Lecture 6
Inflation (II): The Cagan-Sargent-Wallace model of inflation dynamics.

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Outline

1. Introduction

2. The Cagan-Sargent-Wallace model.

3. The Krugman model of currency crises.

References: Cagan (1956), Sargent and Wallace (1973); Jim Costain’s class notes.
1. Introduction

- The *classical* view of macroeconomics has a **dichotomy** between real and nominal variables.

- The classical model assumes that monetary variables have *no effect* on the real economy, because prices adjust instantaneously to the level consistent with real conditions.

- While we study classical inflation dynamics, we will assume that real output $y$ and the real interest $r$ are both constant.

- This model is useful to study money and inflation in a dynamic setup.
Part I

The Cagan-Sargent-Wallace model of inflation dynamics
Jumps and growth rate

- The model of Cagan, Sargent, and Wallace studies how inflation behaves, taking as given the central bank’s money supply decisions.

- It is all based on the money market equilibrium equation:

\[
\frac{M^s}{P} = m^d(y, r + \pi, \gamma)
\]

- We will focus on the effects of two important kinds of changes in the money supply
  - the central bank suddenly increases the money supply by a large amount \( \rightarrow \) a jump in the money supply
A jump in the money supply
the central bank gradually increases the money supply by a little bit each period by some proportion $\mu$:

$$\frac{dM^s}{dt} = \mu M^s_t$$

Notice this means that

$$M^s_t = M_0 \exp(\mu t)$$

or

$$\log M^s_t = \log M^s_0 + \mu t$$
Constant money growth
We can analyze price dynamics in the same way.

- **A jump** in the price level:

- And **constant growth** of the price level:
Recall that inflation $\pi_t$ is

$$\pi_t = \frac{dP_t}{dt}/P_t \quad \text{or} \quad dP_t/dt = \pi_t P_t$$

So when inflation is a constant, $\pi_t = \pi$, price dynamics follow an equation similar to the one we derived for the money supply:

$$\log P_t = \log P_0 + \pi t$$
Example 1: Constant money growth

- Suppose the central bank chooses money supply growth of exactly $\mu$ for all $t$.

- Remember also that we are holding real conditions $y$, $r$, and $\gamma$ fixed.

- Prices and inflation will be consistent with this monetary policy if the money market equilibrium equation is satisfied at all time.

$$\log M_s^t - \log P_t = \log(m^d(y, r + \pi, \gamma))$$

- Rule 1. If we draw $\log M_s$ and $\log P$ in the same graph, then the distance between the two curves represents $\log m^d$. 
Suppose we start looking for a solution by making the reasonable guess that constant money growth should also imply constant inflation. Then both $\log M^s$ and $\log P$ are straight lines:
But this graph *cannot be correct*! Real money demand depends only on $y$, $r$, $\gamma$, and $\pi$, all of which are assumed constant. So $\log m^d$ must be constant:
Conclusions

- **Rule 2.** Constant money growth $\mu$ implies constant inflation $\pi = \mu$ (so the line $\log P$ is has the same slope as $\log M^s$).

- Also real money demand decreases when there is greater inflation. So again, when we observe that the log of real money demand is the distance between the two lines, we conclude:

- **Rule 3.** When $\log M^s$ is steeper (faster money growth $\mu$), the log $P$ line must be closer to $\log M^s$ (less real money demand $m^d$).
2. The Cagan-Sargent-Wallace model of inflation dynamics

How prices respond to the money supply
Example 2. An unexpected increase in money growth

- What happens when the government changes its monetary policy?

- Consider an initially responsible government that maintains a low, constant rate of money growth $\mu_1$.

- Suddenly and *unexpectedly* (expectations are important here!) a corrupt government takes over, and starts printing money quickly, at growth rate $\mu_2 > \mu_1$.

- Result
2. The Cagan-Sargent-Wallace model of inflation dynamics

How prices respond to the money supply

- We are *not* talking about a *jump* in the money supply— only a change in its rate of growth $\mu$.

- Nonetheless, the price level jumps!
  - Before the change in government, everyone expects $\mu_1$ forever,
  - so prices increase in parallel with the money supply,
  - and there is lots of demand for money (log $P$ far from log $M^s$).
  - After the change in government, everyone expects $\mu_2$ forever
  - so prices again increase in parallel with the money supply,
  - and there is *much less* demand for money (log $P$ close to log $M^s$).
  - But this requires a jump in prices at the time of the change in policy.

- Then, an increase in the rate of money growth is even *more inflationary* than we might expect.
Example 3. An unexpected jump in money supply

- Consider a responsible, trusted government that always maintains a low rate of money growth $\mu_0$.

- Imagine that suddenly, unexpectedly, the government prints a lot of extra money.

- Assume everyone understands that the jump in the money supply is due to the emergency situation, and that afterwards the rate of money growth will return to $\mu_0$. 
2. The Cagan-Sargent-Wallace model of inflation dynamics

Result

Rule 4. People cannot expect the price level to jump. That is, prices can only jump in response to news.
Example 4. An anticipated increase in money growth

- Consider a country that is ruled for many years by a party that prefers low inflation.

- Suppose that another party, which prefers high inflation, is unexpectedly elected to power at time $t_0$.

- If the high-inflation party enters government immediately, we are back in Example 2.

- But if the high-inflation party must wait some months before taking power, and only starts governing at time $t_1$, then:
Money supply starts growing at time $t_1$, when the high-inflation party enters power.

But the price level jumps up much earlier—it jumps at time $t_0$, when there is news that money supply will change.

Inflation increases gradually ($\log P$ gets gradually steeper), consistent with a gradual decrease in money demand ($\log P$ gradually gets closer to $\log M^s$).

Except at the time of “news”, the rate of inflation changes smoothly (no kinks in $\log P$), and money demand evolves continuously (no sudden changes in the distance between $\log M^s$ and $\log P$).

Rule 5. In the absence of “news”, there is a kink in $\log P$ (a sudden change in inflation) if and only if there is a jump in $\log M^s$. 
Part II

The Krugman model of currency crises
3. The Krugman model of currency crises

- 1990s: many smaller countries tried to fix their exchange rates relative to some of the major currencies used in international transactions.

- Some of them suffered speculative attacks that led to currency crisis or a balance of payments crisis.

- Krugman showed that exactly the same model of Cagan/Sargent/Wallace also helps us understand what happens when a fixed exchange rate regime collapses.
Implications of fixed exchange rates

- There is only one practical way to fix the nominal exchange rate $e$: the government must promise to buy and sell foreign currency at the fixed price $e$.

- In order to fix the exchange rate, the central bank must maintain a sufficiently large stock of foreign reserves.

- The central bank can either choose its money supply and inflation rate independently, or it can fix its exchange rate to another currency. *It cannot do both!*

- The government’s choice of $e$ determines the price level and hence the money supply.
In graphs

- Money market equilibrium equation

\[ M^s = Pm^d(y, r + \pi, \gamma) \]

- Definition of the real exchange rate

\[ \varepsilon = \frac{P^*e}{P} \]

- \( P^* \) is chosen by the foreign central bank, and \( \varepsilon \) (according to the theory of relative PPP) is determined by real market conditions.

- So if the central bank chooses \( e \), then it is forced to choose

\[ P = \frac{P^*e}{\varepsilon} \]

- Then it is forced to choose exactly the money supply that is consistent with that price level
Obviously, the central bank will never run out of pesos, because it prints pesos! But what is essential is that the central bank must always have a large quantity of dollars available for sale. That is, in order to fix the exchange rate, the central bank must maintain a sufficiently large stock of foreign reserves. The second point we need to remember is that the central bank can either choose its money supply and inflation rate independently, or it can fix its exchange rate to another currency. It cannot do both! To see why, consider the money market equilibrium equation:

\[ M_s = P m d(y, r + \pi, \gamma) \]

and the definition of the real exchange rate:

\[ \varepsilon = \frac{P_f}{P} \]

The foreign price level \( P_f \) is chosen by the foreign central bank, and the real exchange rate \( \varepsilon \) (according to the theory of relative purchasing power parity) is determined by real market conditions. So if the central bank chooses \( \varepsilon \), then it is forced to choose \( P = \frac{P_f}{\varepsilon} \). Then we can see from a money supply / money demand diagram that it is forced to choose exactly the money supply that is consistent with that price level:

\[ M_s \text{ and } M^d \text{ must be consistent with } P \]

Thus we conclude: The government's choice of \( \varepsilon \) determines the price level and hence the money supply. The third fact that we must understand is that fixing the exchange rate will place limits on government fiscal policy, because it places limits on the seignorage that the government can receive. With a flexible exchange rate, the central bank can always print more money and use it to buy government bonds, which is what we mean by seignorage. This gives the government a way of financing spending which avoids other sorts of distorting taxes.
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But what happens if the central bank prints money to buy bonds under a fixed exchange rate?

But what happens if the central bank prints money to buy bonds under a fixed exchange rate? This open market operation would initially, temporarily increase the money supply (this is (1) below):

An attempt to raise the money supply

However, the price level implied by the fixed exchange rate is still:

\[ P = \frac{P_F}{\epsilon} \]

which is unchanged. Therefore, the new, higher money supply is greater than money demand, which is also unchanged. And everyone knows that the central bank promises to buy and sell foreign currency at price \( e \). So when the central bank sells the public more money (pesos) than people want, they can come right back to the central bank and sell those pesos for dollars. When people sell their pesos to the central bank for dollars, the supply of circulating pesos returns to its initial level, and the central bank's foreign reserves decrease (this is (2) in the diagram).

In other words, as we said before, choosing \( e \) means the central bank cannot choose \( M_s \). When it tries to increase \( M_s \), the public immediately decreases it again by buying the bank's dollar reserves. The overall result is that

(1) the central bank holds more government bonds
(2) the central bank has less reserves of foreign currency
and (3) the money supply is unchanged.

So we conclude:

Fact.
If the exchange rate \( e \) is fixed, then the government can obtain seignorage only by selling its foreign reserves, not by increasing the money supply.

What happens when the central bank runs out of reserves?

Krugman studied what happens when a government is trying to fix its exchange rate \( e \), but runs out of foreign reserves.

Consider a government that currently has a fixed exchange rate of \( e \) pesos per dollar, and assume that the inflation rate in the US is zero. Suppose the government is in a difficult financial situation, and is running a constant deficit

\[ D = P(G - T) + iBGP \]

where \( P \) is the price level, \( G \) is government spending, \( T \) is taxes, \( i \) is the nominal interest rate, and \( BGP \) is government debt held by the public. Suppose the government has already borrowed so much that the public is worried about a default, and therefore refuses to buy any more bonds.
Summing up

- Choosing $e$ means the central bank cannot choose $M^s$.

- When it tries to increase $M^s$ the overall result is
  - the central bank holds more government bonds
  - the central bank has less reserves of foreign currency, and
  - the money supply is unchanged.
What happens when the central bank runs out of reserves?

- Krugman studied what happens when a government is trying to fix its exchange rate $e$, but runs out of foreign reserves.

- Consider a government that currently has a fixed exchange rate against the dollar, and assume that the inflation rate in the US is zero.

- Suppose the government is in a difficult financial situation, and is running a constant deficit.

- Suppose the government has already borrowed so much that the public is worried about a default, and therefore refuses to buy any more bonds.

- If a government with a fixed exchange rate runs a deficit, then if it is unable to sell more bonds to the public, its reserves will decrease over time.
- Krugman showed what happens when a government that is unable to control its deficit runs out of reserves and is forced to float its currency.

- As long as the fixed exchange rate regime is in place, the local inflation rate must be the same as the foreign inflation rate (assumed 0 here).

- So nominal interest rates are low, and the real demand for money is large:
Before the central bank runs out of reserves

However, if the government is unable to control its deficit, then after it runs out of foreign reserves, it will have to print money to finance its deficit. Using the government budget constraint again, and assuming that the government cannot sell more bonds, we can calculate the rate of money printing:

\[
\Delta M_s = \Delta (M_s/P) = \frac{D}{P}\mu = \frac{D}{P \cdot m_d}
\]

So now the rate of money printing will be positive, inflation will be positive, the nominal interest rate \(i\) will be higher, and hence real money demand will be lower. So after the fixed exchange rate regime ends, the situation will be as follows:

Notice that \(\log(P_{US}/e)\) is unchanged (assuming relative purchasing power parity, and assuming there is no inflation in the US), but \(\log m_d\) has decreased because real money demand goes down when inflation is higher.

From these two diagrams, we notice a simple fact: real money demand jumps down when the government runs out of reserves! As long as the government has reserves, it can fix \(e\), so inflation is zero and real money demand is high. As soon as the government runs out of reserves, money supply growth will be positive, so real money demand will decrease.

To figure out how the "before" and "after" diagrams fit together, remember from our discussion of the Cagan/Sargent/Wallace model that people cannot expect prices to jump. If everyone expected the price of groceries to double tomorrow, they would all:

\[
\log P = \pi = \mu \log m_d
\]

\[
\log P_{US}/e = \delta = \pi = \mu
\]

\[
\log e = \text{time}
\]

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\[ \Delta M^s = D \rightarrow \left( \frac{\Delta M^s}{M^s} \right) \left( \frac{M^s}{P} \right) = \frac{D}{P} \rightarrow \mu = \frac{D}{Pm^d} \]

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\[ \Delta M_s = D \rightarrow \left( \frac{\Delta M_s}{M_s} \right) \frac{M_s}{P} = \frac{D}{P} \]

\[ \mu = \frac{D}{P_{md}} \]

So now the rate of money printing will be positive, inflation will be positive, the nominal interest rate \( i \) will be higher, and hence real money demand will be lower. So after the fixed exchange rate regime ends, the situation will be as follows:

After the central bank runs out of reserves

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\[ \text{slope } \mu \quad \text{log } M^s \]

\[ \text{log } m^d \quad \text{log } P \]

\[ \text{slope } \pi = \mu \quad \text{log } e \]

\[ \text{slope } \delta = \pi = \mu \quad \text{log } P_{US}/\varepsilon \]

\[ \text{slope } \delta = \pi = \mu \quad \text{log } e \]
Notice: real money demand jumps down when the government runs out of reserves!

As long as the government has reserves, it can fix $e$, so inflation is zero and real money demand is high.

As soon as the government runs out of reserves, money supply growth will be positive, so real money demand will decrease.

CSW model: people cannot expect prices to jump.

If prices or exchange rates jump, this must be in response to unexpected events, not to expected events.
In the Krugman model, everyone knows that the government is running a deficit, so everyone is expecting the reserves of dollars to run out. The end of a fixed exchange rate regime

The end of a fixed exchange rate regime
The supply of money must jump down at the time $T^{**}$ when the fixed exchange rate ends.

The public sells a large quantity of the domestic currency for dollars at time $T^{**}$.

Discussion:

- Fixed $e$ ends with a speculative attack.
- Determining the time of the attack.
- Criticism: usually $e$ does jump up.
- Criticism: why should the government sell all its reserves?