PRACTICE PROBLEMS FOR MIDTERM

1. Simplify: $1 + x + x^2 + x^3 + ...$

2. Simplify: $x + x^2 + x^3 + ...$

3. Simplify: $1 + \frac{1}{1+r} + \left(\frac{1}{1+r}\right)^2 + \left(\frac{1}{1+r}\right)^3 + ...$

4. Simplify: $\frac{1}{1+r} + \left(\frac{1}{1+r}\right)^2 + \left(\frac{1}{1+r}\right)^3 + ...$

5. Simplify: $1 + \frac{1+g}{1+r} + \left(\frac{1+g}{1+r}\right)^2 + \left(\frac{1+g}{1+r}\right)^3 + ...$

6. You expect to earn 300 cookies this year ($t=0$) and every year until $t=10$. Then from $t=11$ to $t=20$ you expect to earn 400 cookies per year. If the annual real interest rate is $r = 5\%$, what is the NPV of all this income?

7. At the beginning of 2007 you have the opportunity to buy a bond that will pay you 10 cookies at the end of 2007, 2008 and 2009, and later it will pay 100 cookies at the end of 2010. If the real interest rate is $r = 4\%$ per year, what is the NPV at the beginning of 2007? If the price of the bond is 85 cookies, should you buy the bond?

8. In 2000, the stocks of Yahoo.com paid a dividend of 2 cookies, and the price of the stock was 150 cookies. Assuming that the real interest rate is $r = 6\%$ per year, and that the dividend will never change, was it worth buying stock in Yahoo.com? (Buying the stock in 2000 means you receive the dividends from 2001 onwards). Assuming that the annual real interest rate is 6\%, at which rate must the dividends grow to justify the price of the stock?

9. We are in year $t = 0$, Mr. More expects to live until $t = 50$. The annual real interest rate is constant, $r = 4\%$. Mr. More expects an income of 20,000 cookies per year from $t = 0$ to $t = 10$; 30,000 cookies per year from $t = 11$ to $t = 35$, and 10,000 cookies per year from $t = 36$ and $t = 50$. Every period, he pays a fraction $\tau = 0.2 = 20\%$ of his income as taxes.

   (a) Using as notation $c_t$ for consumption and $y_t$ for income, write the intertemporal budget constraint for Mr. More.

   (b) What is the NPV of Mr. More’s income, after taxes?

   (c) If he wants to smooth consumption (i.e., have the same consumption level for all periods), how much should he consume? How much should he save at $t = 0$?

   (d) How much could he increase consumption if his income were 1,000 cookies higher, every period, than the income mentioned above?

   (e) How much would his consumption increase if his income increased by 1,000 cookies, at time $t = 0$ only?

10. Ainhoa has the opportunity to buy a piece of land for $300,000 today (at time 0) which she expects to be able to sell at $410,000 4$ years hence. Ainhoa can borrow and lend at 7\% interest per year.
(a) What is the investment opportunity’s NPV?
(b) If Ainhoa buys the land, what is the effect on her present consumption if she keeps all future consumption unchanged?
(c) What is the maximal price Ainhoa is willing to pay for the land?
(d) What will be the optimal consumption decision for Ainhoa?

11. Suppose that you finish your studies this year (time 0) and start to work in 2007 (time 1) at $60,000 per year. Suppose that your earnings will keep growing at the rate of 3% per year, and that the interest rate is constant at 5%.

(a) What is the present value of your lifetime earnings, assuming that you will live forever?
(b) What is the present value of your lifetime earnings, assuming more realistically that you will earn money for the next 40 years?

12. (This exercise follows from one solved in class, exer_npvgo2.pdf). Consider a firm with expected earnings in the coming year of $5 per share. This firm has the opportunity of engaging in several alternative projects, but it can only engage in one of them. For all the cases considered below the required rate of return is $r = 12.5\%$, and the dividend payout ratio is 40%. Let’s analyze each of them.

(a) What is the price of share if the firm does not invest in any of the projects?
(b) One of the projects (Project A) generates a return on investment of 15%. What would the firm’s share price be if it undertook Project A?
(c) Project B generates a return on investment of 12.5%. What would the firm’s share price be if it undertook Project B? Compare the result with the one obtained for Project A and explain the differences.
(d) Project C generates a return on investment of 10%. What is the price of a share for the firm in this case? Will the firm invest in this project? Why?
(e) Given your results in (a-d) give an explanation of the relationship between the return on investment and the required interest rate or required rate of return in this framework.

(Hint: for sections b-d you may want to compute the NPVGO for each period)