Problem set 4
Dynamic Macroeconomic Analysis
Grado Economía y Finanzas 18266
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Due in class on December 5.

1. Consider an economy with overlapping generations in which agents live for two periods. The population is constant. In any period \( t \), \( N \) young agents are born which we will call generation \( t \). Each young agent receives an endowment \( \omega_t = \omega \) of the unique perishable good. Agents receive no endowment in the second period of their life. The preferences of the agents satisfy

\[
U(c_{yt}, c_{ot+1}) = \sqrt{c_{yt}} + \sqrt{c_{ot+1}} \quad (1)
\]

Throughout the exercise we will restrict attention to stationary allocations with constant consumption levels for young agents, \( c_{yt} = c_y \), and old agents, \( c_{ot+1} = c_{ot} = c_o \).

a. Demonstrate that the set of feasible stationary allocations in this economy is defined by the following resource constraint:

\[ c_y + c_o = \omega \]

b. One feasible allocation is to give all agents a constant level of consumption in both periods of their lives, so that \( c_y = c_o = 0.5\omega \). Show that this allocation is consistent with the Golden Rule — i.e. it maximizes the welfare of a representative future generation subject to the above resource constraint.

c. Derive the equilibrium resource allocation without money and explain why this allocation is inefficient.

Now suppose there is some external agency (government, central bank) that introduces a quantity \( M \) of fiat money by distributing an amount of \( m = M/N \) of fiat money to each of the old agents alive in period 1.

d. Let \( m_{t+1} \) denote the nominal value of cash that a representative agent of generation \( t \) decides to bring to next period and let \( P_t \) denote the price level in \( t \). Use this notation to derive the expression for the budget constraints for young and old agents as well as the lifetime budget constraint of generation \( t \).

e. Solve the maximization problem of a young agent of generation \( t \) and demonstrate that the optimal value of money holdings satisfy:

\[
\frac{m_{t+1}}{P_t} = \left[ \frac{(P_t/P_{t+1})}{1 + (P_t/P_{t+1})} \right]
\]

f. Derive the stationary equilibrium with perfect foresight and show that it decentralizes the efficient allocation. [hint: Derive the market clearing condition for the good in periods \( t \geq 1 \) and show that \( P_t \) is constant in equilibrium. Then use the market clearing condition for period 1 to derive the expression for the price level.]

g. Explain how your results change if the money stock grows at rate \( \mu > 0 \). The government uses the proceeds from money creation to finance the purchase of useless public goods. Why do agents choose a consumption profile in which \( c_y > c_o \)?