1 Consider a version of our canonical OLG model with capital in which the agents choose how much to work when young. The preferences of the agents are given by:

\[
U(c_{y,t}, c_{o,t+1}, l_{y,t}) = \log(c_{y,t}) + \beta \log(c_{o,t+1}) + \log(1 - l_t),
\]

where \(l_t\) denotes labour supply. The young agents offer their labour services on a competitive labour market and labour income, \(w_t l_t\) is their only source of income. The rest of the assumptions are the same as in class.

a Solve the optimization problem of the agents and derive the deterministic steady state allocation for the case in which \(A_t = 1\) for all \(t\).

b. Suppose that the economy is steady state when it is hit by a negative productivity shock in period \(T\) that reduces the value of \(A_t\) by 10% during one period. Derive the percentage deviations of \(Y_t\), \(w_t\) and \(K_{t+1}\) for \(t = T, T + 1, T + 2\).

c. Explain why the economy does not immediately return to steady state in period \(T+1\).

d. In this example, optimal labour supply does not respond to shocks to productivity. Explain why this is the case. How would your answers to the previous questions change if the substitution effects from changes in \(w_t\) are larger than the income effects.