PROBLEM SET 2

Due on Monday, April 17th 2006

To get credit for this problem set, it must be handed in on Monday, April 17 in class.

Late homework will not be accepted.

1. Exercises 4.21, 4.26, 4.28, 4.29, 4.30, 4.33, 4.36 of Chapter 4 in textbook.

2. Reviewing indifference curves

Analyze the investment and consumption decisions of an individual that lives for only two periods. At \( t = 0 \) he gets a wage of \( w_0 = $50 \) and consumes. At \( t = 1 \) he gets a wage \( w_1 = $100 \) and what is left after consuming at \( t = 0 \) and consumes. The individual can borrow and lend in the financial markets at an interest rate between time 0 and 1 of \( r = 7\% \).

a. Define and plot in a graph this individual’s budget constraint.

b. For each of the following utility functions, compute and plot in a graph the optimal consumption choice of the individual:

1. \( u(c_0, c_1) = \min\{c_0, c_1\} \);
2. \( u(c_0, c_1) = c_0 c_1 \);
3. \( u(c_0, c_1) = c_0 + c_1 \).

c. Assume instead, that the individual obtains the following investment opportunity: he can invest $20 at time \( t = 0 \) at an 8\% interest rate. How does the optimal consumption choice of this individual change for each of the preferences given above? Is he better or worse off than before? Explain.

3. More on net present values...

You and two of your colleagues at college are about to finish your studies and have the opportunity of investing in a business for the following year. This investment requires renting an office for which you need to deposit $3,200 now (April ’06), and pay $800 per month starting in September. In four months (August ’06), you would need to contract the utilities, that will mean another cost of $200 per month. Finally, in September ’06 you could start doing business and you expect to earn around $1,560 per month for the following year. You plan to continue running the business until August ’07, so you expect all these expenses and earnings to continue until that time.

1. Assume that the real interest rate per month is \( r = 4\% \). Should you undertake the project?

2. Assume that it is already December ’06, and an investor wants to buy your business. At what price would you be willing to sell?

3. Assume that it is already December ’06 and an investor wants to buy your business. But now suppose your earnings have been growing at 2\% per month and you expect them to continue growing at that rate until August ’07. Your earnings in December ’06 are $1,655.48. At what price would you be willing to sell in this case?
4. **Life annuities.** Annie is 50 years old and is planning for her retirement at age 65. Apart from her pension, she wants to invest in some other income source. She has no descendents so she wants to spend all her money in providing herself a good elderly care.

She goes to Fidelious.com to get an estimate of a life annuity. A life annuity is a fixed annuity, that is, a contract in which a life insurance company promises to pay the investor a fixed dollar amount on a regular basis until the investor’s death, starting at a fixed point in time in the future. The size of these payments will depend on how much money the investor gives the insurance company before receiving the annuity payments. Also the investor’s age is important.

In the on-line simulation, Annie is asked about her age, and the investment she would be willing to make. Assume that Fidelious.com is in a competitive market where it makes no profits. And assume that actuaries at Fidelious.com have estimated that Annie will live to be 92 but will not survive to be 93.

1. Suppose the annual interest rate that Fidelious.com uses for discounting is $r = 2.5\%$, and Annie is ready to pay $25,000 each year as long as she works. How much money could Annie get per year from an annuity that starts paying out as soon as she retires?

2. If instead of making this investment decision at age 50, she makes up her mind 10 years later, how much could she get with same data as above? Explain the differences with the result in point 1.

(Note: a life annuity acts as insurance against the investor’s uncertainty on the time of his/her death, since it assures you a constant flow of income until you die. Other types of annuities include variable annuities and deferred annuities.)