Social Security

Dynamic Macroeconomic Analysis

Universidad Autónoma de Madrid

Fall 2012
Dynamic inefficiency

- In previous lectures we have seen that the equilibrium of an OLG model need not be efficient.

- In our OLG model with physical capital the steady state stock of capital may exceed the level consistent with the Golden Rule — steady state consumption would be higher if the agents would reduce savings.

- In our OLG model with perishable goods the introduction of fiat money was shown to lead to a Pareto-improvement — the introduction of a constant money supply moved the equilibrium from autarky to the Golden Rule allocation.

- In this lecture we reconsider efficiency in an economy with a growing population.
Preview of the results

Throughout this lecture we assume that the population grows at rate $n > 0$. Hence, $N_{t+1} = (1 + n)N_t$.

- When $\delta = 0$, the Golden Rule capital stock is defined by $PMK = r = n$.
- We show that the decentralized equilibrium may be dynamically inefficient ($r < n$).
- Whenever this is the case, the introduction of a pay-as-you-go pension system may improve welfare. It reduces the steady state capital stock and raises per capita consumption.
In a Pay-As-You-Go pension system, the pensions of the old are financed with the social security contributions of the young. The system works fine as long as the population is growing and a large proportion of the young is employed. Currently, neither of these conditions is satisfied. The dependency ratio is at a historic low. Other risks: increased life expectancy, relatively high ratio between pension and last salary.
Demographic changes

Population by age group, gender, in 2000 and 2050, in percentage of total population in each group

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in 2000: 0.0 | Total population (in millions) | in 2005: 0.0
in 2000: 27 | Old age dependency ratio (65+ in % 20-64) | in 2050: 73

OLG Fall 2012 5 / 19
Throughout this lecture we assume that the population $N_t = L_t$ grows at the constant rate $n$ per period.

$$N_t = N_{t-1} \left(1 + n\right)$$

We will first study efficiency in a model without pensions.
The resource constraint in a growing economy

The consumption and investment decisions in our economy need to satisfy

\[ N_t c_{y,t} + N_{t-1} c_{o,t} + N_t s_{t+1} = A_t K_t^\alpha N^{1-\alpha} + (1 - \delta) K_t \]

\[ N_t c_{y,t} + N_{t-1} c_{o,t} + K_{t+1} = A_t K_t^\alpha N^{1-\alpha} + (1 - \delta) K_t \]

Define the capital-labor ratio as \( \kappa_t = K_t / N_t \). Dividing both sides by \( N_t \) we obtain

\[ c_{y,t} + \frac{1}{1 + n} c_{o,t} + (1 + n) k_{t+1} = A_t \kappa_t^\alpha + (1 - \delta) \kappa_t \]

So, in steady state

\[ c_y + \frac{1}{1 + n} c_o = A \kappa^\alpha - (n + \delta) \kappa = f(\kappa) - (n + \delta) \kappa \]

where \( f(\kappa_t) = Y_t / N_t = A_t \kappa^\alpha \)
Golden Rule

The Golden Rule capital stock maximizes the value of (per capita) consumption. It solves

$$\max_\kappa f(\kappa) - (n + \delta)\kappa$$

The efficient level of the capital-labor ratio is given by

$$\alpha A^\kappa^{\alpha-1} = n + \delta$$

$$\kappa^{GR} = \left[ \frac{A\alpha}{n + \delta} \right]^{\frac{1}{1-\alpha}}$$

and with full depreciation:

$$\kappa^{GR} = \left[ \frac{A\alpha}{1 + n} \right]^{\frac{1}{1-\alpha}}$$
Decentralized equilibrium without social security

The optimization problem of the households

\[
\max \ log(c_{y,t}) + \beta \log(c_{o,t+1})
\]

s.t. \( c_{y,t} + s_{t+1} = w_t \)

\( c_{o,t+1} = (1 + r_{t+1} - \delta)s_{t+1} \)

and the representative firm

\[
\max A_t K_t^{\alpha} L_t^{1-\alpha} - w_t L_t - r_t K_t
\]

are the same as before.
Optimality conditions

\[ c_{y,t} = \frac{1}{1 + \beta} w_t \]
\[ s_{t+1} = \frac{\beta}{1 + \beta} w_t \]
\[ c_{o,t+1} = (1 + r_{t+1} - \delta) s_{t+1} \]
\[ r_t = \alpha A_t K_t^{\alpha-1} L_t^{1-\alpha} = \alpha A_t \kappa_t^{\alpha-1} \]
\[ w_t = (1 - \alpha) A_t K_t^{\alpha} L_t^{-\alpha} = (1 - \alpha) A_t \kappa_t^{\alpha} \]
\[ = f(\kappa_t) - r_t f'(\kappa_t) \]
Steady state

In a growing economy it does not make sense to analyze a steady state in absolute levels. Instead we should analyze the convergence to a steady state in which $\kappa_t = K_t / N_t$ and $y_t = f(\kappa_t) = Y_t / N_t$ converge to constant levels.

\[
\begin{align*}
K_{t+1} &= N_t s_{t+1} \\
\frac{K_{t+1}}{N_{t+1}} rac{N_{t+1}}{N_t} &= s_{t+1} \\
(1 + n)\kappa_{t+1} &= \frac{\beta}{1 + \beta} w_t \\
\kappa_{t+1} &= \frac{1}{1 + n} \frac{\beta}{1 + \beta} (1 - \alpha) A_t \kappa_{t}^\alpha
\end{align*}
\]
Steady state value of the capital-labor ratio

Given the law of motion for $\kappa_t$

$$\kappa_{t+1} = \frac{1}{1 + n} \frac{\beta}{1 + \beta} (1 - \alpha) A_t \kappa_t^\alpha$$

it is straightforward to calculate the steady state value. Imposing $\kappa_t = \kappa_{t+1} = \kappa$, we obtain

$$\kappa = \left[ \frac{\beta}{1 + \beta} \frac{(1 - \alpha) A}{(1 + n)} \right]^{\frac{1}{1-\alpha}}$$

The steady-state value of $\kappa$ is (i) decreasing in $n$ and (ii) greater than $\kappa^{GR}$ iff $\frac{\beta}{1 + \beta} (1 - \alpha) > \alpha$. 
Steady state value of per capita output

Once we know the steady-state value of the capital-labor ratio we can define all the steady state variables:

\[
y = Ak^\alpha = A \left( \frac{\beta}{1 + \beta} \frac{A(1 - \alpha)}{1 + n} \right)^{\frac{\alpha}{1-\alpha}}
\]

\[
= A^{\frac{1}{1-\alpha}} \left( \frac{\beta}{1 + \beta} \frac{A(1 - \alpha)}{1 + n} \right)^{\frac{\alpha}{1-\alpha}}.
\]

\[K_t = kN_t \text{ and } Y_t = yN_t,\]

Hence, the capital-labor ratio is constant, but the aggregate capital stock and aggregate output grow at rate \(n\).
Below we study the introduction of a PAYGO pension system.

Each young agent contributes a quantity $\tau$ in the first period of their lives.

The government takes these resources and pays a transfer $b$ to the old agents.

In each period, the government needs to satisfy the following budget constraint

$$N_t \tau = N_{t-1} b$$

So,

$$b = \frac{N_t}{N_{t-1}} \tau = (1 + n) \tau$$
The household problem

Max \( \log(c_{y,t}) + \beta \log(c_{o,t+1}) \)

\[ c_{y,t} + s_{t+1} = w_t - \tau \]

\[ c_{o,t+1} = (1 + r_{t+1} - \delta)s_{t+1} + (1 + n)\tau \]

Combining the budget constraints, we obtain the lifetime budget constraint:

\[ c_{y,t} + \frac{c_{o,t+1}}{1 + r_{t+1} - \delta} = w_t - \left[ \frac{r_{t+1} - (n + \delta)}{1 + r_{t+1} - \delta} \right] \tau \]

Note: If \( r_{t+1} < (n + \delta) \), the right-hand side is increasing in \( \tau \).
Social Security

- The optimal consumption choice of the young:

\[ c_{yt} = \frac{1}{1 + \beta} \left[ w_t - \left( \frac{r_{t+1} - (n + \delta)}{1 + r_{t+1}} \right) \tau \right]. \]

Notice that \( r_{t+1} < (n + \delta) \rightarrow \frac{\partial c_{yt}}{\partial \tau} > 0. \)

- Similarly, savings are equal to

\[
\begin{align*}
s_{t+1} &= w_t - \tau - c_{yt} \\
&= w_t - \tau - \frac{1}{1 + \beta} \left[ w_t - \left( \frac{r_{t+1} - (n + \delta)}{1 + r_{t+1}} \right) \tau \right] \\
&= \frac{\beta}{1 + \beta} w_t - \tau \left[ 1 - \frac{1}{1 + \beta} \left( \frac{r_{t+1} - (n + \delta)}{1 + r_{t+1}} \right) \right].
\end{align*}
\]
Social security

- We are now in a position to analyze the long-run consequences of a PAYGO pension scheme. Given that

\[ k_{t+1}(1 + n) = s_{t+1}, \]

we get

\[ k_{t+1}(1 + n) = \frac{\beta}{1 + \beta} w_t - \tau \left[ 1 - \frac{1}{1 + \beta} \left( \frac{r_{t+1} - (n + \delta)}{1 + r_{t+1}} \right) \right] \]

- Or equivalently,

\[ k_{t+1} = \frac{1}{1 + n} \left( \frac{\beta}{1 + \beta} A_t (1 - \alpha) k_t^\alpha - \tau \left[ 1 - \frac{1}{1 + \beta} \left( \frac{r_{t+1} - (n + \delta)}{1 + r_{t+1}} \right) \right] \right) \]

Hence, the effect of the PAYGO system on savings (and the capital stock) depends on the sign of \( Z_t \).
Social security

\[ Z_t = 1 - \frac{1}{1 + \beta} \left( \frac{r_{t+1} - n}{1 + r_{t+1}} \right) \]

\[ = \frac{(1 + \beta)(1 + r_{t+1}) - (r_{t+1} - n)}{(1 + \beta)(1 + r_{t+1})} \]

\[ = \frac{1 + r_{t+1} + \beta(1 + r_{t+1}) - r_{t+1} + n}{(1 + \beta)(1 + r_{t+1})} > 0. \]

Hence, the introduction of a PAYGO system leads to an unambiguous decrease in the per capita capital stock. Note that this is welfare-increasing if \( \kappa > \kappa^{GR} \).
The alternative to a PAYGO-system is a fully funded system in which the contributions of the agents are invested. In this system the future pension is therefore a direct function of the contributions and the returns on the investment.

Suppose that the government introduces a fully funded system and obliges the young to contribute $\tau^k = \frac{\beta}{1+\beta} w_t$. The contributions of the young are invested in capital. What are the welfare implications of this system?