and \( K \) is the SH gain coefficient defined by Equation (2). When light at \( \lambda_{f} \) is coupled into the microresonator, the high circulating fundamental power generates a SH wave at \( \lambda_{f}/2 \) that we desire to overlap with a resonance in the \( P_{\text{circ}}^{\text{f NS}}(\lambda_{f}/2) \) spectrum. Higher quality factors can increase SHG by increasing the resonating powers but make it more difficult to achieve overlap in the \( P_{\text{circ}}^{\text{f NS}}(\lambda_{f}) \) and \( P_{\text{circ}}^{\text{f NS}}(\lambda_{f}/2) \) spectra.

The amount of SH conversion depends on the detuning of the resonances, \( |\lambda^{\text{f NS}}_{\text{f NS}} - 2\lambda_{\text{SH NS}}| \), compared with their linewidths. The resonance linewidths depend on the coupling (\( \Omega_{f} \)) and intrinsic (\( \Omega_{f}^{\text{circ}} \)) quality factors of the microdisk. \( \Omega_{f}^{\text{circ}} \) are typically fixed by...
fabrication, while $Q_i$ can be varied by adjusting coupling to the microdisk. When the resonances are aligned ($|\Delta l_{\text{res}} - 2\Delta Q_{\text{SH}}| = 0$), maximum conversion is obtained at critical coupling ($Q_i = Q_0$ at both wavelengths). When the resonances are not aligned ($|\Delta l_{\text{res}} - 2\Delta Q_{\text{SH}}| \neq 0$), higher SH conversion may be produced when the cavity is overcoupled ($Q_i < Q_0$) where increased coupling broadens the resonance linewidths and produces better spectral overlap, as illustrated in Fig. 2. Using theory presented in Kuo and Solomon11, we calculated SH conversion, $\eta$, for different resonance detunings, $|\Delta l_{\text{res}} - 2\Delta Q_{\text{SH}}|$, as $Q_i$ is varied relative to $Q_0$. In Fig. 2a, $Q_i$ is varied relative to $Q_0$ while we fix $Q_{\text{SH}} = Q_{\text{SH}}$. In addition to the peak conversion at critical coupling $Q_i = Q_0$, a second maximum is obtained at $Q_i/Q_0 < 1$ when the half width at half maximum of the fundamental microdisk resonance equals the detuning. Figure 2b shows the SH conversion when $Q_{\text{SH}}$ is varied relative to $Q_{\text{SH}}$ while we fix $Q_i = Q_0$. Only when $|\Delta l_{\text{res}} - 2\Delta Q_{\text{SH}}| = 0$ is the SH conversion maximized at $Q_i = Q_{\text{SH}}$. With non-zero $|\Delta l_{\text{res}} - 2\Delta Q_{\text{SH}}|$ the conversion efficiency is largest at lower $Q_{\text{SH}}$ where the SH resonance is broadened by coupling. In general, increased detunings lower the peak SH conversion (see Fig. 2 insets); however, reducing the $Q_i/Q_0$ ratio can help mitigate the drop. Overcoupling broadens the resonance linewidths, which can lead to better spectral overlap and improved conversion.

Frequency conversion in a GaAs microdisk cavity. Experimentally, GaAs microdisks were probed using the setup sketched in Fig. 3a. Microdisk whispering-gallery-mode resonances are characterized by three integers ($m_i$, $p_i$, and $q_i$) that count the azimuthal, radial and vertical antinodes, respectively. Using a tapered optical fibre to launch and collect light from the microdisk22 (Fig. 3b), we measured microdisk transmission spectra in the fundamental (1,900–2,015 nm) and SH (950–1,007 nm) wavelength ranges and identified ($m_i$ and $p_i$) values of the resonances ($q_i = q_{\text{SH}} = 1$ for the thin microdisks used here). The resonance wavelengths and free-spectral ranges of different radial-mode families were compared with predictions from two-dimensional (2D) finite-element modelling to determine ($m_i$ and $p_i$). The microdisk thickness and radius were used as fitting parameters in the modelling, and the resulting fitted sizes were consistent with observations by scanning electron microscopy (within ±5%). We characterized $Q_i$ and $Q_{\text{SH}}$ (related to the total quality factor through $1/Q_{\text{res}}^2 = 1/Q_i^2 + 1/Q_{\text{SH}}^2$) by observing the changes in the transmission spectra as the gap between the fibre and microdisk was varied. The microdisks were then pumped with the fundamental beam, and the generated SH light was detected by a silicon detector.

We fabricated GaAs microdisks with 2.6-µm radius and 160-nm thickness using molecular-beam epitaxy, photolithography and wet chemical-etching (see Methods for details). Figures 1,3b (inset) show images of the GaAs microdisk cavities. All fabricated microdisks showed misaligned fundamental and SH resonances ($|\Delta l_{\text{res}} - 2\Delta Q_{\text{SH}}| 
eq 0$). Figure 4 plots the fundamental and SH transmission spectra for the GaAs microdisk with the smallest detuning, measured with the fibre taper contacted to the microdisk. We chose to touch the fibre to the top edge of the microdisk to increase measurement stability and increase coupling, as suggested by Fig. 2 for non-zero detuning. We identified the TE-polarized, fundamental resonance centred at 1,983.8 nm as ($m_i = 13$, $p_i = 1$). At the SH wavelength, we observed two TM-polarized resonances in the vicinity of 992 nm. By probing these resonances with various taper-disk separations and taper sizes, we determined that the resonance at 991.6 nm is ($m_{\text{SH}} = 24$, $p_{\text{SH}} = 2$) and the resonance at 992.25 nm is ($m_{\text{SH}} = 21$, $p_{\text{SH}} = 3$). We also observed the ($m_{\text{SH}} = 28$, $p_{\text{SH}} = 1$) resonance at 997 nm. Nonlinear optical mixing using the $m_i = 13$ and $m_{\text{SH}} = 21$ resonances does not occur since there is no phase-matching ($\Delta m = -5$)11, and the ($m_{\text{SH}} = 28$, $p_{\text{SH}} = 1$) resonance is too far detuned for efficient SHG. We conclude that SHG in the microdisk involves the ($m_i = 13$, $p_i = 1$) fundamental resonance at 1,983.8 nm and the ($m_{\text{SH}} = 24$, $p_{\text{SH}} = 2$) SH resonance at 991.6 nm with $\Delta m = -2$ and detuning $|\Delta l_{\text{res}} - 2\Delta Q_{\text{SH}}| = 2.2$ nm.

In Fig. 5a, we plot the observed SH conversion efficiency ($P_{\text{SH}}/P_{\text{in}}$) as a function of the fundamental pump wavelength. $P_{\text{SH}}/P_{\text{in}}$ are powers inside the fiber taper adjacent to the GaAs microdisk and are calculated based on the measured taper losses and detected powers. For this microdisk with $|\Delta l_{\text{res}} - 2\Delta Q_{\text{SH}}|$, SH conversion is maximized when the fundamental is on-resonance while the SH is off-resonance, as seen in Fig. 5a top inset. Figure 5b,c plots the generated SH power and the SHG conversion efficiency in the tapered fibre, which we call the

**Figure 2 | Dependence of theoretical SH conversion efficiency on resonance detuning and quality-factor ratio.** $\eta$ is normalized to its peak value and shown as a function of the ratio of coupling to intrinsic quality factors ($Q_i/Q_0^2$) for several detunings ($|\Delta l_{\text{res}} - 2\Delta Q_{\text{SH}}| = 0, 0.5, 1.0$ and 2.0 nm). It is calculated for doubling $\Delta l_{\text{t}} \approx 1,990$ nm in a 2.6-µm-radius, 160-nm-thick GaAs microdisk where $\Delta m = -2$. The intrinsic quality factors are $Q_0^2 = Q_{\text{SH}}^2 = 20,000$. In a, critical coupling is the SH resonance ($Q_{\text{SH}} = Q_{\text{SH}}$), while $Q_i$ is varied relative to $Q_0$, while in b, critical coupling is assumed for the fundamental resonance ($Q_i = Q_0$) while $Q_{\text{SH}}$ is varied relative to $Q_{\text{SH}}$. When $|\Delta l_{\text{res}} - 2\Delta Q_{\text{SH}}| \neq 0$, optimum SH conversion may be obtained by lowering $Q_i$ and over-coupling until the half width at half maximum (HWHM) equals the detuning. The insets plot the value of maximum SH conversion efficiency ($\eta_{\text{max}}$) for the different detunings. Larger detunings lead to lower conversion efficiency; however, over-coupling to broaden linewidths can improve conversion.
external conversion efficiency, for $\lambda_l = 1,985.38$ nm. At low pump power, the logarithmic plot of generated SH power (Fig. 5b) has a slope of 2.15 ± 0.10, confirming quadratic growth of the second harmonic. The fitted slope at low power in Fig. 5c indicates normalized external conversion efficiency $\eta = (5.2 ± 0.4) \times 10^{-5}$ mW$^{-1}$. Note that this value of the conversion efficiency corresponds to absolute value and not percentage. Unless otherwise stated, this convention is assumed for all values of conversion efficiency provided hereafter. At higher pump powers, the conversion efficiency at fixed $\lambda_l$ rolls off primarily due to shifting of the fundamental resonance wavelength.

Modelling and simulations. We model the conversion efficiency using two methods. We first used the semi-analytical coupled-mode theory (CMT)\(^{11}\) to calculate the theoretical conversion efficiency (for further details, see the Supplementary Discussion associated with Supplementary Fig. 1 and Supplementary Table 1). From Fig. 4, we observed $Q_{\text{SH}}^{\text{tot}} = 16,000$ and $Q_{\text{SH}}^{\text{tot}} = 4,000$. We estimated intrinsic quality factors from transmission spectra taken with the taper several hundred nanometres separated from the microdisk where $1/Q_l$ is approximately zero, but where weak transmission dips from the modes remain, and found $Q_l^{\text{tot}} = 33,000$ and $Q_{\text{SH}}^{\text{tot}} = 9,000$. Using these quality factors and our observed resonance detuning $|\omega_l^{\text{res}} - \omega_{\text{SH}}^{\text{res}}| = 2.2$ nm, the theoretical $\eta$ is $1 \times 10^{-5}$ mW$^{-1}$. If the resonances of this microdisk were aligned ($|\omega_l^{\text{res}} - \omega_{\text{SH}}^{\text{res}}| = 0$), we expect $\eta = 8 \times 10^{-2}$ mW$^{-1}$, an 80-fold increase in conversion efficiency. By performing a series of repeated trials, we found an uncertainty of approximately ±20% in the experimental determination of $Q_{\text{SH}}^{\text{tot}}$ and $Q_{\text{SH}}^{\text{tot}}$, which in turn yielded an uncertainty of ±20% in the predicted conversion efficiency. Thus, the discrepancy between the CMT predictions and the experimental results cannot be ascribed to the experimental error in the determination of the quality factors in the system.

We investigated the discrepancy between the measured and CMT-derived conversion efficiencies by conducting \textit{ab initio} full three-dimensional (3D) numerical simulations of the second-order nonlinear frequency-mixing process occurring in the fibre-microdisk system. These simulations are based on both 3D finite-elements-method (3D-FEM) and 3D finite-difference-time-domain (3D-FDFTD) modelling (see Methods). The simulations allow us to observe the effect of the fibre lying in close proximity to a GaAs microdisk. The scale bar is 10 μm.

**Figure 3** | Experimental setup. In experimental diagram (a), two fibre-coupled beams at the fundamental and SH wavelengths were produced using a continuous-wave (CW), periodically poled LiNbO$_3$ (PPLN) OPO and a PPLN doubling crystal. The beams were alternately coupled into the GaAs microdisk using a tapered fibre. The monitor ($p_{\text{mon}}'$ and $p_{\text{mon}}''$) and exit ($p_{\text{exit}}''$, $p_{\text{exit}}'''$) powers were recorded by a lockin amplifier: LPF, 1,850-nm long-pass filter; VOA, variable optical attenuator; dichroic filter reflects the fundamental and transmits the second harmonic. (b) Photograph of the fibre taper in close proximity to a GaAs microdisk. The scale bar is 10 μm.

**Figure 4** | Measured transmission spectra of the microdisk cavity. (a) TE-polarized transmission spectra of the ($m_l = 13$, $p_l = 1$) resonance near 1,985 nm at two power levels. (b) TM-polarized, transmission spectrum (black) near 990 nm along with fit (green) to the ($m_{\text{SH}} = 24$, $p_{\text{SH}} = 2$) resonance.
efficiency of $\eta = 6.78 \times 10^{-5} \text{ mW}^{-1}$. This value is in excellent agreement with the experimental results. The simulations show that the discrepancy between the experimental results for the full system and the predictions from our CMT is partially accounted for by our experimental configuration, where the fibre taper is in contact with the top surface of the disk. As seen in our numerical simulations, the taper perturbs the electric field in the microdisk and causes increased scattering, visible as higher electric field densities outside the disk on the left side of Fig. 6d. This effect is not accounted for by the simple CMT but is included in our full 3D simulations. Using the quality-factor values from the experiment, the amount of stored energy in the fundamental

Figure 5 | Measure SHG spectra and dependence of SH conversion on fundamental pump power. (a) Spectrum of SH external conversion efficiency (at two fundamental powers) as a function of fundamental pump wavelength. The top inset shows the low-power resonance spectra for the fundamental and SH (in units of $2\omega_{\text{SH}}$) wavelengths. The spectrum in the region of the SH uses the fitting shown in Fig. 4b for the $m_{\text{SH}} = 24$ resonance. The SH resonance is centred at $2\omega_{\text{SH}} = 1,983.2 \text{ nm}$ (off-scale). (b) Logarithmic plot of generated second-harmonic versus pump power at 1,985.38-nm pump wavelength (blue dots). The fitted slope (red line) is $2.15 \pm 0.10$, indicating quadratic growth of SH power with pump power. (c) SHG conversion efficiency as a function of 1,985.38-nm pump power. The slope at low power indicates $(5.2 \pm 0.4) \times 10^{-5} \text{ mW}^{-1}$ normalized conversion efficiency; however the curve rolls off because of distortion of the resonance.

Figure 6 | Numerical simulations of a tapered fibre-disk system. (a) Magnetic field amplitude at the fundamental resonance evaluated in the $x$-$y$ plane at the centre of the dielectric microdisk ($z = 0$). (b) Electric field amplitude at the second-harmonic resonance evaluated in the same plane as in a. (c) Transverse polarization distribution resulting from the second-order nonlinear interaction shown in a, b. Shaded areas in the bottom region of the panels (a-c) represent the tapered fibre. Red (blue) regions in these panels indicate positive (negative) amplitudes of the corresponding field. (d) Cross-sectional view of the modulus of the fundamental electric field as computed in the $y$-$z$ plane at $x = 0$. 

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We have for the first time experimentally observed 4-quasi-phase matched SHG in a GaAs microdisk cavity. 4-QPM is a new QPM technique that relies on curved propagation geometries rather than engineered domain inversions. The observed normalized conversion efficiency is much higher than that of conventional, non-resonant devices scaled to comparable, micrometre sizes. Through better tuning techniques, the quasi-phase matched GaAs microdisk can operate more readily in the doubly resonance regime (that is, when $|\Delta^2 - 2\Delta_{SH}| = 0$) rather than singly resonant SHG, yielding a significant increase in the internal conversion efficiency. More efficient and robust coupling, similar to that produced in other microresonator systems, will also result in increased output coupling and better integration. GaAs microdisk frequency converters can be attractive, on-chip sources for infrared radiation and sensing, for squeezed light, or for the production of entangled photon pairs. New methods of sensing may be possible because of the integrated nature of infrared generation and the resonator cavity; the presence of absorbing species affects the resonant conditions and, in turn, frequency conversion in the microdisk. From a more fundamental perspective, the experimental observations reported in this work pave the way for a number of novel research avenues based on exploring the unique interplay among nonlinearity, 4-QPM and light-matter interactions at the micro- and nano-length scales.

**Methods**

**Microdisk cavity fabrication.** Molecular-beam epitaxy was used to make the sample, which consisted of a 160-nm-thick GaAs layer on top of a 1.5-μm-thick Al$_{0.75}$Ga$_{0.25}$As. As sacrificial layer. The lateral dimensions of the GaAs microdisk were defined with photolithography and a hydrobromic acid etch. A hydrofluoric acid etch was then used to remove most of the Al$_{0.75}$Ga$_{0.25}$As layer, leaving a narrow pedestal supporting the GaAs microdisk (see Fig. 1). The pedestal had negligible effect on the low-radiation order optical modes used in the experiment.

**Optical measurements.** The setup was shown in Fig. 3a. The probe beam was produced by a tunable, periodically poled LiNbO$_3$ optical parametric oscillator (OPO) (Lockheed Martin Aculight Argo), whose linewidth was <1 MHz. A long-pass filter blocked any leakage pump light from the OPO. To measure the resonances of the GaAs microdisks in the SH-wavelength range, a portion of the OPO light was doubled to the range of 950–1,007 nm in an oven-mounted periodically poled LiNbO$_3$ crystal and then sent through a dichroic filter to reject the residual OPO light. The fundamental and SH beams were launched into two separate optical fibres, which were alternately connected to the fibre taper that addressed the GaAs microdisks. The powers of both beams were monitored using a 1% fibre tap coupler for the fundamental-wavelength beam and an uncoated BK7 pickoff plate placed near Brewster’s angle for the SH-wavelength beam. A fibre polarization controller was placed immediately before the fibre taper to control the polarization of the light at the microdisk.

**Resonance identification.** To identify the $(m, p, q)$ values of the resonances, transmission spectra of the microdisks were measured over extended wavelength ranges in the fundamental and SH bands. The 160-nm-thick GaAs microdisks only supported the lowest order vertical modes $(q_l = q_t = 1)$. Transverse electric (TE) and transverse magnetic (TM) resonances were observed in the SH wavelength range, while only the TE modes were supported at the fundamental wavelength. Different radial mode families were identified by their common lineshapes and spectral separations. Finite-element method numerical modelling was used to...
match the resonance locations by slightly numerically adjusting the microdisk size. We also varied the taper diameter during measurements in order to change the taper-mode coupling conditions, which helped us to order and identify the radial mode families. We observed series of transmission spectra as the taper approached and finally touched the microdisk, which allowed estimation of the intrinsic and coupling quality factors for the microdisk.

**Numerical simulations.** Our 3D simulations are based on the following three-step numerical approach. First, the electromagnetic response of the considered tapered fibre–disk system at the fundamental frequency was simulated by launching a HE11 waveguide mode along the tapered fibre. To ensure that a proper mode excitation is achieved in the fibre, the numerical solution of the corresponding 2D eigenvalue problem was directly fed into the input end of the fibre. This numerical profile was obtained by assuming that the power carried by the HE11 waveguide mode is the same as the fundamental input power used in our experiments. Second, we computed the current density that is induced inside the microdisk as a consequence of the temporal variation of the nonlinear second-order polarization. In this calculation, we assumed $d_\text{q} = 94 \text{ pm V}^{-1}$ (ref. 25) for the non-zero components of the second-order nonlinear susceptibility. By computing the work done by the disk at the second-harmonic frequency, we obtained the corresponding energy stored inside the disk couples back into the tapered fibre. In order to check the accuracy of our numerical results, the above numerical procedure was implemented using both a 2D FDTD approach (using the MIT-MEEP implementation) and a 3D FDTD (using the COMSOL Multiphysics package). Open space in both types of simulations was mimicked by using perfectly matched layers or/and scattering boundary conditions. The corresponding discretization grids were refined until a relative numerical error of <1% is achieved in all reported results. In both cases, the refractive index of the microdisk was modelled by considering the corresponding dispersion and focusing on the production of optical harmonics. The geometry parameters of the microdisk were chosen so that its TM-polarized ($m_{\text{TM}} = 24$, $p_{\text{TM}} = 2$) whispering-gallery-mode is detuned by 2.2 nm with respect to two times the wavelength of the TE-polarized ($m = 13, p = 1$) resonance. The refractive index of the fibre was assumed to be 1.44. No additional approximations, apart from the one introduced by the spatial discretization of the dielectric constant (inherent to any FDTD or FEM implementation), were introduced in the simulations, and no fitting parameters to experimental data were assumed in the calculations.

**References**


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**Author contributions**

P.S.K. and G.S.S. conceived the experiments; G.S.S. made the sample; P.S.K. fabricated and measured the microdisks and fibre tapers; P.S.K. performed the optical experiments and CMT theoretical calculations, and analysed the data; J.B.-A. conducted 3D numerical simulations, and P.S.K., J.B.-A. and G.S.S. wrote the manuscript.

**Additional information**

**Supplementary Information** accompanies this paper at http://www.nature.com/naturecommunications

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