Appendix: The term structure of interest rates

Why do long-term bonds offer higher yields to maturity?

Risk, expectations that interest rates will be higher in time, expectations of higher inflation ...
Example:

Spot rates: the yield to maturity on zero-coupon bonds.

NOTE: Spot rates or yields do NOT equal the one-year interest rates for each year!!!!
The **term structure** describes the spot rates with different maturities.

**Example:** Consider the following zero-coupon bonds with different maturity and same par value

<table>
<thead>
<tr>
<th>Year</th>
<th>Interest Rate ($r_t$)</th>
<th>Par value</th>
<th>Price</th>
<th>Yield to maturity or spot rates ($y_t$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 (Today)</td>
<td>8%</td>
<td>$1,000</td>
<td>$925.93</td>
<td>8%</td>
</tr>
<tr>
<td>1</td>
<td>10%</td>
<td>$1,000</td>
<td>$841.75</td>
<td>8.99%</td>
</tr>
<tr>
<td>2</td>
<td>11%</td>
<td>$1,000</td>
<td>$758.33</td>
<td>9.66%</td>
</tr>
<tr>
<td>3</td>
<td>11%</td>
<td>$1,000</td>
<td>$683.18</td>
<td>9.99%</td>
</tr>
</tbody>
</table>

If we plot the yields to maturity along time we obtain the **yield curve**.
Forward interest rate

Problem: investors do NOT know ST interest rates for the future years.

DO know bond prices & yields to maturity → infer future short term rates.

In our first example:
We can get $r^f$ as follows:

$$(1 + r_2^0)^2 = (1 + r_1^0)(1 + r^f) \rightarrow r^f = \frac{(1 + r_2^0)^2}{(1 + r_1^0)} - 1$$

In general, future interest rates at time $n$ with information at $t = 0$ would be $r^f$:

$$(1 + r_{n-1}^0)^{n-1}(1 + r^f) = (1 + r_n^0)^n,$$

that is,

$$r^f = \frac{(1 + r_n^0)^n}{(1 + r_{n-1}^0)^{n-1}} - 1,$$
Theories of the term structure

The Expectations Hypothesis: the forward rate equals the market consensus expectation of the future short interest rate, i.e. in our example $r^f = E(\tilde{r}_{1}^1)$, and liquidity premiums are zero.

We can use the forward rates derived from the yield curve to infer market expectations of future short rates. Take for example

$$(1 + r_2^0)^2 = (1 + r_1^0)(1 + r^f) = (1 + r_1^0) \left[ 1 + E(\tilde{r}_{1}^1) \right]$$

if the expectations hypothesis is correct.

The yield to maturity would thus be determined solely by current and expected future one-period interest rates.

An upward-sloping yield curve would be clear evidence that investors anticipate increases in interest rates.
Liquidity-preference Hypothesis:

Short term investors will be unwilling to hold long-term bonds unless

\[ r^f > E(\tilde{r}_1^1) \]

Long term investors will be unwilling to hold short bonds unless

\[ r^f < E(\tilde{r}_1^1) \]

I.e.: both groups require a premium to induce them to hold bonds with maturities different from their investment horizons.

Liquidity preference theory: short term investors dominate the market so that the forward rate exceeds the expected short rate, i.e. \( r^f > E(\tilde{r}_1^1) \), and

\[ \text{liquidity premium} = r^f - E(\tilde{r}_1^1) > 0. \]