Sub-wavelength interference in a hyperbolic medium
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Graphene supports the propagation of subwavelength optical solitons
Abstract Nonlinear propagation of light in a graphene monolayer is studied theoretically. It is shown how the large intrinsic nonlinearity of graphene at optical frequencies enables the formation of quasi one-dimensional self-guided beams (spatial solitons) featuring subwavelength widths at moderate electric-field peak intensities. A novel class of nonlinear self-confined modes resulting from the hybridization of surface plasmon polaritons with graphene optical solitons is also demonstrated.

Graphene supports the propagation of subwavelength optical solitons

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The experimental discovery and isolation of graphene monolayers from bulk graphite [1] has attracted great interest during the last years. The study of graphene properties has become a hot topic of research within the physics and nanoscience communities [2] as it promises, among others, a variety of optical and opto-electronical applications [3,4]. Very large values of the nonlinear optical susceptibilities corresponding to multiple harmonic generation were theoretically predicted [5,6] and have been experimentally verified very recently in the case of third-order nonlinearities [7]. Still, it is an open question whether this high nonlinear coefficient, which occurs in a two-dimensional (2D) system, could induce strong nonlinear effects in electromagnetic (EM) modes that extend on the three spatial dimensions.

One of the nonlinear effects with greater potential for controlling light propagation at the micro- and nano-scales is the formation of temporal and spatial EM solitons [8–12]. In this Letter we demonstrate that 2D graphene monolayers support spatial non-diffracted beams (i.e., solitons) of subwavelength width in the optical regime. We illustrate this capability by analyzing two arrangements leading to solitons with different polarizations: a graphene monolayer embedded into a conventional dielectric waveguide and a graphene sheet placed on top of a metal-dielectric structure.

Using feasible values for the beam peak intensity, we also develop a quasi-analytical model that is able to capture the basic ingredients of the numerical results. The first structure in our analysis consists of a single graphene monolayer placed inside a planar linear dielectric waveguide, see Fig. 1 (a). This dielectric waveguide provides vertical confinement in the z-direction for the propagating EM mode. Graphene must be physically considered as a 2D material with nonlinear conductivity. But mathematically we can also approximate graphene by a very thin layer of a finite thickness introducing an effective dielectric constant. Then we can treat graphene using the Maxwell equations for bulk media. We have checked that both 2D and 3D approaches give virtually the same numerical results. Thus, we take directly the nonlinear susceptibility from the experiment, and approximate graphene by a thin graphene layer of a finite thickness introducing an effective dielectric constant.

The second structure in our analysis consists of a single graphene monolayer placed on top of a metal-dielectric structure (i.e., embedding functionalized graphene into a metal-dielectric waveguide). Here, we consider a metal-dielectric waveguide in the form of a metal-air halfspace with a graphene monolayer embedded into it, see Fig. 1 (b). The nonlinear graphene waveguide is a 2D material with nonlinear conductivity. But mathematically we can also approximate graphene by a very thin layer of a finite thickness introducing an effective dielectric constant. Then we can treat graphene using the Maxwell equations for bulk media. We have checked that both 2D and 3D approaches give virtually the same numerical results. Thus, we take directly the nonlinear susceptibility from the experiment, and approximate graphene by a thin graphene layer of a finite thickness introducing an effective dielectric constant.

In this Letter we demonstrate that 2D graphene monolayers support spatial non-diffracted beams (i.e., solitons) of subwavelength width in the optical regime. We illustrate this capability by analyzing two arrangements leading to solitons with different polarizations: a graphene monolayer embedded into a conventional dielectric waveguide and a graphene sheet placed on top of a metal-dielectric structure. We analyze in detail the formation of spatial solitons with different polarizations: a graphene monolayer support spatial non-diffracted beams (i.e., solitons) of subwavelength widths at moderate electric-field peak intensities. A novel class of nonlinear self-confined modes resulting from the hybridization of surface plasmon polaritons with graphene optical solitons is also demonstrated.
I tons. This is illustrated in Fig. 1(c), computed for compensating diffraction leading to the formation of EM solitons. In a bulk nonlinear waveguide [10], in contrast to a conventional nonlinear waveguide, in which the nonlinear index, similarly to what happens to a beam traveling within high-index dielectric media is several orders of magnitude smaller than the one characterizing graphene.

We turn now to analyze the dependence of soliton width (characterized by the full width at half maximum (FWHM), \(a\), of the soliton \(E\)-field) on the external intensity illuminating the system. In order to do this, we have computed the nonlinear eigenmodes for several peak \(E\)-field amplitudes in the graphene layer. The results, in terms of the corresponding intensity distributions (which in each case have been normalized to the maximum beam intensity), are summarized in the inset of Fig. 3. As the maximum value of the \(E\)-field in the graphene layer (\(|E|_{\text{max}}\)) is increased from \(0.8 \times 10^{10} \text{ V/m}\) (bottom panel) to \(4.2 \times 10^{10} \text{ V/m}\) (top panel), \(a\) decreases from \(a = 2 \mu\text{m}\) (more than 2 times the wavelength of the external illumination) to \(a = 0.253 \mu\text{m}\) (well inside the subwavelength regime). The results displayed in Fig. 3 represent a novel instance, in a strict 2D system,
of the soliton mode for the case in which the peak intensity is the normalized E-field (panel b) display a conventional soliton profile. For the vertical cross-sections (panel c), the E-field at $v_2$ has the standard profile of a linear waveguide mode. For the cross-section evaluated at $v_1$ the shape of the beam corresponds to that of a nonlinear system. The filled area on the panel (c) shows the location of the dielectric waveguide.

Figure 2 (online color at: www.lpr-journal.org) Soliton profile analysis. Panel (a) shows the transversal E-field distribution, $|E|$, of the soliton mode for the case in which the peak intensity is $I = 1.8 \times 10^{10}$ W/cm². The horizontal cross-sections $h_1$ and $h_2$ of the normalized E-field (panel b) display a conventional soliton profile proportional to sech$(y)$. For the vertical cross-sections (panel c), the E-field at $v_2$ has the standard profile of a linear waveguide mode. For the cross-section evaluated at $v_1$ the shape of the beam corresponds to that of a nonlinear system. The filled area on the panel (c) shows the location of the dielectric waveguide.

Figure 3 (online color at: www.lpr-journal.org) Dependence of the soliton width with the input intensity. Main panel presents the peak electric field dependence with the soliton width. Circular dots represent the values obtained with the full nonlinear calculation whereas the solid line renders the results from the quasi-analytical treatment (see main text). When the profile $A(x)$ and the propagation constant $\beta$ corresponding to the numerical calculation is included (triangular dots), the agreement between analytics and numerics is improved. The inset shows the normalized intensity distributions for the five soliton widths considered in the main panel. The peak intensity $I_{\text{max}}$ is indicated for each profile. The operating wavelength in all cases is $\lambda_0 = 850$ nm.

on how the balance between nonlinearity and diffraction can yield self-guided propagating beams with subwavelength lateral confinement. In this context, it is important to point out that when graphene losses are incorporated into the calculations (these losses stem from the linear part of the graphene conductivity), the propagation length $L$ of the soliton, defined as $L = 1/|\beta_{\text{NL}}|$, $\beta_{\text{NL}}$ being the complex propagation constant of the nonlinear mode, is barely dependent on $a$. In the case considered in Fig. 3, we find that the propagation length slightly decreases with $a$ in a monotonous manner, being $L = 20 \, \mu$m for $a = 2 \, \mu$m and $L = 15 \, \mu$m for $a = 0.25 \, \mu$m. This virtual independence of the propagation length on the field confinement is very different to what is observed in other subwavelength-confined EM modes as, for example, surface plasmon polaritons.

To account for the physical origin of the above described dependence of the soliton width on the peak electric field amplitude, we have adapted to this problem the theoretical approaches used to describe soliton formation in conventional 3D nonlinear optical materials [8–10]. For this quasi-analytical treatment, we employ the 3D modeling of the graphene layer, which, as mentioned before, gives virtually the same results as a description based on a strictly 2D conductivity. Within this approach the propagation of light inside the graphene layer is formulated in terms of the non-homogeneous vector Helmholtz’s equation,

$$c^2 \varepsilon_0 \left[ \left( \frac{n_s}{c} \right)^2 \frac{\partial^2}{\partial t^2} - \nabla^2 \right] A(r, t) = j_{\text{NL}}(r, t)$$

where $A(r, t)$ is the magnetic potential vector (i.e., $E(r, t) = -\partial A(r, t)/\partial t$, choosing the gauge $\nabla \cdot A = 0$) and $n_s$ is the linear refractive index of graphene. To solve Eq. ((1)), we start by assuming that its solutions are of the form

$$A(r, t) = \frac{1}{2} [\hat{\Lambda}(x) F(z, y) \exp[\i (\beta z - \omega t)] + c.c.]$$

where $\hat{\Lambda}(x)$ is, in principle, an arbitrary function that governs the confinement of the EM field along the $x$-direction (see definition of axes in Fig. 1). As deduced from Eq. ((2)), $\hat{\Lambda}(x)$ also defines the polarization of the considered modal profile. The EM field profile in the graphene plane is controlled by the complex function $F(z, y)$, whereas the corresponding propagation constant along the $z$-direction is given by $\beta$.

Now, we insert Eq. ((2)) into Eq. ((1)), we apply the slowly varying amplitude approximation, and we project
the left and right-hand-side of the resulting equation over $[\hat{A}/(x)]^T$ (where $[1]^T$ stands for the transpose conjugate). Then, we define the auxiliary function $f(z, y) = F(z, y) \exp(-i\phi(z))$ (where $\phi \equiv (k_{z}^2 - \beta^2 + I_{2}/I_{1})/2$, $k_{z} = \omega a/2c^2$, $I_{1} \equiv \int_{-\infty}^{\infty} dx |\hat{A}_{e}(x)|^2$ and $I_{2} \equiv \int_{-\infty}^{\infty} dx |\hat{A}_{e}(x)]^T \hat{A}_{e}(x)/3x^2$). Using these definitions, after some algebra, one finds that Eq. ((1)) can be rewritten in terms of the function $f(z, y)$ as

$$2i\beta \frac{\partial f(z, y)}{\partial z} + \frac{\partial^2 f(z, y)}{\partial y^2} + g[f(z, y)]^2 f(z, y) = 0$$

(3)

where $g \equiv \frac{\omega^2}{2} \chi^{(3)}_{gr} I_3/I_1 c^2$, with $I_3 \equiv \int_{-\infty}^{\infty} dx |\hat{A}_e(x)|^4$. The crucial point to realize is that Eq. ((3)) corresponds to the standard form of the nonlinear Schrödinger equation, whose solutions have a canonical first-order soliton form [8, 10],

$$f(y, z) = \frac{1}{w} \sqrt{2} \frac{\text{sech}(y/w)}{g} \exp(i/2\beta w^2)$$

(4)

where $w$ is the conventional definition of the soliton width, which in terms of the soliton FWHM is given by $w \approx a/2.64$. Physically, Eq. ((3)) and its corresponding solution given in Eq. ((4)) can be interpreted as those governing the propagation of light in a special class of index-guided waveguide in which the refractive index contrast between the core and the cladding is induced by the intensity of the propagating beam itself. Importantly, Eq. ((4)) confirms the existence of soliton solutions in graphene, as observed in the numerical experiments reported in Figs. (1)–(3). Notice that the strength of the effective nonlinearity is characterized by the parameter $g$, which is proportional to both $\chi^{(3)}_{gr}$ (related to the intrinsic nonlinearity of graphene in a free-standing configuration) and $I_3/I_1$, which provides a measure of the fraction of EM energy that flows inside the graphene sheet.

Inspired by the theoretical approaches used traditionally in nonlinear optics [8–10], we assume that both the vector function $\hat{A}_e(x)$ and the propagation constant $\beta$ of the modal profile correspond to those obtained numerically for the linear counterpart of the structure sketched in Fig. 1(a). The results computed within this approximation are displayed in Fig. 3 (see solid line), showing a qualitative agreement between the analytical results and the full numerical calculations. We emphasize that no fitting parameters are used in this comparison. The discrepancy between analytic and full numerics becomes larger as the value of $a$ decreases. This fact can be ascribed to the difference between the profile $\hat{A}_e(x)$ obtained for the linear case and that computed numerically for the full nonlinear problem, which increases as $a$ decreases. This point is confirmed by the additional results displayed in Fig. 3 (triangular points), which show how the agreement between analytics and numerics improves when we introduce in Eq. ((2)) both the self-consistent profile, $\hat{A}_e(x)$ (obtained at $y = 0$), and the propagation constant $\beta$ corresponding to our nonlinear simulations. The remaining difference can be traced back to the separability in the $x$ and $y, z$ coordinates implied by Eq. ((2)) that cannot fully account for the complexity of the graphene EM solitons.

Finally, we show that TM-polarized optical solitons can also propagate along a graphene monolayer. A graphene structure that is able to support these TM optical solitons is rendered on Fig. 4. Here the vertical confinement is provided by a surface plasmon polariton (SPP) mode that is...
propagating on the interface between gold and a dielectric film. The graphene monolayer, which is characterized by a large nonlinear third-order susceptibility, must be separated from the metal surface by a dielectric spacer. We have chosen a 100 nm silicon dioxide layer, just for proof-of-principles purposes. Our calculations show that this system supports the propagation of a very peculiar class of TM soliton, which results from the hybridization between the SPP supported by the metal-dielectric interface and the soliton propagating in the graphene sheet. The computed distributions of the transversal $|E_x|$ and in-plane $|E_y|$ components of the electric field of this hybrid SPP-soliton solution is plotted in Fig. 4(a,b) and displays exactly the conventional solitonic profile $\sim \text{sech}(y)$ along the y-direction. The in-plane $|E_y|$ component of the electric field of hybrid SPP-soliton is responsible for nonlinear effect and provides diffraction compensation. This component reaches its maximum value at the graphene layer, see Figs. 4(b) and 4(c).

The dependence of the soliton width with the peak E-field amplitude rendered in Fig. 4(d) is very similar to that found for TE optical solitons, predicting the existence of subwavelength optical solitons also for this polarization. Similarly to the case of the TE geometry, the propagation length of the hybrid SPP-soliton is weakly dependent on the soliton width, the points on the inset of Fig. 4(d). In contrast to the TE case, in the case of hybrid SPP-soliton the propagation length is almost independent on soliton width when this is larger than the wavelength (reflecting that, in this case, losses occur mainly in the metal). For smaller soliton widths the propagation length decreases with $a$, and our calculations show that the losses occur mainly in the graphene layer.

To analyze the effect of losses on the beam shape we calculated how the beam width changes as the soliton propagates. In the lossless case, the soliton width remains constant while propagating. When realistic losses in both metal and graphene were considered, the soliton beam broadens with distance. As a representative example, when the incident beam width is 0.7 $\mu$m, the computed soliton broadening is 30% after traveling a distance of 4 $\mu$m. In comparison, in the linear regime the corresponding value is 150%. This behavior is similar to that found for SPP-solitons appearing at the interface between a metal and a semi-infinite Kerr dielectric [18]. Notice that considering a dielectric with a higher dielectric constant would allow increasing the confinement in the direction normal to the surface. However, this would push the field towards the metal, with a corresponding increase in losses and a decrease in the propagation length of the soliton.

In conclusion, we have demonstrated that graphene monolayers can support both TE and TM spatial optical solitons due to the extremely large magnitude of its nonlinear third-order susceptibility. Moreover, we have shown that for feasible values of the input intensity these quasi-one dimensional optical solitons can have a subwavelength lateral width. We have also developed a quasi-analytical model that has a semi-quantitative value and that is able to predict the field intensities needed for soliton formation. The existence of subwavelength optical solitons adds a new capability to the already broad range of optical phenomena associated with graphene structures.

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[13] We have used the implementation of the finite element method provided by the commercial software COMSOL Multiphysics.