Money

Dynamic Macroeconomic Analysis

Universidad Autónoma de Madrid

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Price flexibility and the neutrality of money

- So far, we have only considered flexible-price models of the real economy without money.
- The reason is that money tends to be *neutral* if prices are perfectly flexible.
- **Definition:** Money is said to be neutral if a one-time increase in the level of the money supply does not affect any of the relevant equilibrium quantities.
- In this lecture we will illustrate the neutrality of money in a simplified version of our OLG model.
- The analysis will also serve to illustrate some of the peculiarities of an economy with finite lifetimes.
A model of money

Any meaningful model of money should reflect its peculiar features and explain why agents choose to hold this asset:

**Medium of exchange** Agents want goods for consumption. On the contrary, agents hold money to purchase the goods that provide utility.

- In other words, money is valued not because it provides utility, but because we expect to be able to exchange money for goods at a later date.

**Necessary ingredients of monetary models**

- Some form of trading frictions — the absence of a centralized market mechanism so that the double coincidence of wants is not guaranteed.
- Agents must be willing to hold money for some time although it is intrinsically worthless and pays no interest.
Trading frictions

- Search models of money capture trading frictions by assuming that goods are traded in bilateral transactions.
  - Agents hold one type of product and need to search for the agents who sell their desired product.
  - In this environment barter trade is very expensive — double coincidence of wants is an exception rather than the rule.
  - The introduction of fiat money may reduce transaction costs and increase the number of transactions.

- OLG models provide a natural scope for trading frictions
  - With perishable goods the young of generation $t$ would like to trade with the unborn young of $t + 1$.
  - The introduction of pure fiat money may alleviate this trading friction and make all generations better off.
Sneak preview

- In our OLG model with perishable goods, the decentralized equilibrium with money leads to autarky. The young consume their entire endowment and they starve when old.

- In the equilibrium with money the outcome may be efficient - the decentralized consumption levels coincide with the golden rule consumption levels. In addition, money is neutral.

- The monetary equilibrium is extremely fragile. All generations must be convinced that all future generations will accept the money.

- In an extension we will study the role of inflation and seigniorage.
The environment

In this lecture we consider a basic OLG model without production and with perishable goods.

- In each period there are $N$ young agents and $N$ old agents in the economy.
- Young agents receive an endowment $\omega > 0$ of the unique good. This is the only source of income of the agents.
- The unique good is perishable and the agents have logarithmic preferences
  \[ U(c_{yt}, c_{ot+1}) = \ln(c_{yt}) + \beta \ln(c_{ot+1}) \]
- By assumption there are $N\omega$ goods in the economy in every period.
Feasible allocations

- Any feasible allocation needs to satisfy the resource constraint:

\[ Nc_yt + Nc_{ot} = N\omega \]
\[ c_yt + c_{ot} = \omega \]

in all periods.

- We will be interested in stationary or steady state allocations in which the consumption levels of young and old agents are constant over time, i.e. \( c_{yt} = c_y \) and \( c_{ot} = c_o \) \( \forall t \). Notice that feasible steady-state allocations satisfy

\[ c_y + c_o = \omega \]
Decentralized equilibrium without money

- Young agents of any generation $t$ face the following trivial problem:

$$\max ln(c_{yt}) + \beta ln(c_{ot+1})$$

s.t. $c_{yt} = \omega$

$c_{ot+1} = 0$

- Without money the only feasible (steady-state) equilibrium is autarky:

$$c_{yt} = c_y = \omega$$

$$c_{ot+1} = c_o = 0$$

- The young would like to save $b = \frac{\beta}{1+\beta} \omega$ but there is no demand from agents who can repay next period.
What prevents the emergence of credit markets?

The young agents in any period $t$ want to lend to agents who can repay the loan in $t + 1$. However, there are no such agents in the economy.

- The young in $t$ do not want to lend to the current old because they will be dead in $t + 1$.
- The young in $t$ do not want to lend to another young agent because he/she will not have any resources in $t + 1$.
- The young in $t$ would love to lend to the young of $t + 1$ but these agents are not yet alive.

In the rest of the lecture we will analyze how the introduction of pure fiat money may alleviate this trading friction.
Fiat money

- Fiat money is an intrinsically worthless commodity (piece of paper) that derives its value from the fact that it allows agents to buy goods.

- The amount of fiat money is determined by an external agent (central bank, government...). This is only possible if it cannot (easily) be counterfeited.

- Money can act as a storage of wealth from one period to the next.

- Governments can impose the obligation that private agents accept money in court settlements (“legal tender”). Yet in private transactions agents are free to choose the currency.
The introduction of fiat money

- In period $t$ the government introduces an amount $M$ of pure fiat money by transferring an amount $M/N$ of money to each old agent of generation $t-1$.
- Money is perfectly divisible and cannot be counterfeited.
- Money is an additional item and so we need an additional price.
- Let $P_t$ denote the nominal price of a single unit of the good in period $t$.
- One unit of money buys $v_t = 1/P_t$ goods in $t$ — and so money loses value as the price level increases.
- The return on money is $v_{t+1}/v_t = P_t/P_{t+1} = 1/(1 + \pi_t)$. Or approx. the real interest rate on money is $\tilde{r} = -\pi_t$. 
The value of money

- The return on money in $t$ depends on the expectations of agents about the entire sequence $\{v_{t+j}\}_{j=1}^{\infty}$.

- Suppose that agents expect that in some period $v_{t+j} = 0$ with probability 1. In that case agents will reject to accept money in period $t + j - 1$ with prob. 1 because money will not have any value next period. But anticipating this, young agents in $t + j - 2$ will reject to accept money as it will be worthless in $t + j - 1$. And so forth...

**Conclusion:** Money is worthless today if agents expect money to loose value in the (distant) future.

- In the sequel we will analyze stationary equilibria with perfect foresight — i.e. agents have perfect knowledge about the entire sequence of future prices.
The individual problem

Consider the problem of an individual household of generation $t$ who decides to bring a quantity $m_{t+1}$ of money to the next period.

\[
\begin{align*}
\text{max } & \ln(c_{yt}) + \beta \ln(c_{ot+1}) \\
\text{s.t. } & P_t c_{yt} + m_{t+1} = P_t \omega \\
& P_{t+1} c_{ot+1} = m_{t+1}
\end{align*}
\]

Or equivalently,

\[
\begin{align*}
\text{max } & \ln(c_{yt}) + \beta \ln(c_{ot+1}) \\
\text{s.t. } & P_t c_{yt} + P_{t+1} c_{ot+1} = P_t \omega
\end{align*}
\]
Optimality conditions

\[ L = \ln(c_{yt}) + \beta \ln(c_{ot+1}) + \lambda [P_t \omega - P_t c_{yt} - P_{t+1} c_{ot+1}] \]

FOC’s:

\[ \frac{1}{c_{yt}} = P_t \lambda \]
\[ \frac{\beta}{c_{ot+1}} = P_{t+1} \lambda \]

\[ P_t c_{yt} + P_{t+1} c_{yt+1} = P_t \omega \]

The Consumption Euler eqn.:

\[ P_{t+1} c_{ot+1} = \beta P_t c_{yt} \]
\[ c_{ot+1} = \beta \frac{P_t}{P_{t+1}} c_{yt} = \beta \frac{v_{t+1}}{v_t} c_{yt} \]
Optimal consumption levels

Inserting the Euler consumption condition $P_{t+1}c_{ot+1} = \beta P_t c_{yt}$ into the budget constraint we obtain:

\[ P_t c_{yt} + \beta P_t c_{yt} = P_t \omega \]

\[ P_t c_{yt} = \frac{1}{1 + \beta} P_t \omega \]

\[ c_{yt} = \frac{1}{1 + \beta} \omega \]

\[ m_{t+1} = \frac{\beta}{1 + \beta} P_t \omega \]

\[ c_{ot+1} = \frac{m_{t+1}}{P_{t+1}} = \frac{\beta}{1 + \beta} \frac{P_t}{P_{t+1}} \omega \]
A stationary equilibrium with perfect foresight is a sequence of positive and finite prices \( \{P_t\}_{t=1}^{\infty} \) and stationary allocations:

\[
c_{yt} = c_y, \quad c_{ot+1} = c_o, \quad m_{t+1} = m \quad \forall t \geq 1
\]

1. The allocations of the agents of all generations \( t \geq 1 \) solve their maximization problem.

2. The goods market clears in every period:

\[
Nc_{yt} + Nc_{ot} = N\omega
\]
Market clearing

Our optimality conditions imply that:

\[ c_{ot} = \frac{\beta}{1 + \beta} \frac{P_{t-1}}{P_t} \omega \]

Hence our market clearing condition requires that

\[ Nc_{yt} + Nc_{ot} = N\omega \]
\[ c_{yt} + c_{ot} = \omega \]
\[ \frac{1}{1 + \beta} \omega + \frac{\beta}{1 + \beta} \frac{P_{t-1}}{P_t} \omega = \omega \]

The necessary condition: \( \frac{P_{t-1}}{P_t} = 1 \), i.e. the price level is constant over time.
Equilibrium price level

- To solve for the equilibrium price level, we need to look at the market clearing condition in period 1.
- The old of generation $t$ each receive a transfer of $m = M/N$ units of money.
- With this money they can buy $c_{01} = m/P_1$ units of the endowment of generation 1.

Thus market clearing in period 1 requires

$$\frac{m}{P_1} + \frac{1}{1 + \beta} \omega = \omega$$

$$\frac{m}{P_1} = \frac{\beta}{1 + \beta} \omega$$

$$P_1 = \frac{1 + \beta}{\beta} \frac{m}{\omega}$$

Notice that money is neutral. Multiplying $M$ by 2, the price level doubles while all else remains the same.
Efficiency

Due to price stability, the maximization problem of the agent reduces to

$$\max \ln(c_y) + \beta \ln(c_o)$$

s.t. $c_y + c_o = \omega$

In other words, the agents maximize their expected lifetime utility subject to the feasibility constraint for stationary allocations.

This problem coincides with the maximization problem of a benevolent social planner whose objective is to maximize utility of all future generations and who is constrained to treat all generations alike ("The Golden Rule").

Conclusion: A monetary equilibrium with perfect foresight and a constant money supply decentralizes the Golden Rule.
Golden rule

Asignación Eficiente: Óptimo según el Planificador Social
The distortionary effects of inflation

The government (or the central bank) has a monopoly on the creation of paper money. The revenue from the printing of fresh money is called seigniorage.

Formally, if the government injects a quantity $M_{t+1} - M_t$ of money in the economy during period $t$ it can buy

$$G_t = \frac{M_{t+1} - M_t}{P_t} = \mu_t \frac{M_t}{P_t}$$

of goods where $\mu_t$ denotes the growth rate of money.

It seems as if the government does not need to levy taxes in order to finance $G$. However, money creation leads to inflation and this is an implicit tax on money holdings as the existing money stock loosens part of its value.
Budget constraints

The budget constraints of young and old agents can be written as before:

\[ P_t c_{yt} + m_{t+1} = P_t \omega \]
\[ P_{t+1} c_{ot+1} = m_{t+1} \]

Equivalently

\[ c_{yt} + \frac{P_{t+1}}{P_t} c_{ot+1} = \omega \]

This intertemporal budget constraint coincides with the feasibility constraint when \( P_t \) is constant. But this condition is violated when the money stock grows.
Constant money growth and inflation

Suppose that the money stock grows at the constant rate $\mu_t = \mu$. In any period the young must voluntarily hold the money stock and so

$$\frac{M_{t+1}}{P_t} = N(\omega - c_{yt}) = N\frac{\beta}{1 + \beta}\omega$$

Hence, for any two consecutive periods we find

$$\frac{M_{t+2}}{P_{t+1}} = \frac{M_{t+1}}{P_t} \rightarrow \frac{P_{t+1}}{P_t} = \frac{M_{t+2}}{M_{t+1}} \rightarrow (1 + \pi_t) = 1 + \mu$$

Thus, the lifetime budget constraint of the individuals can be written as:

$$c_{yt} + \frac{1}{1 + \mu}c_{ot+1} = \omega$$

The slope of this budget constraint becomes flatter as $\mu$ increases.
Resource constraint

The resource constraint of the economy can now be written as:

\[ Nc_{yt} + Nc_{ot+1} + G_t = N\omega \]
\[ c_{yt} + c_{ot+1} + g_t = \omega \]
\[ c_y + c_o + g = \omega \]

Let’s suppose that the public goods do not provide utility to the agents. The decentralized equilibrium with inflation is different from the one with a constant money supply because

- The government “appropriates” part of the available goods.
- Money creation distorts the relative price of the goods - agents need to hold money to consume at old age, but inflation erodes the value of money.
  - In response the higher opportunity cost \( \frac{P_t}{P_{t+1}} = \frac{1}{1+\mu} \), agents optimally reduce \( c_o \).
Efficient allocation
Equilibrium with seigniorage
Equilibrium with seigniorage

Asignación Óptima con Gasto Público en Mercancías

Equilibrio con Señoreaje

\[ U_{Optimo} \]

\[ U_{Señoreaje} \]