Social Security

Dynamic Macroeconomic Analysis

Universidad Autónoma de Madrid

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In previous lectures we have seen that the equilibrium of an OLG model need not be efficient.

In our OLG model with physical capital the steady state stock of capital may exceed the level consistent with the Golden Rule — steady state consumption would be higher if the agents would reduce savings.

In our OLG model with perishable goods the introduction of fiat money was shown to lead to a Pareto-improvement — the introduction of a constant money supply moved the equilibrium from autarky to the Golden Rule allocation.

In this lecture we reconsider efficiency in an economy with a growing population.
Preview of the results

Throughout this lecture we assume that the population grows at rate \( n > 0 \). Hence, \( N_{t+1} = (1 + n)N_t \).

- When \( \delta = 0 \), the Golden Rule capital stock is defined by \( PMK = r = n \).
- We show that the decentralized equilibrium may be dynamically inefficient \( (r < n) \).
- Whenever this is the case, the introduction of a pay-as-you-go pension system may improve welfare. It reduces the steady state capital stock and raises per capita consumption.
Sustainability of the pension system

In a Pay-As-You-Go pension system, the pensions of the old are financed with the social security contributions of the young.

The system works fine as long as the population is growing and a large proportion of the young is employed.

Currently, neither of these conditions is satisfied. The dependency ratio is at a historic low.

Other risks: increased life expectancy, relatively high ratio between pension and last salary
Demographic changes

Population by age group, gender, in 2000 and 2050, in percentage of total population in each group

MEN
- 85+
- 80 - 84
- 75 - 79
- 70 - 74
- 65 - 69
- 60 - 64
- 55 - 59
- 50 - 54
- 45 - 49
- 40 - 44
- 35 - 39
- 30 - 34
- 25 - 29
- 20 - 24
- 15 - 19
- 10 - 14
- 5 - 9
- 0 - 4

SPAIN

WOMEN
- 85+
- 80 - 84
- 75 - 79
- 70 - 74
- 65 - 69
- 60 - 64
- 55 - 59
- 50 - 54
- 45 - 49
- 40 - 44
- 35 - 39
- 30 - 34
- 25 - 29
- 20 - 24
- 15 - 19
- 10 - 14
- 5 - 9
- 0 - 4

Total population (in millions)
- in 2000: 0.0
- in 2000: 27
- in 2050: 0.0
- in 2050: 73

Old age dependency ratio (65+ in % 20-64)
- in 2000: 0.0
- in 2050: 73
Throughout this lecture we assume that the population $N_t = L_t$ grows at the constant rate $n$ per period.

$$N_t = N_{t-1} (1 + n)$$

We will first study efficiency in a model without pensions.
The resource constraint in a growing economy

The consumption and investment decisions in our economy need to satisfy

\[ N_t c_y, t + N_{t-1} c_o, t + N_t s_{t+1} = A_t K_t^\alpha N_t^{1-\alpha} + (1 - \delta) K_t \]

\[ N_t c_y, t + N_{t-1} c_o, t + K_{t+1} = A_t K_t^\alpha N_t^{1-\alpha} + (1 - \delta) K_t \]

Define the capital-labor ratio as \( \kappa_t = K_t / N_t \). Dividing both sides by \( N_t \) we obtain

\[ c_y, t + \frac{1}{1+n} c_o, t + (1 + n) k_{t+1} = A_t \kappa_t^\alpha + (1 - \delta) \kappa_t \]

So, in steady state

\[ c_y + \frac{1}{1+n} c_o = A \kappa^\alpha - (n + \delta) \kappa = f(\kappa) - (n + \delta) \kappa \]

where \( f(\kappa_t) = Y_t / N_t = A_t \kappa^\alpha \)
Golden Rule

The Golden Rule capital stock maximizes the value of (per capita) consumption. It solves

$$\max_\kappa f(\kappa) - (n + \delta)\kappa$$

The efficient level of the capital-labor ratio is given by

$$\alpha A\kappa^{\alpha - 1} = n + \delta$$

$$\kappa_{GR} = \left[\frac{A\alpha}{n + \delta}\right]^{\frac{1}{1-\alpha}}$$

and with full depreciation:

$$\kappa_{GR} = \left[\frac{A\alpha}{1 + n}\right]^{\frac{1}{1-\alpha}}$$
Decentralized equilibrium without social security

The optimization problem of the households

$$\max \ log(c_{y,t}) + \beta log(c_{o,t+1})$$

s.t. \( c_{y,t} + s_{t+1} = w_t \)

\[ c_{o,t+1} = (1 + r_{t+1} - \delta) s_{t+1} \]

and the representative firm

$$\max A_t K_t^\alpha L_t^{1-\alpha} - w_t L_t - r_t K_t$$

are the same as before.
Optimality conditions

\[ c_{y,t} = \frac{1}{1 + \beta} w_t \]

\[ s_{t+1} = \frac{\beta}{1 + \beta} w_t \]

\[ c_{o,t+1} = (1 + r_{t+1} - \delta) s_{t+1} \]

\[ r_t = \alpha A_t K_t^{1-\alpha} L_t^{1-\alpha} = \alpha A_t \kappa_t^{1-1} \]

\[ w_t = (1 - \alpha) A_t K_t^{\alpha} L_t^{-\alpha} = (1 - \alpha) A_t \kappa_t^{\alpha} \]

\[ = f(\kappa_t) - r_t f'(\kappa_t) \]
Steady state

In a growing economy it does not make sense to analyze a steady state in absolute levels. Instead we should analyze the convergence to a steady state in which $\kappa_t = K_t / N_t$ and $y_t = f(\kappa_t) = Y_t / N_t$ converge to constant levels.

\[ K_{t+1} = N_t s_{t+1} \]
\[ \frac{K_{t+1} N_{t+1}}{N_{t+1} N_t} = s_{t+1} \]
\[ (1 + n)\kappa_{t+1} = \frac{\beta}{1 + \beta} w_t \]
\[ \kappa_{t+1} = \frac{1}{1 + n} \frac{\beta}{1 + \beta} (1 - \alpha) A_t \kappa_t^\alpha \]
Steady state value of the capital-labor ratio

Given the law of motion for $\kappa_t$

$$\kappa_{t+1} = \frac{1}{1 + n} \frac{\beta}{1 + \beta}(1 - \alpha)A_t \kappa_t^{\alpha}$$

it is straightforward to calculate the steady state value. Imposing $\kappa_t = \kappa_{t+1} = \kappa$, we obtain

$$\kappa = \left[ \frac{\beta}{1 + \beta} \frac{(1 - \alpha)A}{(1 + n)} \right]_{1-\alpha}^{1}$$

The steady-state value of $\kappa$ is (i) decreasing in $n$ and (ii) greater than $\kappa^{GR}$ iff

$$\frac{\beta}{1 + \beta}(1 - \alpha) > \alpha.$$
Steady state value of per capita output

Once we know the steady-state value of the capital-labor ratio we can define all the steady state variables:

\[ y = A k^\alpha = A \left( \frac{\beta}{1 + \beta} \frac{A(1 - \alpha)}{1 + n} \right)^{\frac{\alpha}{1-\alpha}} \]

\[ = A^{\frac{1}{1-\alpha}} \left( \frac{\beta}{1 + \beta} \frac{A(1 - \alpha)}{1 + n} \right)^{\frac{\alpha}{1-\alpha}}. \]

\[ K_t = kN_t \text{ and } Y_t = yN_t, \]

Hence, the capital-labor ratio is constant, but the aggregate capital stock and aggregate output grow at rate \( n \).
Below we study the introduction of a PAYGO pension system.

Each young agent contributes a quantity $\tau$ in the first period of their lives.

The government takes these resources and pays a transfer $b$ to the old agents.

In each period, the government needs to satisfy the following budget constraint

$$ N_t \tau = N_{t-1} b $$

So,

$$ b = \frac{N_t}{N_{t-1}} \tau = (1 + n) \tau $$
The household problem

\[ \text{Max } \log(c_y,t) + \beta \log(c_o,t+1) \]
\[ c_y,t + s_{t+1} = w_t - \tau \]
\[ c_o,t+1 = (1 + r_{t+1} - \delta)s_{t+1} + (1 + n)\tau \]

Combining the budget constraints, we obtain the lifetime budget constraint:

\[ c_y,t + \frac{c_o,t+1}{1 + r_{t+1} - \delta} = w_t - \left[ \frac{r_{t+1} - (n + \delta)}{1 + r_{t+1} - \delta} \right] \tau \]

Note: If \( r_{t+1} < (n + \delta) \), the right-hand side is increasing in \( \tau \).
The optimal consumption choice of the young:

$$c_{yt} = \frac{1}{1 + \beta} \left[ w_t - \left( \frac{r_{t+1} - (n + \delta)}{1 + r_{t+1}} \right) \tau \right].$$

Notice that $r_{t+1} < (n + \delta) \rightarrow \frac{\partial c_{yt}}{\partial \tau} > 0$.

Similarly, savings are equal to

$$s_{t+1} = w_t - \tau - c_{yt}$$

$$= w_t - \tau - \frac{1}{1 + \beta} \left[ w_t - \left( \frac{r_{t+1} - (n + \delta)}{1 + r_{t+1}} \right) \tau \right]$$

$$= \frac{\beta}{1 + \beta} w_t - \tau \left[ 1 - \frac{1}{1 + \beta} \left( \frac{r_{t+1} - (n + \delta)}{1 + r_{t+1}} \right) \right].$$
We are now in a position to analyze the long-run consequences of a PAYGO pension scheme. Given that

\[ k_{t+1}(1 + n) = s_{t+1}, \]

we get

\[ k_{t+1}(1 + n) = \frac{\beta}{1 + \beta} w_t - \tau \left[ 1 - \frac{1}{1 + \beta} \left( \frac{r_{t+1} - (n + \delta)}{1 + r_{t+1}} \right) \right] \]

Or equivalently,

\[ k_{t+1} = \frac{1}{1 + n} \left( \frac{\beta}{1 + \beta} A_t (1 - \alpha) k_t^\alpha - \tau \left[ 1 - \frac{1}{1 + \beta} \left( \frac{r_{t+1} - (n + \delta)}{1 + r_{t+1}} \right) \right] \right) \]

Hence, the effect of the PAYGO system on savings (and the capital stock) depends on the sign of \( Z_t \).
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\[
Z_t = 1 - \frac{1}{1 + \beta} \left( \frac{r_{t+1} - n}{1 + r_{t+1}} \right)
= \frac{(1 + \beta)(1 + r_{t+1}) - (r_{t+1} - n)}{(1 + \beta)(1 + r_{t+1})}
= \frac{1 + r_{t+1} + \beta(1 + r_{t+1}) - r_{t+1} + n}{(1 + \beta)(1 + r_{t+1})} > 0.
\]

Hence, the introduction of a PAYGO system leads to an unambiguous decrease in the per capita capital stock. Note that this is welfare-increasing if \( \kappa > \kappa^{GR} \).
The alternative to a PAYGO-system is a fully funded system in which the contributions of the agents are invested. In this system the future pension is therefore a direct function of the contributions and the returns on the investment.

Suppose that the government introduces a fully funded system and obliges the young to contribute $\tau^k = \frac{\beta}{1+\beta} w_t$. The contributions of the young are invested in capital. What are the welfare implications of this system?