6. Market equilibrium

Researchers have shown that the best measure of the risk of a security in a large portfolio is the beta (\( \beta \)) of the security.

**Beta** measures the responsiveness of a security to movements in the market portfolio.

\[
\beta_i = \frac{Cov(r_i, r_M)}{\sigma^2_M} \quad (1)
\]

Clearly, your estimate of beta will depend upon your choice of a proxy for the market portfolio.

The average beta across all securities is one, that is

\[
\sum_{i=1}^{N} x_i \beta_i = 1,
\]
where $x_i$: the proportion of security $i$'s market value to that of the entire market, and $N$: the number of securities in the market.
7. Risk and expected return (CAPM)

Expected return on the market:

\[ E(r_M) = r_F + \text{Market Risk Premium} \]

Expected return on individual security: Capital Asset Pricing Model (CAPM)

\[ E(r_i) = r_F + \beta_i \times \left[ \frac{E(r_M) - r_F}{\text{Market Risk Premium}} \right] \]

This applies to individual securities held within well-diversified portfolios.

- If \( \beta_i = 0 \), then the expected return is \( r_F \)
- If \( \beta_i = 1 \), then \( E(r_i) = E(r_M) \)
If we plot $E(r_i) = r_F + \beta_i \times [E(r_M) - r_F]$ in the axis $(\beta_i, E(r_i))$, we obtain

CAPM: returns (and therefore risk premia) for any stock (or portfolio) are related to beta, the systematic risk that cannot be diversified away.
IMPLICATIONS:

1. Linear relationship between $E(r_i)$ and $\beta_i$.

2. CAPM is valid for valuing portfolios as well as securities.

3. Notice that the Security Market Line is NOT the same as the Capital Market Line.
SUMMARY AND CONCLUSIONS

- This chapter sets forth the principles of modern portfolio theory.

- The expected return and variance on a portfolio of two securities A and B are given by

\[ E(r_P) = w_A E(r_A) + w_B E(r_B) \]
\[ \sigma_P^2 = (w_A \sigma_A)^2 + (w_B \sigma_B)^2 + 2 (w_A \sigma_A) (w_B \sigma_B) \rho_{AB} \]

- By varying \( w_A \), one can trace out the efficient set of portfolios. We graphed the efficient set for the two-asset case as a curve, pointing out that the degree of curvature reflects the diversification effect: the lower the correlation between the two securities, the greater the diversification.
• The same general shape holds in a world of many assets.

• The efficient set of risky assets can be combined with riskless borrowing and lending. In this case, a rational investor will always choose to hold the portfolio of risky securities represented by the market portfolio.

• The contribution of a security to the risk of a well-diversified portfolio is proportional to the covariance of the security’s return with the market’s return. This contribution is called the beta.

• The CAPM states that the expected return on a security is positively related to the security’s beta.