Problem set 2  
Dynamic Macroeconomic Analysis  
Máster en Economía Internacional  
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1. Consider a pure exchange economy that lasts for two periods. Agents are indexed by $i = a, b$ and have a known endowment $y_{i,t}$ in both periods. The agents have access to a perfectly competitive credit market. The agents’ objective is to maximize discounted lifetime utility

$$U(c_{i,1}, c_{1,2}) = \frac{c_{i,1}^{1-\sigma}}{1-\sigma} + \beta \frac{c_{i,2}^{1-\sigma}}{1-\sigma}$$

a. Characterize the optimal consumption and savings decisions of a representative agent for an arbitrary interest rate $R$.

b. Derive the response of the agent’s optimal consumption ratio $c_2/c_1$ to a 10% increase in the value of $1 + R$, the relative price of future consumption.

c. Calculate the equilibrium interest rate when $\beta = 1$ and there are $N_a = N_b = N$ agents of both types with income streams $y_{a,1} = y_{b,2} = 1$ and $y_{a,2} = y_{b,1} = 0$.

2. Consider an economy that lasts for two periods. There is a unit mass of identical agents with a fixed income of $y$ in both periods. The government purchases a constant amount of public goods, $g$, in both periods. By assumption, both the government and the agents have access to an international credit market allowing them to borrow or lend any desired quantity at the fixed rate $R > 0$. Suppose that the preferences of the agents are given by

$$U(c_1, c_2) = \ln(c_1) + \beta \ln(c_2)$$

Suppose that the government levies a lump-sum tax $\tau_t$ in both periods.

a. Suppose for the moment that the government needs to run a balanced budget in both periods. Calculate the value of the tax and the optimal consumption levels in both periods for any arbitrary $R$.

b. Now assume that the government is allowed to run a deficit in the first period as long as it repays its debt in the second period. Show how the optimal consumption decisions of the agents change if the government reduces the first-period tax rate by 10%. *hint: Calculate the required change in $\tau_2$ and determine how this change in tax rates affect the discounted value of disposable income.]*

3. Consider an agent that lives for two periods. In the first period the agent receives a fixed endowment $y_1 = y$ while his second-period endowment is given by $y_2 = y + \epsilon$, where $\epsilon$ is a random variable with mean zero en variance $\sigma^2$. The objective of the agent is to maximize expected lifetime utility

$$U(c_1, c_2) = u(c_1) + \beta E u(c_2)$$

where $u(c_1) = \frac{c_1^{1-\gamma-1}}{1-\gamma}$. 

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a. Derive the expression for marginal utility in the second period for any arbitrary value of \( c_2, \beta \) and \( R \).

b. Derive the expression for a second-order Taylor expansion of \( u'(c_2) \) for small deviations of \( y_2 \) from its expected value \( E_{y_2} = E(y + \epsilon) = y \).

c. Use your solution in \( b \) to obtain an expression for \( E(u'(c_2)) \) and show that is strictly increasing in \( \sigma^2 \).

d. Discuss the implications of a mean-preserving spread, i.e. an increase in the variance of the shock for a given value of the mean, for the optimal consumption level in the first period. \( \text{[hint: Recall that } u'(c_1^*) = \beta(1 + R)E(u'(c_2^*))] \).