Problem set 3
Dynamic Macroeconomic Analysis
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1. Elasticity of Intertemporal Substitution Consider the consumption-savings problem of an agent who has the following preferences over consumption in the two periods of his life:

\[ U(c_1, c_2) = \frac{c_1^{1-\sigma} - 1}{1 - \sigma} + \beta \frac{c_2^{1-\sigma} - 1}{1 - \sigma} \]

a. Derive the expression for the elasticity of intertemporal substitution.

b. Demonstrate that the agent will prefer to consume the same amount in both periods if \( \beta(1 + R) = 1 \).

c. Demonstrate the equivalence between the case of logarithmic utility and the above preferences for the limit in which \( \sigma \) approaches 1.

d. Empirical studies for the US suggest that the EIS is around 0.5. What value should we choose for \( \sigma \) if we want to have a EIS equal to 0.5?

2. Ricardian equivalence Consider an economy that lasts for two periods. The \( N \) households in the economy have a disposable income of \( y_t - \tau_t \) units in the two periods, where \( \tau_t \) is a lump-sum tax. Suppose that the government faces the following inter-temporal budget constraint

\[ G_1 + \frac{G_2}{1 + R} = T_1 + \frac{T_2}{1 + R} \]

where \( T_t = N\tau_t \). The minister of finance is considering two options. in option a the government only levies a tax in the first period. In option b the government borrows an amount \( G_1 \) in the first period and levies a tax in the second period.

a. Describe the differences in the optimal consumption patterns of the households when they pay the same interest rate as the government and agent’s preferences are given by

\[ U(c_1, c_2) = \ln c_1 + \beta \ln c_2 \]

3 N periods Consider an agent who lives for 3 periods and who faces the following sequence of one-period budget constraints:

\[ c_1 + b_1 = y_1 \]
\[ c_2 + b_2 = y_2 + (1 + R)b_1 \]
\[ c_3 = y_3 + (1 + R)b_2 \]

a. Show that this agent faces the following lifetime budget constraint (when \( b_3 = 0 \)).

\[ c_1 + \frac{c_2}{1 + R} + \frac{c_3}{(1 + R)^2} = y_1 + \frac{y_2}{1 + R} + \frac{y_3}{(1 + R)^2} \]
b. Suppose the agent’s preferences are given by

\[ U(c_1, c_2, c_3) = \sum_{t=1}^{N} \beta^{t-1} \log(c_t) = \log(c_1) + \beta \log(c_2) + \beta^2 \log(c_3) \]

Derive the first-order conditions that characterize the optimal solution and demonstrate that \( c_1^* = c_2^* = c_3^* \) when \( \beta(1 + R) = 1 \).

4 Endogenous income Consider an agent who lives for two periods. During the first period the agent offers his labour services on a perfectly competitive labour market. His labour income \( w_1l_1 \) serves to pay for consumption in both periods. Let \( R_1 \) denote the real interest rate and assume that the agent’s preferences are given by

\[ U(c_1, c_2, 1-l_1) = \ln(c_1) + \beta \ln(c_2) + \ln(1-l_1) \]

a. Derive the expression for the optimal labour supply and consumption decisions of the agent for arbitrary values of \( \beta, R \)