1. 

(i) \[ \sigma^p = \sqrt{(X_T \sigma_T)^2 + (X_w \sigma_w)^2 + 2(X_T \sigma_T)(X_w \sigma_w)\rho_{TW}} \]
\[ = \sqrt{(0.5 \times 0)^2 + (0.5\sigma_w)^2 + 2(0.5 \times 0)(0.5\sigma_w)^2} \times 0 \]
\[ = \sqrt{(0.5\sigma_w)^2} = 0.5\sigma_w = 0.5(\sigma_w + \sigma_T) \quad \text{Since } \sigma_T = 0 \]

The standard deviation of the portfolio is the midway of that of the two securities.

(ii) \[ \sigma^p = \sqrt{(X_w \sigma_w)^2 + (X_X \sigma_X)^2 + 2(X_w \sigma_w)(X_X \sigma_X)\rho_{WX}} \]
\[ = \sqrt{(0.5 \times \sigma_w)^2 + (0.5\sigma_X)^2 + 2(0.5 \times \sigma_w)(0.5\sigma_X) \times 1} \]
\[ = \sqrt{(0.5\sigma_w + 0.5\sigma_X)^2} = 0.5(\sigma_w + \sigma_X) \]

The standard deviation of the portfolio is the midway of that of the two securities.

(iii) \[ \sigma^p = \sqrt{(X_X \sigma_X)^2 + (X_Y \sigma_Y)^2 + 2(X_X \sigma_X)(X_Y \sigma_Y)\rho_{XY}} \]
\[ = \sqrt{(0.5 \times \sigma_X)^2 + (0.5\sigma_Y)^2 + 2(0.5 \times \sigma_X)(0.5\sigma_Y) \times 0} \]
\[ < \sqrt{(0.5\sigma_X + 0.5\sigma_Y)^2} \]

So, \( \sigma^p < 0.5(\sigma_X + \sigma_Y) \), the standard deviation of the portfolio is not the midway of that of the two securities.

(iv) \[ \sigma^p = \sqrt{(X_Y \sigma_Y)^2 + (X_Z \sigma_Z)^2 + 2(X_Y \sigma_Y)(X_Z \sigma_Z)\rho_{YZ}} \]
\[ = \sqrt{(0.5 \times \sigma_Y)^2 + (0.5\sigma_Z)^2 + 2(0.5 \times \sigma_Y)(0.5\sigma_Z) \times (-1)} \]
\[ = \sqrt{(0.5\sigma_Y - 0.5\sigma_Z)^2} = 0.5(\sigma_Y - \sigma_Z) \]

The standard deviation of the portfolio is not the midway of that of the two securities.

2. (a) 

![return-risk trade off](image)
(b) Portfolio A, D and G are not efficient portfolios.

A is dominated by B, since \( r^A < r^B \), but \( \sigma^A > \sigma^B \).

D is dominated by E, since \( r^D < r^E \), but \( \sigma^D = \sigma^E \).

G is dominated by F, since \( r^G = r^F \), but \( \sigma^G < \sigma^F \).

(c)

If you can borrow or lend at risk free rate 12% and you choose one risky portfolio (named I) from above, then the expected return and risk for the new portfolio (NP) consisting of risky portfolio I and riskless asset (borrowing or lending) can be indicated by a straight line. The equation for this line is:

\[
E(r^{NP}) = r + \frac{E(r^I) - r}{\sigma_I} \sigma_{NP} = 12\% + \frac{E(r^I) - 12\%}{\sigma_I} \sigma_{NP}
\]

Portfolio F is the best choice since it maximizes the slope \( \frac{E(r^I) - 12\%}{\sigma_I} \).

So, \( E(r^{NP}) = r + \frac{E(r^F) - r}{\sigma_F} \sigma_{NP} = 12\% + \frac{18\% - 12\%}{32\%} \sigma_{NP} = 12\% + 0.1875\sigma_{NP} \)

(d) You can only choose portfolio C which delivers 15% rate of return, if you can’t borrow or lend.

(e) \( E(r^{NP}) = 12\% + 0.1875\sigma_{NP} = 12\% + 0.1875\times25\% = 16.68\% \)

So you can get 16.68% rate of return if you can borrow or lend.

10.19

(a) \( E(r^{WD}) = r + \frac{E(r^M) - r}{\sigma_M} \sigma_{WD} = 5\% + \frac{12\% - 5\%}{10\%} \sigma_{WD} = 5\% + 0.7\sigma_{WD} \)
\[ \sigma_{WD} = 7\% \Rightarrow E(r_{WD}) = 5\% + 0.7 \times 7\% = 9.9\% \]

(b) \( E(r_{WD}) = 5\% + 0.7 \sigma_{WD} \)

\[ E(r_{WD}) = 20\% \Rightarrow \sigma_{WD} = 21.43\% \]

10.23

\[ E(r^H) - r = \beta(r^H) \times [E(r^M) - r] \]

\[ E(r^H) - 6\% = 1.2 \times 8.5\% \Rightarrow E(r^H) = 16.2\% \]

10.26

\[ E(r^T) - r = \beta(r^T) \times [E(r^M) - r] \]

\[ 14.2\% - 3.7\% = \beta(r^T) \times 7.5\% \Rightarrow \beta(r^T) = 1.4 \]

10.27

\[ E(r^{MP}) - r = \beta(r^{MP}) \times [E(r^M) - r] \Rightarrow 25\% - r = 1.4 \times [E(r^M) - r] \]  

(1)

\[ E(r^{PDC}) - r = \beta(r^{PDC}) \times [E(r^M) - r] \Rightarrow 14\% - r = 0.7 \times [E(r^M) - r] \]  

(2)

(1)-2*(2) \Rightarrow (25\%-r)-(28\%-2r)=0 \Rightarrow r=3\%, \ E(r^M) = 18.71\% \]

10.29

(a) \( E(r^M) = \text{risk free rate} + \text{market risk premium} = 5\% + 7\% = 12\% \)

(b) According to CAPM, the expected rate of return of a security with a beta of 0.8 is

\[ E(r^A) = r + \beta(r^A) \times [E(r^M) - r] = 7\% + 0.8 \times 5\% = 11\% \]

Its actual expected rate of return, as indicated in question, is 9\%, which is lower than 11\%. This means this security is overpriced. Once the investors realize it, the demand for this security will
decrease, the price will be lowered and the expected rate of return will increase to 11%.

(c) According to CAPM, the expected rate of return of a security with a beta of 3 is

\[ E(r^A) = r + \beta(r^A) \cdot [E(r^M) - r] = 7\% + 3 \cdot 5\% = 22\% \]

Its actual expected rate of return, as indicated in question, is 25%, which is higher than 22%. This means this security is underpriced. Once the investors realize it, the demand for this security will increase, the price will be higher and the expected rate of return will lower to 22%.

10.32
The expected rate of return and standard deviation of any portfolio consisting of risky securities and risk-free asset can be illustrated as:

\[ E(r^p) = r + \frac{E(r^M) - r}{\sigma_M} \cdot \sigma_p \]

25\% = 5\% + \frac{20\% - 5\%}{\sigma_M} \cdot 4\% \Rightarrow \sigma_M = 3\%

\[ \beta_{r(\text{new portfolio})} = \frac{\text{Cov}(r_{\text{new portfolio}}, r^M)}{\text{Var}(r^M)} = \frac{\beta_{r(\text{new portfolio}), r^M} \cdot \sigma_{r(\text{new portfolio})}}{\sigma_M} = \frac{0.5 \cdot 2\%}{3\%} = 1/3 \]

\[ E(r_{\text{new portfolio}}) = r + \beta_{r(\text{new portfolio})} \cdot [E(r^M) - r] = 5\% + (1/3) \cdot (20\% - 5\%) = 10\% \]

10.35
(a) \[ E(r^A) = r + \beta(r^A) \cdot [E(r^M) - r] = 4.9\% + 9.4\% \cdot \beta(r^A) \]

(b) \[ E(r^p) = r + \beta(r^p) \cdot [E(r^M) - r] = 4.9\% + 9.4\% \cdot \beta(r^p) \]

\[
= 4.9\% + 9.4\% \cdot \frac{\text{Cov}(r^M, r^p)}{\text{Var}(r^M)} = 4.9\% + 9.4\% \cdot \frac{0.0635}{0.04326} = 18.70\% 
\]

10.38

\[ \beta(r^p) = X_4 \beta_A + X_5 \beta_B + \ldots + X_D \beta_D \]

\[
= \frac{\$5,000}{\$30,000} \cdot 0.75 + \frac{\$10,000}{\$30,000} \cdot 1.1 + \frac{\$8,000}{\$30,000} \cdot 1.36 + \frac{\$7,000}{\$30,000} \cdot 1.88 = 1.293 
\]

\[ E(r^p) = r + \beta(rP) \cdot [E(r^M) - r] = 4\% + 1.293 \cdot (15\% - 4\%) = 18.22\% \]