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So far, we have studied consumption and savings decisions and labor supply decisions in a two-period setting. However, in the case of labor supply, we analyzed a partial equilibrium setting with exogenous wages.

In this theme we derive the general equilibrium allocations in two periods with endogenous labor supply, savings and consumption.

Second, we will analyze the response of the equilibrium to transitory and permanent shocks to TFP.

Our model is still incomplete (we ignore the role of capital and the economy only lasts for two periods), but it neatly illustrates essential features of RBC theory.
The environment

- The economy lasts for two periods.
- There are two groups of agents with different discount rates. All agents value consumption and leisure and need to decide labor supply in both periods. The labor market is competitive.
- At the other side of the labor market there is a representative firm that hires labor services taking the competitive wage in each period as given.
- At the end of the first period the agents can access the credit market to borrow or lend resources.
The Household Problem

The households choose \((l_1, l_2, c_1, c_2)\) taking prices \((w_1, w_2, R)\) as given. Formally,

\[
\max_{c_1, c_2, l_1, l_2} \left\{ \ln c_1 + \ln(1 - l_1) + \beta (\ln c_2 + \ln(1 - l_2)) \right. \\
+ \left. \lambda \left[ w_1 l_1 + \frac{1}{1 + R} w_2 l_2 - c_1 - \frac{1}{1 + R} c_2 \right] \right\},
\]

where \(\lambda\) is the Lagrange multiplier.

Notice that we have solved this problem in the previous lecture.
The Household Problem

The first-order conditions

\[
\frac{1}{c_1} - \lambda = 0 \quad (c_1)
\]

\[
\beta \frac{1}{c_2} - \lambda \frac{1}{1 + R} = 0 \quad (c_2)
\]

\[
-\frac{1}{1 - l_1} + \lambda w_1 = 0 \quad (l_1)
\]

\[
-\beta \frac{1}{1 - l_2} + \lambda \frac{w_2}{1 + R} = 0. \quad (l_2)
\]
Consumption decisions

In a first step we derive the consumption Euler condition:

\[ c_2 = \beta (1 + R) c_1, \]

Inserting this condition in the budget constraint we obtain:

\[ (1 + \beta) c_1 = x \implies c_1 = \frac{x}{1 + \beta}, \]  \hspace{1cm} (1)

\[ c_2 = c_1 \beta (1 + R) = \frac{\beta x (1 + R)}{1 + \beta}. \]  \hspace{1cm} (2)
Consumption decisions

- In the next step we combine the FOCs for $c_1$ and $l_1$:

$$
\frac{1}{1 - l_1} = \frac{1}{c_1} w_1 \implies w_1 l_1 = w_1 - c_1. \quad (3)
$$

- Similarly, inserting $\lambda = u'(c_1)$ into the FOC for $l_2$

$$
-\beta \frac{1}{1 - l_2} + \lambda \frac{w_2}{1 + R} = 0.
$$

we obtain the following condition:

$$
\beta \frac{1}{1 - l_2} = \frac{1}{c_1} \frac{w_2}{1 + R}.
$$

- Hence,

$$
c_1 \beta (1 + R) = w_2 - w_2 l_2 \implies \frac{w_2 l_2}{1 + R} = \frac{w_2}{1 + R} - c_1 \beta. \quad (4)
$$
Consumption decisions

- The value of the agent’s lifetime resources can now be written as

\[ x = w_1 l_1 + \frac{w_2 l_2}{1 + R} = w_1 - c_1 + \frac{w_2}{1 + R} - c_1 \beta. \]

- Given that \( c_1 = \frac{x}{1 + \beta} \), we can thus write

\[ c_1 = \frac{w_1 - c_1 + \frac{w_2}{1 + R} - c_1 \beta}{1 + \beta}, \]

leading to the following solutions

\[ c_1^* = \frac{1}{2(1 + \beta)} \left[ w_1 + \frac{w_2}{(1 + R)} \right]. \]

\[ c_2^* = \beta(1 + R) \frac{1}{2(1 + \beta)} \left[ w_1 + \frac{w_2}{(1 + R)} \right]. \]
Labor supply decisions

Given that we have demonstrated that \( w_1 l_1 = w_1 - c_1 \), we have

\[
w_1 l_1^* = w_1 - c_1^* \\
= w_1 - \frac{1}{2(1 + \beta)} \left[ w_1 + \frac{w_2}{(1 + R)} \right]
\]

\[
w_1 l_1 = w_1 \left[ 1 - \frac{1}{2(1 + \beta)} \right] - \frac{1}{2(1 + \beta)} \frac{w_2}{(1 + R)}
\]

\[
l_1^* = \frac{1 + 2\beta}{2(1 + \beta)} - \frac{1}{2(1 + \beta)} \frac{1}{1 + R} \frac{w_2}{w_1}.
\]
Labor supply decisions

Similarly, our previous results imply that \( w_2 l_2^* = w_2 - c_1^* \beta (1 + R) \),

\[
w_2 l_2 = w_2 - c_1 \beta (1 + R)
= w_2 - \frac{\beta (1 + R)}{2(1 + \beta)} \left[ w_1 + \frac{w_2}{(1 + R)} \right]
= w_2 \left[ 1 - \frac{\beta (1 + R)}{2(1 + \beta)} \frac{1}{1 + R} \right] - \frac{\beta (1 + R)}{2(1 + \beta)} w_1.
\]

\[
l_2^* = \frac{2 + \beta}{2(1 + \beta)} - \frac{\beta (1 + R)}{2(1 + \beta)} w_1.
\]
Summary household decisions

In equilibrium, the household decisions concerning \((c_1, c_2, l_1, l_2)\) satisfy

\[
c_1^* = \frac{1}{2(1 + \beta)} \left[ w_1 + \frac{w_2}{(1 + R)} \right]
\]

\[
c_2^* = \beta(1 + R) \frac{1}{2(1 + \beta)} \left[ w_1 + \frac{w_2}{(1 + R)} \right]
\]

\[
l_1^* = \frac{1 + 2\beta}{2(1 + \beta)} - \frac{1}{2(1 + \beta)} \frac{1}{1 + R} \frac{w_2}{w_1}
\]

\[
l_2^* = \frac{2 + \beta}{2(1 + \beta)} - \beta(1 + R) \frac{w_1}{2(1 + \beta)} \frac{1}{w_2}
\]
The representative firm

- Like before we assume that there is a representative firm that acts as a price taker in the labor and the product market.

- For simplicity, we assume that production function is linear in labor:

\[ Y_j^f = A_j L_j^f, \]

where \( A_j \) denotes the value of TFP in period \( j \), \( Y_j^f \) is output in period \( j \) and \( L_j^f \) is the firm’s labor demand in \( j \).
The problem of the representative firm

- In each period the representative firm solves the following static problem:

\[ \pi_j = \max_{l_f} \left[ A_j L_j^f - w_j L_j^f \right], \]

where \( w_j \) denotes the competitive wage in period \( j \).

- The FOC associated with the firm’s problem implies that

\[ A_j = w_j. \]

- In other words, in each period wages are uniquely determined by the value of TFP, i.e. \( w_1 = A_1 \) and \( w_2 = A_2 \).
To generate a credit market, we will once again assume that there are various groups of agents with different discount rates.

Let's define $\beta_i$ as the discount factor of agent $i$, where $i = 1, \ldots, N$.

Accordingly,

$$c_{1,i}^* = \frac{1}{2(1 + \beta_i)} \left[ w_1 + \frac{w_2}{(1 + R)} \right]. \quad (5)$$

$$c_{2,i}^* = \beta_i (1 + R) \frac{1}{2(1 + \beta_i)} \left[ w_1 + \frac{w_2}{(1 + R)} \right]. \quad (6)$$
Recall that 

\[ b_{1,i} = w_{1}l_{1,i} - c_{1,i} \]

Furthermore, we know that \( w_{1}l_{1,i} = w_{1,i} - c_{1,i} \). Hence, 

\[ b_{1,i}^* = w_{1}l_{1,i} - c \]

\[ = w_{1} - 2c_{1,i} \]

\[ = w_{1} - 2 \frac{1}{2(1 + \beta_i)} \left[ w_{1} + \frac{w_{2}}{(1 + R)} \right] \]

\[ = w_{1} - \frac{1}{1 + \beta_i} \left[ w_{1} + \frac{w_{2}}{(1 + R)} \right] \]

\[ = \frac{w_{1}\beta_i - \frac{w_{2}}{1+R}}{1 + \beta_i} \]
Labor supply

Finally, recall that labor supply in both periods satisfies:

$$I_{1,i}^* = \frac{1 + 2\beta_i}{2(1 + \beta_i)} - \frac{1}{2(1 + \beta_i)} \frac{1}{1 + R} \frac{w_2}{w_1}.$$  \hspace{1cm} (8)

and in the second period

$$I_{2,i}^* = \frac{2 + \beta_i}{2(1 + \beta_i)} - \frac{\beta_i(1 + R)}{2(1 + \beta_i)} \frac{w_1}{w_2}.$$  \hspace{1cm} (9)
**Definition of equilibrium**

Following standard conventions, the competitive equilibrium in this two-period economy can be defined as follows:

- Given $A_1$, $A_2$ and $\beta_i$ for $i = 1, \ldots, N$, the equilibrium of this economy consists of a set of prices $w_1$, $w_2$, and $R$ and allocations $c_{1,i}^*$, $c_{2,i}^*$, $b_{1,i}^*$, $l_{1,i}^*$, and $l_{2,i}^*$ for the households and $L_1^f$ and $L_2^f$ for the firm.

- Given $w_1$, $w_2$, and $R$, the allocations of the household solve the household problem, i.e. $c_{1,i}^*$, $c_{2,i}^*$, $b_{1,i}^*$, $l_{1,i}^*$, and $l_{2,i}^*$ están dadas por las ecuaciones (5), (6), (7), (8) y (9).

- The firms maximize profits given $A_1$ and $A_2$, i.e.

  $$w_1 = A_1 \text{ and } w_2 = A_2.$$

- All markets clear.
Market clearing conditions

- Credit market:

\[ \sum_{i}^{N} b_{1,i}^* = 0, \]

- Goods market:

\[ \sum_{i}^{N} c_{1,i}^* = A_1 L_1^f \quad \text{and} \quad \sum_{i}^{N} c_{2,i}^* = A_2 L_2^f \]

- Labor market:

\[ \sum_{i}^{N} l_{1,i}^* = L_1^f \quad \text{and} \quad \sum_{i}^{N} l_{2,i}^* = L_2^f \]
Walras’ Law

- Remember that the final good is perishable. Hence, in every period we expect that the agents consume the entire value of aggregate output.

- Notice also that the equilibrium in the labor market is satisfied trivially: the firm is willing to hire any amount of labor at the equilibrium wages $w_j^* = A_j$.

- Given the equilibrium in the labor market, we can show that the equilibrium in the goods market implies equilibrium in the credit market, and vice versa.
Proof

- For each agent $i$ the first-period budget constraint can be written as
  \[ b_{1,i} = w_1 l_{1,i} - c_1. \]

- Summing over all agents we get
  \[
  \sum_{i} b_{1,i} = \sum_{i} w_1 l_{1,i} - \sum_{i} c_{1,i} = w_1 \sum_{i} l_{1,i} - \sum_{i} c_{1,i}.
  \]

- Finally, if the labor market and the goods market are in equilibrium, then aggregate savings satisfies
  \[
  \sum_{i} b_{1,i}^* = A_1 \sum_{i} l_{1,i}^* - \sum_{i} c_{1,i}^* \\
  = A_1 L_1^f - \sum_{i} c_{1,i}^* = 0,
  \]
  Hence, the credit market is also in equilibrium.
Aggregate demand

- Let’s start with period 1.
- We have shown that
  \[
  c^*_{1,i} = \frac{1}{2(1 + \beta_i)} \left[ A_1 + \frac{A_2}{(1 + R)} \right].
  \]
- Hence, aggregate demand for the final good is equal to
  \[
  C^d_1 = \sum_i c^*_{1,i} = \left[ A_1 + \frac{A_2}{(1 + R)} \right] \frac{1}{2(1 + \beta_i)}.
  \]
- According to the above equation, aggregate demand in period 1 is a decreasing function of the interest rate.
Aggregate supply

Now let’s derive the solution for aggregate supply:

\[ Y_1^s = A_1 L_1^f = A_1 \sum_{i} I_{1,i}^* = A_1 I_{1,i}^* \]

In this case we find a positive relationship between the level of aggregate supply and the interest rate. The reason is that agents choose to supply more labor in period 1 if the interest rate rises for some reason.
Equilibrium in the product market

The intersection of the aggregate demand and supply schedules uniquely determines the equilibrium interest rate. Notice that $1 + R$ reflects the relative price of future consumption in terms of current consumption!
Labor supply

- Remember that individual’s labor supply in period 1 is given by:
  \[
  l_{1,i}^* = \frac{1 + 2\beta_i}{2(1 + \beta_i)} - \frac{1}{2(1 + \beta_i)} \frac{1}{1 + R \frac{w_2}{w_1}}.
  \]

- Summing over all agents we obtain the expression for aggregate labor supply
  \[
  L_1^s = \sum_{i} l_{1,i}^* = \sum_{i} \left[ \frac{1 + 2\beta_i}{2(1 + \beta_i)} - \frac{1}{2(1 + \beta_i)} \frac{1}{1 + R \frac{w_2}{w_1}} \right]
  \]

- Not surprisingly aggregate labor supply in period 1 is an increasing function of the wage rate \( w_1 \).
Equilibrium in the labor market

Given that the maximization problem of the firm uniquely determines the wage rate \( w_1 = A_1 \), labor demand is perfectly elastic.
A numerical example

- Consider an economy with 100 agents, so that $N = 100$.
- Suppose that 20 of these agents have a discount factor $\beta_1 = 0.8$ while the rest of the agents have a discount factor equal to $\beta_2 = 0.9$. Notice that the type-2 agents are more patient than the type-1 agents.
- Suppose for simplicity that $A_1 = A_2 = 1$.
- Hence in equilibrium $w_1 = 1$ and $w_2 = 1$ and so $w_2/w_1 = 1$. 
In any equilibrium aggregate saving is equal to zero and so:

\[ N_1 \beta_1 A_1 - \frac{A_2}{1+R} + N_2 \beta_2 A_1 - \frac{A_2}{1+R} = 0, \]

which implies that

\[ 20 \frac{0.8 - \frac{1}{1+R}}{1+0.8} + 80 \frac{0.9 - \frac{1}{1+R}}{1+0.9} = 0, \]

and so

\[ 20 \frac{0.8 - \frac{1}{1+R}}{1.8} = 80 \frac{1}{1+R} - 0.9 \]
Equilibrium in the credit market

\[
20 \frac{0.8 - \frac{1}{1+R}}{1.8} = 80 \frac{1}{1+R} - 0.9
\]

From the above equation it follows that

\[
(0.8)20(1.9) - \frac{1.9(20)}{1 + R} = \frac{80(1.8)}{1 + R} - (80)0.9(1.8),
\]

or equivalently

\[
30.4 - \frac{38}{1 + R} = \frac{144}{1 + R} - 129.6.
\]

Collecting terms we get:

\[
30.4 + 129.6 = \frac{144}{1 + R} + \frac{38}{1 + R}
\]

\[
160 = \frac{182}{1 + R} \implies 1 + R = 1.1375 \implies R = 0.1375, (13.75%).
\]
Consumption

- Given our solutions for \( w_1, w_w \) and \( R \), we can now derive the values of consumption:

\[
c_{1,1}^* = \frac{1}{2(1 + \beta_1)} \left[ w_1 + \frac{w_2}{(1 + R)} \right] = \frac{1}{2(1 + 0.8)} \left( 1 + \frac{1}{1.1375} \right) = 0.522.
\]

- Also,

\[
c_{2,1}^* = \beta_1 (1 + R) \frac{1}{2(1 + \beta_1)} \left[ w_1 + \frac{w_2}{(1 + R)} \right] = 0.8(1.1375) \frac{1}{2(1 + 0.8)} \left( 1 + \frac{1}{1.1375} \right) = 0.475
\]
Consumption

- For the type-2 agents, consumption in period 1 is given by

\[ c_{1,2}^* = \frac{1}{2(1 + \beta_2)} \left[ w_1 + \frac{w_2}{(1 + R)} \right] \]

\[ = \frac{1}{2(1 + 0.9)} \left( 1 + \frac{1}{1.1375} \right) \]

\[ = 0.494. \]

- Similarly, in period 2 these agents consume a quantity:

\[ c_{2,2}^* = \beta_2(1 + R) \frac{1}{2(1 + \beta_2)} \left[ w_1 + \frac{w_2}{(1 + R)} \right] \]

\[ = 0.9(1.1375) \frac{1}{2(1 + 0.9)} \left( 1 + \frac{1}{1.1375} \right) \]

\[ = 0.506 \]
Consumption

- Note that the relatively impatient agents of type 1 consume more in the first period than the patient agents of type 2.
- Summarizing, aggregate demand in the first period is equal to:

\[ C_1^d = 20c_{1,1}^* + 80c_{1,2}^* \]

\[ = 20(0.52198) + 80(0.49451) \]

\[ = 50 \]
Labor supply

- Again let’s start with the agents of type 1:

\[
l_{1,1}^* = \frac{1 + 2\beta_1}{2(1 + \beta_1)} - \frac{1}{2(1 + \beta_1)} \frac{1}{1 + R} \frac{A_2}{A_1}
\]

\[
= \frac{1 + 2(0.8)}{2(1 + 0.8)} - \frac{1}{2(1 + 0.8)} \frac{1}{1.1375} = 0.47802
\]

- and

\[
l_{2,1}^* = \frac{2 + \beta_1}{2(1 + \beta_1)} - \frac{\beta_1(1 + R)}{2(1 + \beta_1)} \frac{A_1}{A_2}
\]

\[
= \frac{2 + 0.8}{2(1 + 0.8)} - \frac{0.8(1.1375)}{2(1 + 0.8)} = 0.525.
\]
Similarly, for the agents of type 2 we find the following values:

\[ l_{1,2}^{*} = \frac{1 + 2\beta_2}{2(1 + \beta_2)} - \frac{1}{2(1 + \beta_2)} \frac{1}{1 + R} \frac{A_2}{A_1} \]

\[ = \frac{1 + 2 \times 0.9}{2(1 + 0.9)} - \frac{1}{2(1 + 0.9)} \frac{1}{1.1375} \]

\[ = 0.50549 \]

and

\[ l_{2,1}^{*} = \frac{2 + \beta_2}{2(1 + \beta_2)} - \frac{\beta_2(1 + R)}{2(1 + \beta_1)} \frac{A_1}{A_2} \]

\[ = \frac{2 + 0.9}{2(1 + 0.9)} - \frac{0.9 \times 1.1375}{2(1 + 0.9)} \]

\[ = 0.4935. \]
The labor market

- Hence, labor market equilibrium in both periods requires that:

\[
L_1^f = L_1^s \\
= (20l_{1,1}^* + 80l_{1,2}^*) \\
= 20(0.47802) + 80(0.50549) \\
= 50
\]

- and

\[
L_2^f = L_2^s \\
= (20l_{2,1}^* + 80l_{2,2}^*) \\
= 20(0.525) + 80(0.4935) \\
= 50
\]
Aggregate demand and supply

- The aggregate supply of goods is given by

\[ Y_1^s = A_1 \left( 20l_{1,1}^* + 80l_{1,2}^* \right) \]
\[ = 20(0.47802) + 80(0.50549) \]
\[ = 50. \]

which is equal to aggregate consumption (demand)

\[ C_1^d = 20c_{1,1}^* + 80c_{1,2}^* \]
\[ = 20(0.52198) + 80(0.49451) \]
\[ = 50 \]
Similarly, in the second period aggregate demand and supply are given by

\[ C_2^d = 20c_{2,1}^* + 80c_{2,2}^* \]
\[ = 20(0.475) + 80(0.50625) \]
\[ = 50 \]

and

\[ Y_2^s = A_2 \left( 20l_{2,1}^* + 80l_{2,2}^* \right) \]
\[ = 20(0.525) + 80(0.4935) \]
\[ = 50. \]

Hence, the goods market is in equilibrium in both periods.
Productivity shocks

In the rest of this lecture we will consider the effects of two different types of productivity shocks:

- **Permanent productivity shocks**: an increase in TFP in both periods.

- **Transitory productivity shocks**: an increase in TFP in one of the two periods.
  - Contemporary shock in the first period
  - Anticipated shock in the second period
A permanent productivity shock

Given our assumption of a linear production technology

\[ Y_j^f = A_j L_j^f, \]

A permanent productivity shock is a rise of equal size in the values of \( A_1 \) and \( A_2 \).
A Permanent Productivity Shock

Período 1

\[ y_1 = A_1 l_1 \]

\[ L^s \]

Período 2

\[ y_2 = A_2 l_2 \]

\[ Y_2 \]
Intra-temporal substitution

Given that the productivity shock leads to higher wages in both periods, the agents have an incentive to work more — the marginal benefit of work rises in both periods:

\[
\frac{1}{c_1} w_1 = \frac{1}{1 - l_1} \quad \text{and} \quad \frac{1}{c_2} w_2 = \frac{1}{1 - l_2}
\]
Income Effects

- The permanent rise in wages produces a positive income effect.
- The richer agents will choose to enjoy more leisure and consumption.
Intertemporal substitution

- The wage increase is identical in both periods. Hence, there is no incentive for the inter-temporal substitution of labor.

- The overall change in hours worked will therefore depend on the relative strength of the income and (intra-period) substitution effects.
Our numerical example

- Consider an economy with 100 agents, \( N = 100 \).
- 20 agents have a low discount factor \( \beta_1 = 0.8 \) while the other 80 have a relatively high discount factor \( \beta_2 = 0.9 \).
- Suppose that initially \( A_1 = A_2 = 1 \).
- Below we consider the effects of a 10\% increase in the values of \( A_1 \) and \( A_2 \) from 1 to 1.10 (+10\%)
- The above productivity changes are reflected in a 10\% increase in the wages to \( w_1 = 1.10 \) and \( w_2 = 1.10 \)
The credit market equilibrium

- Inspection of the equilibrium condition for the credit market shows that:

\[ N_1 \beta_1 \uparrow A_1 - \frac{\uparrow A_2}{1+R} + N_2 \beta_2 \uparrow A_1 - \frac{\uparrow A_2}{1+R} = 0, \]

\[ \frac{0.8 (1.1) - \frac{1.1}{1+R}}{1.8} = \frac{80 \frac{1.1}{1+R} - 0.9 (1.1)}{1.9} \]

- Notice that all terms are multiplied by 1.1. We therefore expect no change in the interest rate.
Our claim is confirmed below:

\[ (0.8)20(1.9)(1.1) - \frac{1.9(20)(1.1)}{1 + R} = \frac{80(1.8)(1.1)}{1 + R} - (80)0.9(1.8)(1.1), \]

\[ 33.44 - \frac{41.8}{1 + R} = \frac{158.4}{1 + R} - 142.56 \]

\[ 33.44 + 142.56 = \frac{158.4}{1 + R} + \frac{41.8}{1 + R} \]

\[ 176 = \frac{200.2}{1 + R} \implies 1 + R = \frac{200.2}{176} \implies 1 + R = 1.1375 \implies R = 13.75\% \]
Consumption

- Given the equilibrium values of \( R \), \( w_1 \) and \( w_2 \), we can calculate the consumption levels. Let’s start with the type-1 agents:

\[
c_{1,1}^* = \frac{1}{2(1 + \beta_1)} \left[ \uparrow w_1 + \frac{\uparrow w_2}{(1 + R)} \right]
\]

\[
= \frac{1}{2(1.8)} \left[ 1.10 + \frac{1.10}{1.1375} \right]
\]

\[
= 0.277 (2.067) = 0.5725 \uparrow
\]

\[
c_{2,1}^* = \beta_1 (1 + R) \frac{1}{2(1 + \beta_1)} \left[ \uparrow w_1 + \frac{\uparrow w_2}{(1 + R)} \right]
\]

\[
= 0.8(1.1375) \frac{1}{2(1.8)} \left[ 1.10 + \frac{1.10}{1.1375} \right]
\]

\[
= (0.91) 0.277 (2.067) = 0.5210 \uparrow
\]
Consumption

- Equivalently, for the type-2 agents:

\[
c_{1,2}^* = \frac{1}{2(1 + \beta_2)} \left[ w_1 + \frac{w_2}{(1 + R)} \right]
\]

\[
c_{1,2}^* = \frac{1}{2(1.9)} \left[ 1.10 + \frac{1.10}{1.1375} \right]
\]

\[
c_{1,2}^* = 0.263 (2.067) = \uparrow 0.5436
\]

\[
c_{2,2}^* = \beta_2 (1 + R) \frac{1}{2(1 + \beta_2)} \left[ w_1 + \frac{w_2}{(1 + R)} \right]
\]

\[
c_{2,2}^* = 0.9 (1.1375) \frac{1}{2(1.9)} \left[ 1.10 + \frac{1.10}{1.1375} \right]
\]

\[
c_{2,2}^* = (1.023) 0.263 (2.067) = 0.556 \uparrow
\]
Equilibrium consumption decisions

- The agents of both types raise their consumption in both periods by 10%.

- At the aggregate level this is reflected by a 10% rise in both aggregate demand and supply.
Aggregate demand

\[
C_1^d = 20c_{1,1}^* + 80c_{1,2}^* \\
= 20(0.5725) + 80(0.5436) \\
= 55 \uparrow
\]

\[
C_2^d = 20c_{2,1}^* + 80c_{2,2}^* \\
= 20(0.5210) + 80(0.556) \\
= 55 \uparrow
\]
Labor supply

- Optimal labor supply depends on the wage ratio $w_2/w_1$. This ratio remains constant.

$$l^*_{1,1} = \frac{1 + 2\beta_1}{2(1 + \beta_1)} - \frac{1}{2(1 + \beta_1)} \frac{1}{1 + R} \frac{A_2 \uparrow}{A_1 \uparrow} = 0.478$$

$$l^*_{2,1} = \frac{2 + \beta_1}{2(1 + \beta_1)} - \frac{\beta_1(1 + R)}{2(1 + \beta_1)} \frac{A_1 \uparrow}{A_2 \uparrow} = 0.525.$$
The Labor Market

Período 1

Período 2

Dynamic Macroeconomic Analysis (UAM)
## Summary

### Benchmark economy

<table>
<thead>
<tr>
<th></th>
<th>type-1</th>
<th>type-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_1$</td>
<td>0.52198</td>
<td>0.4945</td>
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<tr>
<td>$c_2$</td>
<td>0.475</td>
<td>0.5062</td>
</tr>
<tr>
<td>$l_1$</td>
<td>0.478</td>
<td>0.5055</td>
</tr>
<tr>
<td>$l_2$</td>
<td>0.525</td>
<td>0.49375</td>
</tr>
</tbody>
</table>

### Increase in $A_1$ and $A_2$

<table>
<thead>
<tr>
<th></th>
<th>type-1</th>
<th>type-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_1$</td>
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<td>0.54396</td>
</tr>
<tr>
<td>$c_2$</td>
<td>0.5225</td>
<td>0.5569</td>
</tr>
<tr>
<td>$l_1$</td>
<td>0.478</td>
<td>0.5055</td>
</tr>
<tr>
<td>$l_2$</td>
<td>0.525</td>
<td>0.49375</td>
</tr>
</tbody>
</table>
Aggregate Demand and Supply

- Aggregate supply increases by exactly 10% in both periods:

\[
Y_1^s = \uparrow A_1 \left( 20l_{1,1}^* + 80l_{1,2}^* \right) \\
= 1.1 \left( 20 \times 0.478 + 80 \times 0.505 \right) \\
= 55 \uparrow 
\]

\[
Y_2^s = \uparrow A_2 \left( 20l_{2,1}^* + 80l_{2,2}^* \right) \\
= 1.10 \left( 20 \times 0.525 + 80 \times 0.4935 \right) \\
= 55 \uparrow 
\]

- In sum

\[
C_1^d = Y_1^s = 55 \\
C_2^d = Y_2^s = 55 
\]
Oferta y Demanda Agregadas
Summary

- The permanent change in productivity does not affect the eq. interest rate. Since the increase is of the same size in both periods, the agents have no incentive to substitute current work for future work, or reversely.

- In both periods the agents consume the entire rise in income. This is optimal since income rises by the same amount in both periods while the interest rate remains unchanged.

- Labor supply is unaffected because the income and substitution effects cancel out against each other.
A transitory increase in productivity

- Contrary to before, we now consider an increase in $A_1$ while $A_2$ stays unchanged.

- The rise in first-period productivity makes it more interesting for all agents to work during period 1.

- However, the effects are not (necessarily) uniform, because the two types of agents value current and future leisure differently.
A transitory increase in productivity
A transitory increase in productivity

Below we will calculate the effects of a 10% increase in $A_1$. What results should we expect to find?

- The rise in the relative wage ratio $w_1/w_2$ should give rise to a rise in the ratio $l_1/l_2$.
- The rise in productivity makes agents richer, but the size of the income effect is smaller than before.
- The rise in first-period income induces the agents to raise their supply of savings.
- The change in the interest rate induces a intra-period substitution between consumption and leisure plus income effects of different signs for lenders and borrowers.
Credit market equilibrium

\[ N_1 \frac{\beta_1 \uparrow A_1 - \frac{A_2}{1+R}}{1 + \beta_1} + N_2 \frac{\beta_2 \uparrow A_1 - \frac{A_2}{1+R}}{1 + \beta_2} = 0, \]

\[ 20 \frac{0.8(1.1) - \frac{1}{1+R}}{1.8} = 80 \frac{\frac{1}{1+R} - 0.9(1.1)}{1.9} \]

\[ (0.8)20(1.9)(1.1) - \frac{1.9(20)}{1 + R} = \frac{80(1.8)}{1 + R} - (80)0.9(1.8)(1.1), \]
Credit market equilibrium

\[ 33.44 + 142.56 = \frac{144}{1 + R} + \frac{38}{1 + R} \]

\[ 176 = \frac{182}{1 + R} \implies 1 + R = \frac{182}{176} \implies 1 + R = 1.034 \implies R = 3.40\% \downarrow. \]

Hence, in line with our prior, we observe a fall in the interest rate.
In our previous example we obtained a rise in consumption in both periods due to the positive income effect.

In this example we also have a substitution effect as the rise in first-period productivity leads to a fall in the interest rate.

The fall in $R$ creates a positive income effect on the borrowers (type-2) and a negative income effect on the lenders (type-1).
Consumption

Let’s start with the type-1 agents:

\[
c^{*,1}_{1} = \frac{1}{2(1 + \beta_1)} \left[ \uparrow w_1 + \frac{w_2}{1 + R \downarrow} \right] \\
= \frac{1}{2(1.8)} \left[ 1.10 + \frac{1}{1.034} \right] \\
= 0.277 [2.067] = 0.5742 \uparrow
\]

\[
c^{*,1}_{2} = \beta_1 (1 + \downarrow R) \frac{1}{2(1 + \beta_1)} \left[ \uparrow w_1 + \frac{w_2}{1 + R \downarrow} \right] \\
= 0.8(1.034) \frac{1}{2(1.8)} \left[ 1.10 + \frac{1}{1.034} \right] \\
= (0.8272) 0.277 [2.067] = 0.475
\]

First-period consumption increases, while \( c^{*,1}_{2} \) remains constant.
Consumption

- Similarly, for the type-2 agents we find

\[
c_{1,2}^* = \frac{1}{2(1 + \beta_2)} \left[ \uparrow w_1 + \frac{w_2}{(1 + R \downarrow)} \right]
\]

\[
= \frac{1}{2(1.9)} \left[ 1.10 + \frac{1}{1.034} \right]
= 0.263 [2.067] = 0.5436 \uparrow
\]

\[
c_{2,2}^* = \beta_2 (1 + \downarrow R) \frac{1}{2(1 + \beta_2)} \left[ \uparrow w_1 + \frac{w_2}{(1 + R \downarrow)} \right]
\]

\[
= 0.9 (1.034) \frac{1}{2(1.9)} \left[ 1.10 + \frac{1}{1.034} \right]
= (0.93) 0.263 [2.067] = 0.506
\]
Agregate demand

• Summing over all agents we obtain the value of aggregate product demand in both periods

\[ C_1^d = 20c_{1,1}^* + 80c_{1,2}^* \]
\[ = 20(0.5726) + 80(0.5436) = 55 \uparrow \]

\[ C_2^d = 20c_{2,1}^* + 80c_{2,2}^* \]
\[ = 20(0.4736) + 80(0.506) = 50 \]

• In sum, aggregate demand rises by 10% in the first period, and remains unchanged in the second period.
Labor supply

Now let’s have a look at labor supply, starting with the case of type-1 agents:

\[
\begin{align*}
l^{*,1}_{1} &= \frac{1 + 2\beta_1}{2(1 + \beta_1)} - \frac{1}{2(1 + \beta_1)} \left(1 + \Downarrow R\right) \frac{1}{A_1} A_2 \\
&= \frac{1 + 2(0.8)}{2(1.8)} - \frac{1}{2(1.8)} \frac{1}{1.034} 0.909 \\
&= 0.478
\end{align*}
\]

\[
\begin{align*}
l^{*,1}_{2} &= \frac{2 + \beta_1}{2(1 + \beta_1)} - \frac{\beta_1(1 + \Downarrow R)}{2(1 + \beta_1)} \frac{A_1}{A_2} \\
&= \frac{2.8}{2(1.8)} - \frac{0.8(1.034)}{2(1.8)} 1.1 \\
&= 0.5172
\end{align*}
\]
Similarly, for the type-2 agents we obtain the following equilibrium values:

\[ l_{1,2}^* = \frac{1 + 2\beta_2}{2(1 + \beta_2)} - \frac{1}{2(1 + \beta_2)} \frac{1}{(1 + \downarrow R)} A_2 \]

\[ \uparrow A_1 \]

\[ = 0.5046 \]

\[ l_{2,2}^* = \frac{2 + \beta_2}{2(1 + \beta_2)} - \frac{\beta_2 (1 + \downarrow R)}{2(1 + \beta_1)} \uparrow A_1 \]

\[ \downarrow A_2 \]

\[ = \frac{2.9}{2(1.9)} - \frac{0.9(1.034)}{2(1.9)} 1.1 \]

\[ = 0.763 - 0.269 \]

\[ = 0.50 \]
Labor market equilibrium

\[ L_1^f = L_1^s \]
\[ = (20l_{1,1}^* + 80l_{1,2}^*) \]
\[ = 20(0.478) + 80(0.5046) \]
\[ = 50 \]

\[ L_2^f = L_2^s \]
\[ = (20l_{2,1}^* + 80l_{2,2}^*) \]
\[ = 20(0.5172) + 80(0.4936) \]
\[ = 50 \]
Labor supply

- As always, labor supply is affected by two opposing forces:
  - **The substitution effect:**
    1. The rise in $w_1$ makes current leisure in the first (second) period more expensive (attractive).
    2. The fall in $R$, on the contrary, makes work in the second period more attractive.
  - **Income effect:** The rise in first-period wages creates a positive income effect, which leads to a reduction in labor supply. But this effect is (much) weaker than in the case of a permanent productivity shock.

- Once again, however, the income and substitution effects cancel out due to the assumption of logarithmic preferences!
Labor market

Período 1

Período 2

ES (sube w1) →
ES (cae R) ←
ER ←

ES (sube w1) ←
ES (cae R) →
ER →
Aggrega\texttildelow te demand and supply

- First-period aggregate demand is:

\[
Y_1^s = \uparrow A_1 \left( 20l_{1,1}^* + 80l_{1.2}^* \right) \\
= 1.1 \left( 20 \times 0.478 + 80 \times 0.505 \right) \\
= 55 \uparrow
\]

- Similarly, in period 2 aggregate demand is equal to:

\[
Y_2^s = A_2 \left( 20l_{2,1}^* + 80l_{2.2}^* \right) \\
= (20 \times 0.525 + 80 \times 0.4935) \\
= 50
\]

- and the product market clears in both periods

\[
C_1^d = Y_1^s = 55 \\
C_2^d = Y_2^s = 50
\]
Agregate demand and supply
Summary

- As we have shown in the previous slides, a temporary rise in productivity leads to a fall in the interest rate. The increase in productivity shifts the aggregate supply curve to the right.

- Since the agents know that the rise in productivity is temporary, they try to divide the rise in consumption over the two periods. Hence, the aggregate demand curve also shifts to the right, but less far than the supply curve.

- At the original interest rate, there is an excess supply of goods. The interest rate has to fall to restore equilibrium.

- The reduction in the interest rate provokes a rise in current consumption and a fall in labor supply which eliminates the excess supply of goods during the first period.
<table>
<thead>
<tr>
<th>Benchmark</th>
<th>Permanent</th>
<th>Transitory</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_1 = A_1$</td>
<td>1</td>
<td>1.20</td>
</tr>
<tr>
<td>$w_2 = A_2$</td>
<td>1</td>
<td>1.20</td>
</tr>
<tr>
<td>$R$</td>
<td>13.75%</td>
<td>13.75%</td>
</tr>
<tr>
<td>$L_1^s = L_1^f$</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>$Y_1^s = A_1 L_1^f$</td>
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</tr>
<tr>
<td>$C_1^d = Y_1^s$</td>
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</tr>
<tr>
<td>$L_2^s = L_2^f$</td>
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</tr>
<tr>
<td>$Y_2^s = A_2 L_2^f$</td>
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<td>55</td>
</tr>
<tr>
<td>$C_2^d = Y_2^s$</td>
<td>50</td>
<td>55</td>
</tr>
</tbody>
</table>
An anticipated future rise in productivity

Período 1

\[ y_1 = A_1 l_1 \]

Período 2

\[ y_2 = A_2 l_2 \]
Numerical experiment

This time we consider the effects of an anticipated 10% rise in $A_2$ and $w_2$.

Following the reasoning of before we should expect:

- A rightward shift in the aggregate supply curve in the second period.
- A rise in the interest rate as the agents will try to consume part of this future rise in income during the first period.
- The rise in $R$ will reduce labor supply in the second period and consumption demand in the first period.
- Once again we expect the income and substitution effects to cancel out against each other.
Credit market equilibrium

\[ N_1 \frac{\beta_1 A_1 - \uparrow A_2}{1 + \beta_1} + N_2 \frac{\beta_2 A_1 - \uparrow A_2}{1 + \beta_2} = 0, \]

\[ 20 \frac{0.8 - \frac{1.10}{1+R}}{1.8} = 80 \frac{1.10}{1+R} - 0.9 \]

\[ (0.8)20(1.9) - \frac{1.9(20)(1.1)}{1+R} = \frac{80(1.8)(1.1)}{1+R} - (80)0.9(1.8), \]

\[ 30.4 + 129.6 = \frac{158.4}{1+R} + \frac{41.8}{1+R} \]

\[ 160 = \frac{200.2}{1+R} \implies 1 + R = \frac{200.2}{160} \implies 1 + R = 1.2512 \implies R = 25.12\% \uparrow. \]
The substitution effect has the same implications for all agents: given the higher interest rate the agents want to consume less in period 1 and more in period 2.

The rise in $R$ creates a negative income effect on the borrowers (type-1 agents). But the anticipated rise in $A_2$ creates a positive income effect.

The rise in $R$ creates a positive income effect on the lenders (type-2 agents). These agents also benefit from the positive income effect due to the rise in $A_2$. 
Consumption

\[
c_{1,1}^* = \frac{1}{2(1 + \beta_1)} \left[ w_1 + \frac{\uparrow w_2}{(1 + R \uparrow)} \right]
\]

\[
= \frac{1}{2(1.8)} \left[ 1 + \frac{1.10}{1.2512} \right]
\]

\[
= 0.277 [1.8791] = 0.522
\]

Hence, the first-period consumption of the type-1 agents is the same as in our benchmark economy.

\[
c_{2,1}^* = \beta_1 (1+ \downarrow R) \frac{1}{2(1 + \beta_1)} \left[ w_1 + \frac{\uparrow w_2}{(1 + R \uparrow)} \right]
\]

\[
= 0.8(1.2512) \frac{1}{2(1.8)} \left[ 1 + \frac{1.10}{1.2512} \right]
\]

\[
= 0.277 [1.8791] = 0.5210
\]
Consumption

\[ c_{1,2}^* = \frac{1}{2(1+\beta_2)} \left[ w_1 + \frac{\uparrow w_2}{(1 + R \uparrow)} \right] \]

\[ = \frac{1}{2(1.9)} \left[ 1 + \frac{1.10}{1.2512} \right] \]

\[ = 0.263 [1.8791] = 0.494 \]

Hence, second-period consumption of the type-2 agents is also the same as in our benchmark economy.

\[ c_{2,2}^* = \beta_2 (1 + \downarrow R) \frac{1}{2(1+\beta_2)} \left[ w_1 + \frac{\uparrow w_2}{(1 + R \uparrow)} \right] \]

\[ = 0.9(1.2512) \frac{1}{2(1.9)} \left[ 1 + \frac{1.10}{1.2512} \right] \]

\[ = (1.12608) 0.263 [2.067] = 0.556 \]
Labor supply

\[ l_{1,1}^{*} = \frac{1 + 2\beta_1}{2(1 + \beta_1)} - \frac{1}{2(1 + \beta_1)} \frac{1}{(1 + R)} \frac{A_2 \uparrow}{A_1} \]

\[ = \frac{1 + 2(0.8)}{2(1.8)} - \frac{1}{2(1.8)} \frac{1}{1.2512} 1.10 \]

\[ = 0.478 \]

\[ l_{2,1}^{*} = \frac{2 + \beta_1}{2(1 + \beta_1)} - \frac{\beta_1 (1 + R)}{2(1 + \beta_1)} \frac{A_1}{A_2 \uparrow} \]

\[ = \frac{2.8}{2(1.8)} - \frac{0.8(1.2512)}{2(1.8)} 0.909 \]

\[ = 0.525 \]
Labor supply

\[ l_{1,2}^* = \frac{1 + 2\beta_2}{2(1 + \beta_2)} - \frac{1}{2(1 + \beta_2)} \frac{1}{(1 + \uparrow R)} \frac{A_2}{A_1} \]

\[ = \frac{1 + 2(0.9)}{2(1.9)} - \frac{1}{2(1.9)} \frac{1}{1.2512} 1.10 \]

\[ = 0.506 \]

\[ l_{2,2}^* = \frac{2 + \beta_2}{2(1 + \beta_2)} - \frac{\beta_2(1 + \uparrow R)}{2(1 + \beta_1)} \frac{A_1}{\uparrow A_2} \]

\[ = \frac{2.9}{2(1.9)} - \frac{0.9(1.2512)}{2(1.9)} 0.909 \]

\[ = 0.494 \]
Labor supply

- Again we find that labor supply remains constant because the various income and substitution effects cancel out against each other.

- **Substitution effects:**
  1. Future leisure becomes more expensive relative to current leisure. The agents will therefore in principle prefer to work more in period 2 and less in period 1.
  2. The rise in the interest rate lowers labor supply in period 1 and raises labor supply in period 2.

- **Income effects:** the agents are richer and will try to raise consumption and leisure in both periods.
### Summary

<table>
<thead>
<tr>
<th></th>
<th>Benchmark (type-1)</th>
<th>Benchmark (type-2)</th>
<th>Increase in $A_2$ (type-1)</th>
<th>Increase in $A_2$ (type-2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_1$</td>
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<td>0.522</td>
<td>0.494</td>
</tr>
<tr>
<td>$c_2$</td>
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<td>0.506</td>
<td>0.522</td>
<td>0.557</td>
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<tr>
<td>$l_1$</td>
<td>0.478</td>
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<td>0.478</td>
<td>0.506</td>
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<tr>
<td>$l_2$</td>
<td>0.525</td>
<td>0.494</td>
<td>0.525</td>
<td>0.494</td>
</tr>
</tbody>
</table>
Labor market equilibrium

\[ L_1^f = L_1^s = (20l_{1,1}^* + 80l_{1,2}^*) = 20(0.4778) + 80(0.506) = 50 \]

\[ L_2^f = L_2^s = (20l_{2,1}^* + 80l_{2,2}^*) = 20(0.525) + 80(0.4936) = 50 \]
Labor market equilibrium

Período 1

Período 2

ES (sube w2) ──
ES (sube R) ──
ER ──

ES (sube w2) ──
ES (sube R) ──
ER ──
Aggregate demand and supply in period 1

\[ C_1^d = 20c_{1,1}^* + 80c_{1,2}^* \]
\[ = 20(0.5205) + 80(0.494) = 50 \]

\[ Y_1^s = A_1 \left( 20l_{1,1}^* + 80l_{1,2}^* \right) \]
\[ = (20 \times 0.478 + 80 \times 0.505) \]
\[ = 50 \]
Aggregate demand and supply in period 1

Exceso de demanda de créditos
Aggregate demand and supply in period 2

\[ Y_2^s = A_2 \left( 20l_{2,1}^* + 80l_{2,2}^* \right) \]

\[ = 1.10 \left( 20 \times 0.525 + 80 \times 0.4935 \right) \]

\[ = 55 \uparrow \]

\[ C_2^d = 20c_{2,1}^* + 80c_{2,2}^* \]

\[ = 20(0.5210) + 80(0.556) = 55 \uparrow \]
The anticipated rise in productivity causes a rise in the interest rate, but in equilibrium first-period production and consumption are unaffected.

Since the shock is anticipated the agents will try to raise their consumption in the first period. The net effect is a rise in $R$ and no change in aggregate demand or supply.

At the initial interest rate there is an excess demand for the good. The interest rate needs to increase to clear the goods market

The rise in $R$ offsets the rise in consumption demand and the fall in labor supply.
## Overall predictions

### Benchmark, Permanent, Transitory, Anticipated

<table>
<thead>
<tr>
<th></th>
<th>Benchmark</th>
<th>Permanent</th>
<th>Transitory</th>
<th>Anticipated</th>
</tr>
</thead>
<tbody>
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<td>1.10</td>
<td>1</td>
</tr>
<tr>
<td>( w_2 = A_2 )</td>
<td>1.00</td>
<td>1.10</td>
<td>1.00</td>
<td>1.10</td>
</tr>
<tr>
<td>( R )</td>
<td>13.75%</td>
<td>13.75%</td>
<td>3.40%</td>
<td>25.12%</td>
</tr>
<tr>
<td>( L_1^s = L_1^f )</td>
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<tr>
<td>( Y_1^s = A_1 L_1^f )</td>
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<tr>
<td>( C_2^d = Y_2^s )</td>
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</table>