Example of the NPVGO model (2)

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(This problem is adapted from Bodie et al. 1993 Investments) Consider a firm with expected earnings in the coming year of $5 per share. This firm engages in projects that generate a return on investment of 15%, greater than the required rate of return $r = 12.5\%$. If this firm retains or plows back some of its earnings into its highly profitable projects, it can earn a 15\% of return for its shareholders. Suppose that the dividend payout ratio for this firm is 40\%. What is the price of a share for this firm?

- Dividend growth model

$$P = \frac{Div}{r - g},$$

Dividend payout ratio is 40\%, then

$$Div = 0.4 \times 5 = 2$$

Retention ratio = 1 - Dividend payout ratio = 1 − 0.4 = 0.6.

Growth rate, $g$,

$$g = ROE \times retention\ ratio = 0.15 \times 0.6 = 0.09$$

Then

$$P = \frac{Div}{r - g} = \frac{2}{0.125 - 0.09} = 57.14$$

- NPVGO model

1.- Value of a cash-cow firm

$$p = \frac{EPS}{r} = \frac{5}{0.125} = 40$$
2.- Value of growth opportunities

Time 1: investment is 60% $5 = $3, at 15% return

\[-$3 + \frac{\$3 \times 0.15}{1.125} + \frac{\$3 \times 0.15}{(1.125)^2} + \frac{\$3 \times 0.15}{(1.125)^3} + ... = -$3 + \frac{\$3 \times 0.15}{0.125}\]

Time 2: investment is 60% $5 = $3 that grows at a \( g = 9\% \), that is, $3 \times 1.09, at 15% return

\[-$3 \times 1.09 + \frac{\$3 \times 1.09 \times 0.15}{1.125} + \frac{\$3 \times 1.09 \times 0.15}{(1.125)^2} + ... = \left(-$3 + \frac{\$3 \times 0.15}{0.125}\right) \times 1.09\]

Time 3: investment is 60% $5 = $3 \times 1.09 that grows at a \( g = 9\% \), that is, \( \$3 \times 1.09^2 \), at 15% return

\[-$3 \times 1.09^2 + \frac{\$3 \times 1.09^2 \times 0.15}{1.125} + \frac{\$3 \times 1.09^2 \times 0.15}{(1.125)^2} + ... = \left(-$3 + \frac{\$3 \times 0.15}{0.125}\right) \times 1.09^2\]

and so on and so forth.

If we sum all the NPV of growing opportunities discounted at time 0 we have

\[NPV_{GO} = \frac{(-$3 + \frac{\$3 \times 0.15}{0.125})}{1.125} + \frac{(-$3 + \frac{\$3 \times 0.15}{0.125}) \times 1.09}{1.125^2} + ... \]

factoring out \( \frac{(-$3 + \frac{\$3 \times 0.15}{0.125})}{1.125} \), we have

\[NPV_{GO} = \frac{(-$3 + \frac{\$3 \times 0.15}{0.125})}{1.125} \left[ 1 + \frac{1.09}{1.125} + \frac{1.09^2}{1.125^2} + ... \right] = \]

\[= \frac{(-$3 + \frac{\$3 \times 0.15}{0.125})}{1.125} \left[ \frac{1}{1 - \frac{1.09}{1.125}} \right] = \frac{(-$3 + \frac{\$3 \times 0.15}{0.125})}{1.125} \frac{1.125}{1.125 - 1.09} = \]

\[= \frac{(-$3 + \frac{\$3 \times 0.15}{0.125})}{1.125} \frac{1.125}{0.035} = \frac{(-$3 + \frac{\$3 \times 0.15}{0.125})}{0.035} = \$17.14\]

So summing (1) and (2) we obtain

\[P = \frac{EPS}{r} + NPV_{GO} = \$40 + \$17.14 = \$57.14\]