Name:
DNI:

Instructions: The exam consists of five short questions and one problem. The relative weights are in the heading of each problem. The grade is equal to the total number of points. Please respect the indicated space for the answers to both questions. All mobile devices should be switched off during the exam and the use of notes is not permitted. Students who infringe the general rules for exams will be requested to hand in their exam and will receive a grade of 0.

Part 1. Consider the following five statements. You should indicate whether the statement is true or false and provide a concise explanation for your answer. There is no need to provide calculations or formal demonstrations. Answers that come without an explanation will receive zero points. All five questions refer to our basic OLG model with physical capital in which agents have logarithmic preferences, $U(c_{y,t},c_{o,t+1}) = \log(c_{y,t}) + \beta \log(c_{o,t+1})$, while the production technology of the representative firm is given by $Y_t = A_t K_t^\alpha L_t^{1-\alpha}$. All young agents inelastically supply one unit of labor and for the moment we assume that the population is constant.

1. In this setup the equilibrium savings rate is an increasing function of the interest rate (or rental price of capital) $r_t$. True or false? (0.5 points)

2. The steady-state growth rate of the economy depends positively on the savings rate of the young. True or false? (0.5 points)

3. Suppose we compare two economies that are identical except for the value of $\beta$. In steady state the economy with the highest value of $\beta$ accumulates a higher capital stock and the agents in this economy can unambiguously afford a higher level of consumption than in the second economy with a lower discount rate. True or false? (0.5 points)

4. Suppose the economy is in steady state when it is hit by a negative TFP shock that lasts for one period. In the next period output will continue below its steady-state value because the capital stock will not immediately return to its steady-state value after the shock has died away. True or false? (0.5 points)

5. Suppose that the population grows at rate $n > 0$ and denote the depreciation rate by $\delta$. When the steady-state value of $r$ is such that $r - \delta > n$ the economy is dynamically inefficient and the introduction of a Pay-As-You-Go pension system would raise social welfare. True or false? (0.5 points)
Part 2. Consider an economy with overlapping generations in which each agent lives for two periods. In each period $N_t = N$ young agents are born who obtain a fixed endowment $\omega > 0$ of the unique perishable good. This is the only source of income of the agents whose preferences are given by

$$U(c_{y,t}, c_{o,t+1}) = \frac{c_{y,t}^{1-\sigma}}{1-\sigma} + \beta \frac{c_{o,t+1}^{1-\sigma}}{1-\sigma}$$

with $\sigma \in (0,1]$ and $\beta \in (0,1]$. Throughout the problem we will restrict attention to steady-state allocations and so we can write $c_{y,t} = c_y$ and $c_{o,t} = c_o$. Your answers should contain all the necessary steps to arrive at the desired result.

a. Define the set of feasible steady-state allocations. (1 point)

b. Derive the allocation that maximizes the welfare of a representative generation (golden rule allocation) and explain when consumption in this efficient allocation is constant over time, i.e. $c_y = c_o$. (2.5 points)

c. Explain why the decentralized equilibrium does not coincide with the golden rule. (1 point)

d. Now suppose that the agents have a storage technology that permits them to store part or all of their endowment for consumption at old age. That is each unit stored in $t$ permits the agent to consume 1 unit in $t+1$. Demonstrate that the equilibrium allocation coincides with the efficient allocation defined in b. (2 points)

e. Is the storage technology still efficient when the economy grows at rate $n > 0$? Illustrate your answer with a figure that compares the set of feasible allocations and the budget set of the agents. (1 point)