Problem set 4: Money in OLG
Dynamic Macroeconomic Analysis
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Due in class on Friday, November 22.

1. In this problem we study same environment as in class with the only difference that the
agents receive a positive endowment of the perishable good in both periods of their lives. To
be more precise, we consider an economy with overlapping generations that lasts forever.
In each period \( N \) young agents are born who live for two periods. The agents born in
period \( t \) are called generation \( t \) and they choose consumption when young and old given
by \( c_{yt} \) and \( c_{ot+1} \), respectively. All agents are identical and they receive 1 unit of the good
in the first period and \( \omega < 1 \) units in the second period of their lives. The preferences of
a representative agent of generation \( t \) are given by:

\[
U(c_{yt}, c_{ot+1}) = \log(c_{yt}) + \log(c_{ot+1})
\]

a. Characterize the efficient (golden rule) allocation
b. Compare your answer in a to the equilibrium allocation without money (or any other
financial asset).

Now assume that the government introduces a fixed amount \( M \) of pure fiat money by
handing an amount of \( m = \frac{M}{N} \) units of money to each old agent of generation 0. Let \( P_t \)
denote the price level in \( t \).

c. Formalize the individual optimization problem and demonstrate that there exists an
efficient competitive equilibrium (with perfect foresight) in which \( P_t = P_{t+1} \) \( \forall t \).
d. Demonstrate that there exists a second equilibrium path in which prices are growing at
the rate \( 1/\omega \), i.e. \( P_{t+j} = \left(\frac{1}{\omega}\right)^j \). [You may use a guess-and-verify strategy: substitute
\( P_{t+1}/P_t = 1/\omega \) into the equilibrium conditions and show that the resulting conditions
satisfy the market clearing condition.]
e. Which of the two equilibria is preferred by the agents?