Problem set 2
Dynamic Macroeconomic Analysis
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Due in class on October 15.

1. Elasticity of Intertemporal Substitution Consider the consumption-savings problem of an agent who has the following preferences over consumption in the two periods of his life:

\[ U(c_1, c_2) = \frac{c_1^{1-\sigma}}{1-\sigma} + \beta \frac{c_2^{1-\sigma}}{1-\sigma} \]

a. Derive the expression for the elasticity of intertemporal substitution around the optimal consumption choice (EIS).

b. Demonstrate that the agent will prefer to consume the same amount in both periods if \( \beta (1 + R) = 0 \). Calculate the value of the EIS around the optimal value of \( c_2/c_1 \).

c. Demonstrate the equivalence between the case of logarithmic utility and the above preferences for the limit in which \( \sigma \) approaches 1.

d. Empirical studies for the US suggest that the EIS is around 0.5. What value should we choose for \( \sigma \) if we want to have a EIS equal to 0.5?

2. Consumption smoothing Consider an economy composed of agents of two types called A and B. All agents have the same logarithmic preferences over consumption

\[ U(c_1, c_2) = \log(c_1) + \beta \log(c_2), \]

but they have different income streams. The agents of type A have a income of 1 unit in the first period and 0 units in the second period of their life, while the agents of type B have 0 units of income in the first period and 1 unit in the second period of their life. By assumption all agents have access to a perfectly competitive credit market with real interest rate \( R \).

a. Derive the expressions for the optimal levels of consumption for both types \( (c_{1A}, c_{2A}) \) for \( i = A, B \) for arbitrary values of \( \beta \) and \( R \).

b. Calculate the optimal solutions for the special case in which \( \beta = 1 \) and \( R = 0 \).

c. Repeat the exercise in b for the case in which \( \beta = 0.5 \) and \( R = 1 \) and explain the differences with respect to your answers in b.

d. Suppose that agents cannot borrow against future income. They can only lend resources at the market interest rate. How does this credit constraint affect the agents of both types?

3 General equilibrium Consider the same economy as in problem 2, but now assume that there are \( N_A \) agents of type A and \( N_B \) agents of type B.

a. Derive the equilibrium condition for the credit market. Explain in words why aggregate savings needs to be equal to zero in any equilibrium.

b. Derive the equilibrium interest rate for the case in which \( N_A = N_B = N \).
c. Characterize the expressions for the equilibrium savings and consumption decisions when $N_A = N_B = N$ for some $0 < \beta < 1$.

d. Repeat the above exercises for the case in which $\beta = 1$. How does this assumption change your answers?

e. Finally consider the case in which $\beta = 1$ and $N_B = 2N_A$. Explain why the equilibrium interest rate is strictly positive and why full consumption smoothing by all agents is no longer possible.

4 [N periods] Consider an agent who lives for 3 periods and who faces the following sequence of one-period budget constraints:

\[
\begin{align*}
c_1 + b_1 &= y_1 \\
c_2 + b_2 &= y_2 + (1 + R)b_1 \\
c_3 &= y_3 + (1 + R)b_2
\end{align*}
\]

a. Show that this agent faces the following lifetime budget constraint

\[
c_1 + \frac{c_2}{1 + R} + \frac{c_3}{(1 + R)^2} = y_1 + \frac{y_2}{(1 + R)} + \frac{y_3}{(1 + R)^2}
\]

b. Suppose the agent’s preferences are given by

\[
U(c_1, c_2, c_3) = \sum_{t=1}^N \beta^{t-1} \log(c_t) = \log(c_1) + \beta \log(c_2) + \beta^2 \log(c_3)
\]

Derive the first-order conditions that characterize the optimal solution and demonstrate that $c_1^* = c_2^* = c_3^*$ when $\beta(1 + R) = 1$. 