Problem set 1
Dynamic Macroeconomic Analysis
Grado Economía y Finanzas 18266
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Due in class on September 28.

1. **Robinson Crusoe** Robinson Crusoe produces units of a unique perishable good using labor as the only input. Robinson’s preferences over consumption and work are given by:

\[ U(c, l) = \log(c) - \rho l^\sigma, \]

where \( c \) denotes his consumption, \( l \) reflects the time devoted to work and \( \rho > 0 \) and \( \sigma \geq 1 \) are parameters. Robinson’s production function is given by

\[ f(l) = Al^\alpha, \]

with \( \alpha \in (0, 1) \) and \( A > 0 \).

a. Derive the expression for the marginal rate of substitution (MRS) between consumption and leisure (\( dc/dl \)) and show that the MRS is increasing in the level of \( c \) and \( l \).

b. Formalize the optimization problem of Robinson and derive the expressions for his optimal choices, \( l^* \) and \( c^* \).

c. Analyze the (comparative static) effects of an increase in \( A \) on \( l^* \) and explain your results using the concepts of income and substitution effects.

d. Suppose that Robinson lives eternally. How would this change his optimization problem?

2. **The Laffer Curve** Consider the static problem of an agent who is subject to an income tax and who needs to decide how much to work and consume. Suppose for the moment that his preferences over consumption and leisure are given by

\[ U(c, l) = \ln(c) - \phi l, \]

with \( \phi > 0 \). The disposable income of the agent is equal to \( wl(1 - \tau) \), where \( \tau \in [0, 1] \) is the marginal tax rate.

a. Find the optimal labor supply of the agent and calculate his total tax payments:

\[ T(\tau) = wl\tau. \]

b. Derive the effect of an increase on \( \tau \) on \( T(\tau) \).

c. Now suppose that \( U(c, 1 - l) = 2\sqrt{c} - \phi l \). Show that \( T^*(\tau) = w\tau l^* \) is no longer monotonically increasing in \( \tau \). What is the relation between this result and the so-called Laffer curve? Explain your answers.
3. Autarky vs Market Equilibrium

Consider an individual with preferences over consumption, $c$, and leisure, $1-l$ given by:

$$u(c, 1-l) = \frac{c^{1-\gamma}}{1-\gamma} + a(1-l)$$

Suppose that the agent is able to produce $A$ units of the perishable final good per unit of time, i.e. $y = f(l) = Al$.

a. Solve for the optimal choices $(c^*, l^*)$ using the Lagrange method.

Now suppose that the economy is populated by a large number $N$ of identical agents with the same preferences like the individual in a. The agents offer their labor services on a competitive labor market. At the other side of the labor market there is a representative firm that maximizes its profits $\Pi = AL - wL^f$, where $L^f$ denotes the firm’s labor demand and $w$ denotes the wage rate (notice that the final price of the good is normalized to 1).

b. Solve the problem of a representative worker assuming that the income of a worker is given by $y = wl$.

c. Solve the problem of the representative firm and derive the equilibrium values of output, employment and the wage rate.

d. Explain why the agents end up making the same choices in autarky and in the market equilibrium [Hint: try to draw a picture that describes the optimal choices in both cases.].

e. Is there any unemployment in the market equilibrium? If so, how would you define this unemployment?