Exercise 1, Lesson 5

Suppose you have $10,000 you want to invest for one year. You can invest this money either in a mutual fund, a savings account or in a portfolio that combines both. The mutual fund is a portfolio of stocks. Like any other stock, investors can buy shares of this mutual fund, which give right to receive dividends and which can be sold at any time. The price of a share of the mutual fund today is $70. The guaranteed interest rate on a savings account is 5%.

Looking at historical data and your perception of the current economic situation, you expect the following scenario for next year. With 0.3 of probability the economy will boom, firms will make high profits, will distribute generous dividends and stock market prices will increase. In this scenario, dividends per share of the mutual fund next year would be $5 per share, and the price of a share in one year from now will be $110.

Another possibility is that the economy follows its current trend. You assign this scenario a 0.4 of probability. In this case, dividends per share will be $4 next year, and the price of a share in one year from now will be $80.

Finally, an economic downturn is also possible. In this case, dividends per share will still be $4, but the price one year from now will be lower, $50.

1. Compute the expected rate of return on the mutual fund and its standard deviation.

2. Compute the expected rate of return and standard deviation of your portfolio if you invest a proportion $y$ of your money in the mutual fund and $1 - y$ in the savings account.

3. In a graph, represent the different combinations ($\sigma, E$) of standard deviation expected returns that you can obtain by choosing an appropriate proportion $y$. Draw the graph as precisely as you can. Insert the points $A$ corresponding to $y = 0$, $B$ corresponding to $y = 1/4$, $C$ corresponding to $y = 1/2$, and $D$ corresponding to $y = 1$.

4. Let $(\sigma_A, E_A)$ and $(\sigma_D, E_D)$ denote the coordinates of $A$ and $D$. Consider the point $F$ such that $(\sigma_F, E_F) = (2\sigma_D, E_A + 2(E_D - E_A))$. Insert this point on the graph and explain precisely which portfolio strategy leads to a rate of return with characteristics $(\sigma_F, E_F)$.

5. Actually you are quite risk averse and hate the possibility of losing money. You promise yourself to never tolerate a standard deviation that is larger than the expected value of the risky rate of return. What is the maximum proportion $y_{\text{max}}$ that you can invest in the mutual fund to respect your rule? Indicate how much you would have in the three possible states of the economy if you invest a proportion $y_{\text{max}}$ of your money in the mutual fund. Does your rule guarantee that you are not going to lose money? Explain.