Instructions: The exam consists of two problems with a total of 7 questions. The weight of each of the questions is indicated in the heading of the problem. The grade is equal to the total number of points divided by 10. Please respect the indicated space for the answers to both questions. Your answers should be concise but exhaustive and should clearly state all the necessary steps. All mobile devices should be switched off during the exam and the use of notes is not permitted. Students who infringe the general rules for exams will be requested to hand in their exam and will receive a grade of 0.

1. Consider a version of our canonical OLG model with endogenous labour supply. In each period \( N \) agents are born who live for two periods. The young agents of generation \( t \) work an endogenous fraction of their time \( l_t \in [0,1] \) and consume an amount \( c_{y,t} \) when young and \( c_{o,t+1} \) when old. The agents’ preferences are given by

\[
U(c_{y,t}, c_{o,t+1}, l_t) = \log(c_{y,t}) + \beta \log(c_{o,t+1}) + \log(1 - l_t)
\]

There is also a representative firm with a neo-classical production function

\[
F(A_t, K_t, L_t) = A_t K_t^\alpha L_t^{1-\alpha}
\]

The firm hires labour and capital on competitive markets. We denote the wage rate and the rental price of capital by \( w_t \) and \( R_t \), respectively, and we assume that capital depreciates at rate \( \delta \).

a. Solve the maximization problems of the firm and a representative agent of generation \( t \). (20 points)

b. Derive the expression for the (deterministic) steady state level of capital, output and wages for \( A_t = 1 \ \forall t \). (10 points)

c. Suppose that the economy is in steady state when it is hit by an unanticipated productivity shock at the start of period \( t = T \) that reduces the value of \( A_T \) to 0.95. In the next period the value of \( A_t \) returns to its normal value. Derive the expressions for \( Y_T, Y_{T+1}, Y_{T+2} \) and explain why the economy does not immediately return to steady state. (10 points)

d. In the optional solution the agents decide to work a constant fraction of their time. Explain why this is the case and what is needed to generate changes in labour supply that amplify the effects of a shock to productivity. (10 points)
2. Consider an OLG model in which agents live for two periods. In each period \( N \) agents are born. Each young agent receives a fixed endowment \( \omega \) of the unique *perishable* good which can only be consumed in the same period. Old agents do not receive any endowment. The agents’ preferences are given by

\[
U(c_{y,t}, c_{o,t+1}) = u(c_{y,t}) + u(c_{o,t+1})
\]

The function \( u() \) is a strictly concave function of consumption in each period.

a. Derive the Pareto-efficient allocation with the help of a Social Planner who attaches an equal Pareto-weight to agents of all generations. [hint: all generations are treated equally.] (20 points)

b. Derive the resource allocation in the competitive equilibrium and explain why it fails to replicate the efficient allocation. (10 points)

c. Demonstrate that the introduction of a pay-as-you-go pension system may help to yield an efficient equilibrium outcome. (20 points).