Problem set 3  
Dynamic Macroeconomic Analysis  
Professor Marcel Jansen  
Due in class on Friday, November 8.

1. Consider a two-period economy with a large number of identical agents who work and consume in both periods of their lives. The preferences of a representative agent are given by

\[ U = \sqrt{c_1} + \beta \sqrt{c_2} + \sqrt{1 - l_1} + \beta \sqrt{1 - l_2} \]

where \( c_t, t = 1, 2 \) denote the agent’s consumption levels in both periods and \( l_t \) denotes the share of time devoted to work. We are interested in the solutions under autarky. Each agent has a production technology given by \( y_t = A_t l_t \). Moreover, we assume that the good is storable. The storage technology allows the agent to transform 1 unit of the unique final good produced in period 1 into one unit of consumption in period 2. Finally, we assume that the agent has perfect information about the values of \( A_1 \) and \( A_2 \) at the start of period 1.

a. Formalize the optimization problem of a representative agent and derive the first-order conditions.

b. Derive the expressions for the optimal choices of consumption and labour supply for arbitrary values of \( (\beta, A_1, A_2) \).

c. Derive the equilibrium allocation for the special case in which \( \beta = A_1 = A_2 = 1 \).

d. Describe the effects of a permanent productivity shock that doubles the values of \( A_1 \) and \( A_2 \) when \( \beta = 1 \).

e. Describe the effects of a temporary productivity shock in the first period, so that \( A_1 = 2, A_2 = 1 \), when \( \beta = 1 \).

f. Describe the effects of a temporary productivity shock in period 2 on the consumption levels in \( t = 1 \). Explain why the optimal solution differs from the solution in e.