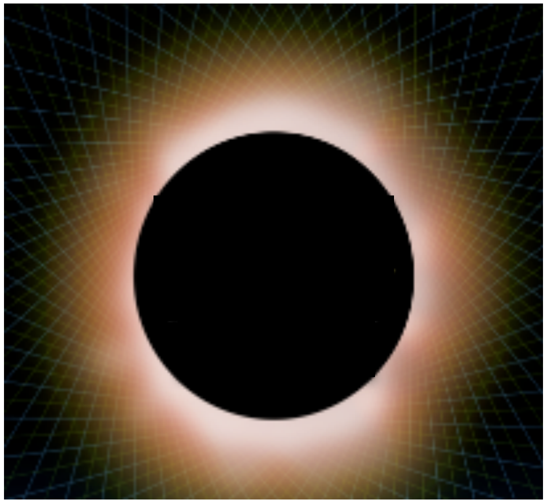
Black Holes
and
Quantum Mechanics



Juan Maldacena

Institute for Advanced Study

Letter from Schwarzschild to Einstein. 22 December 1915,

$$ds^2 = \left(1 - \frac{\gamma}{R}\right) dt^2 - \frac{dR^2}{\left(1 - \frac{\gamma}{R}\right)} - R^2(d\theta^2 + \sin^2\theta d\phi^2)$$

It as been very confusing ever since…

Classically…

$$ds^2 = \left(1 - \frac{\gamma}{R}\right) dt^2 - \frac{dR^2}{\left(1 - \frac{\gamma}{R}\right)} - R^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Extreme slow down of time

The coordinate ``singularity”

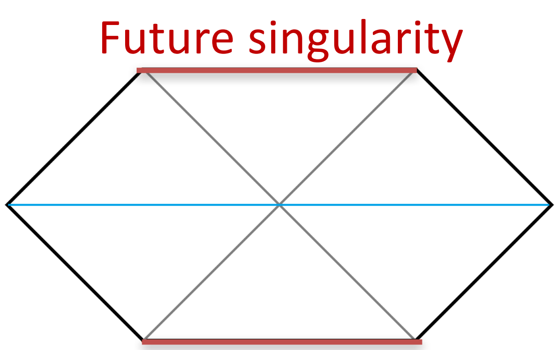
Eddington 1924 finds a non singular coordinate system but did not recognize (or comment on its significance).

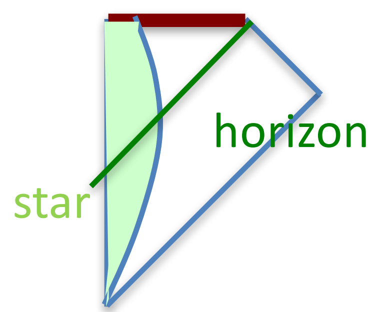
Lemaitre 1933 First published statement that the horizon is not singular.

Einstein Rosen 1935 (Still call it a ``singularity’’)

Szekeres, Kruskal 59-60 coordinates cover the full spacetime

Wheeler Fulling 62 It is a wormhole !





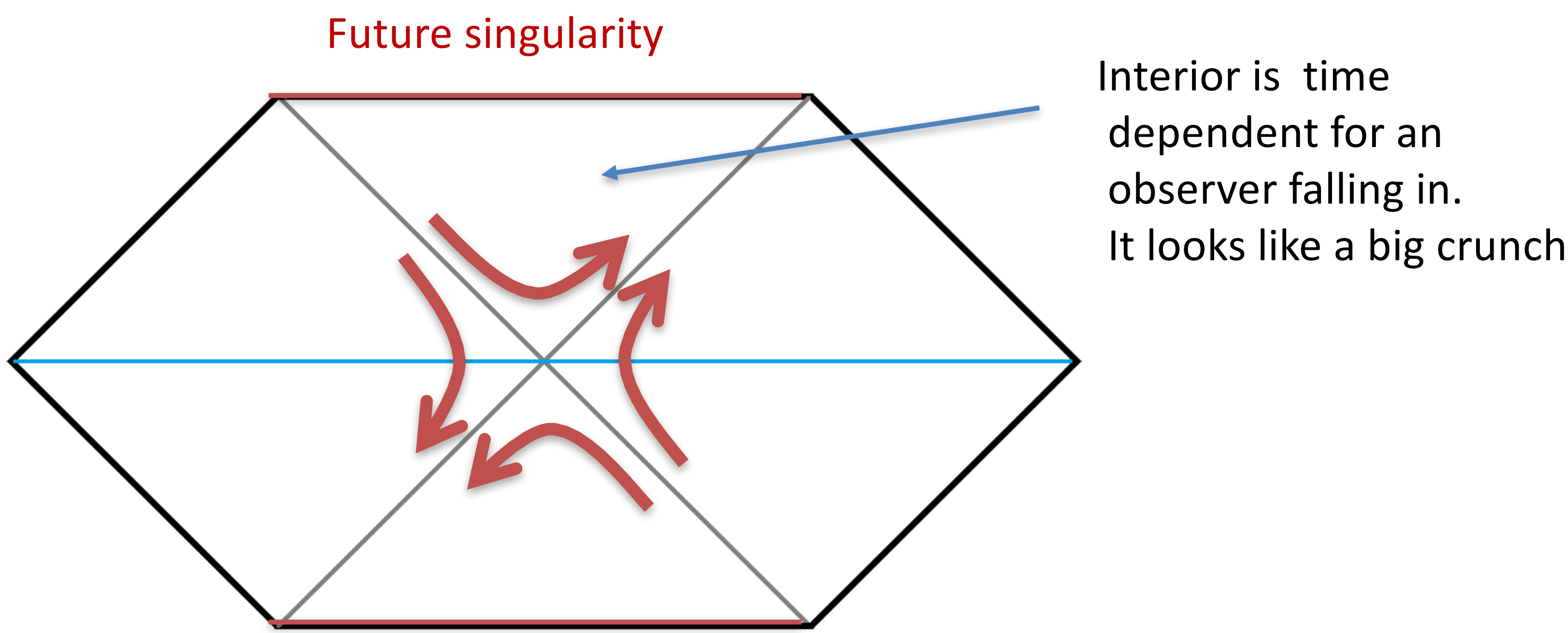
Oppenheimer Snyder 1939

Observer on the surface does not feel anything special at the horizon. Finite time to the singularity

It took 45 years to understand a classical solution…
Why ?

The symmetries are realized in a funny way.

The time translation symmetry → boost symmetry at the bifurcation surface



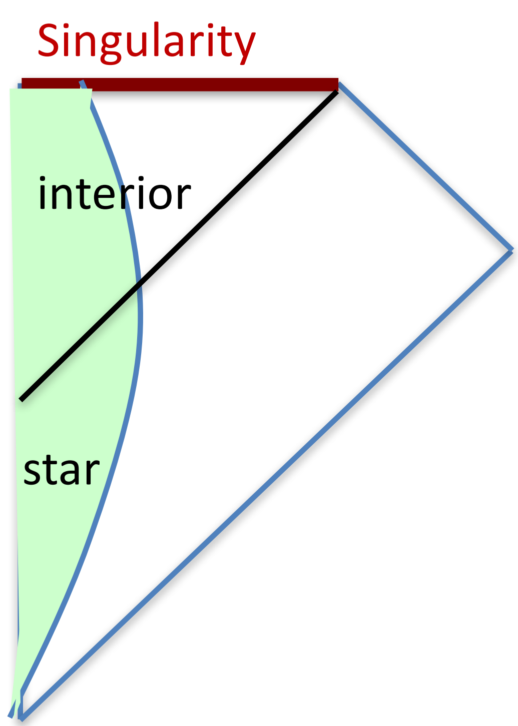
Black holes in astrophysics

- Quasars (most efficient energy sources)
- Stellar mass black holes
- Sources for gravity waves !
- Our own supermassive black hole in the Milky Way…

Interior and singularity

Quantum gravity is necessary at the singularity!

What signals ?



Singularity is behind the horizon.

It is shielded behind the black hole horizon that acts as a Schwarz-Shild.

Black holes and quantum mechanics

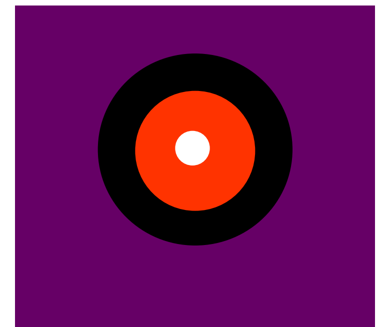
- Black holes are one of the most surprising predictions of general relativity.
- Incorporating quantum mechanics leads to a new surprise:

Black holes are not black !

Hot black holes

- Black holes have a temperature

$$T \sim \frac{\hbar}{r_H}$$



We can have white ``black holes''

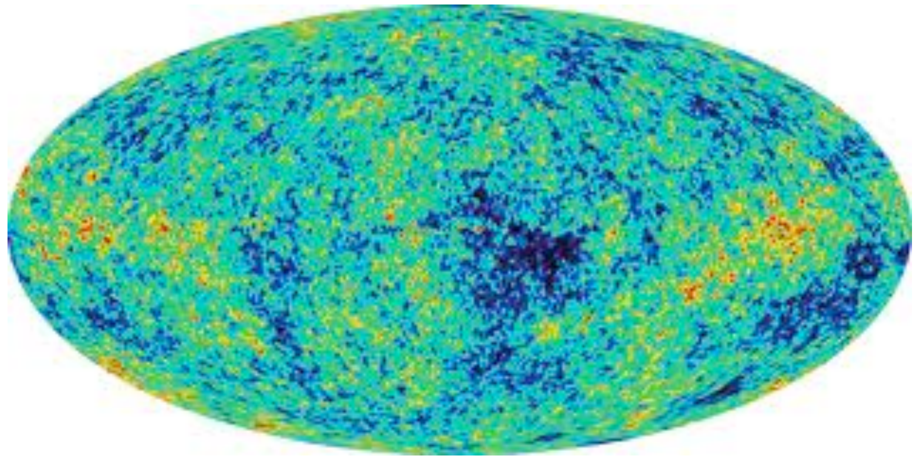
Hawking

- An accelerated expanding universe also has a temperature

$$T \sim \hbar H = \frac{\hbar}{R_H}$$

Chernikov, Tagirov,
Figari, Hoegh-Krohn, Nappi,
Gibbons, Hawking,
Bunch, Davies, ….

Very relevant for us!



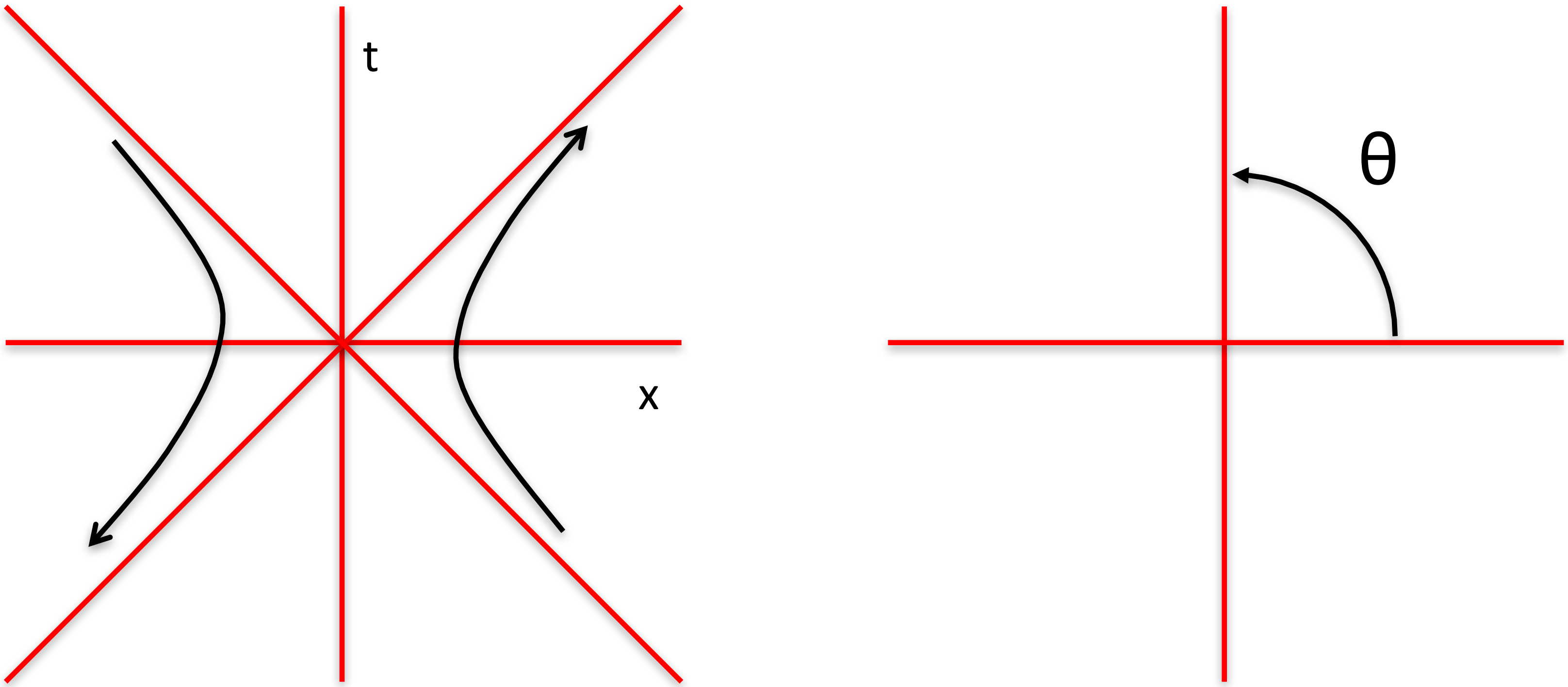
Quantum mechanics is crucial for understanding the large scale geometry of the universe.

Why a temperature ?

- Consequence of special relativity + quantum mechanics.

Flat space first

Why a temperature ?



Lorentzian

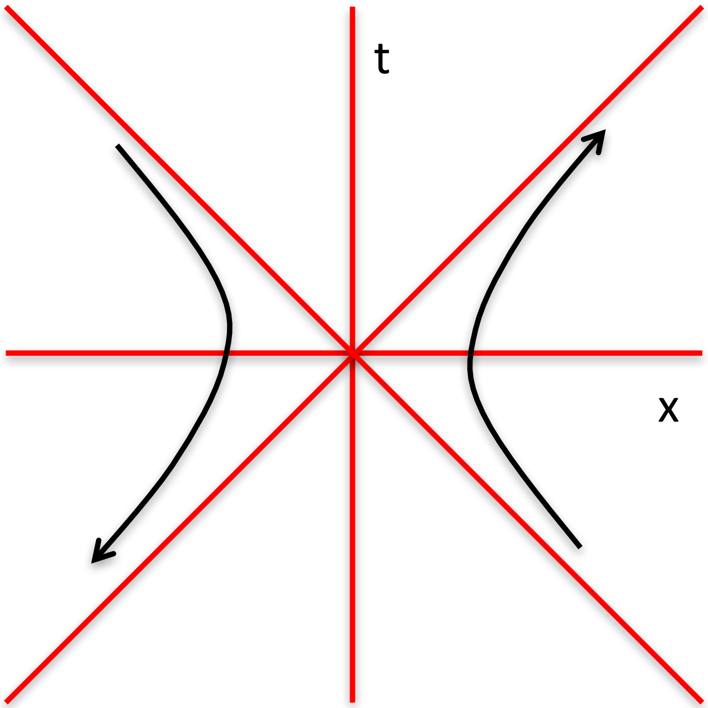
Euclidean

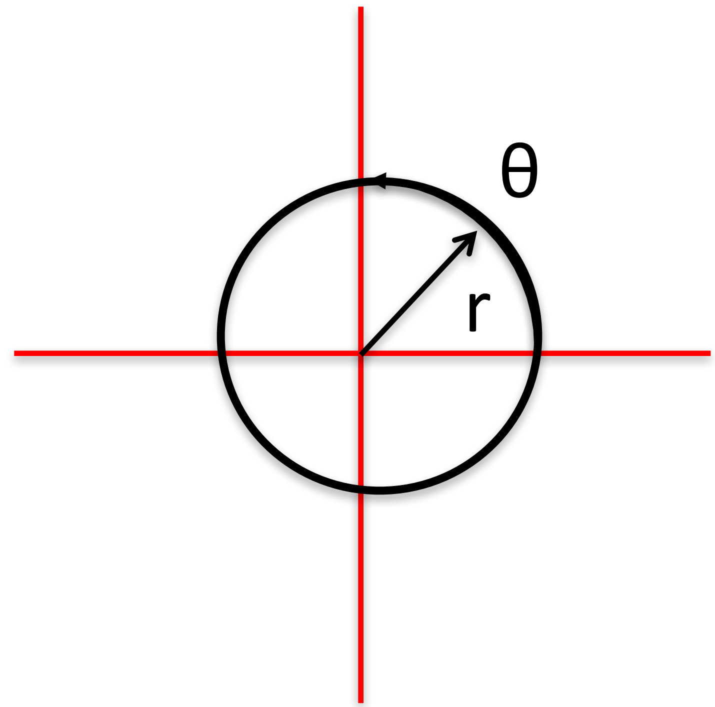
Accelerated observer → energy = boost generator.

Time translation → boost transformation

Continue to Euclidean space → boost becomes rotation.

Why a temperature ?





Continue to Euclidean space → boost becomes rotation.

Angle is periodic → temperature

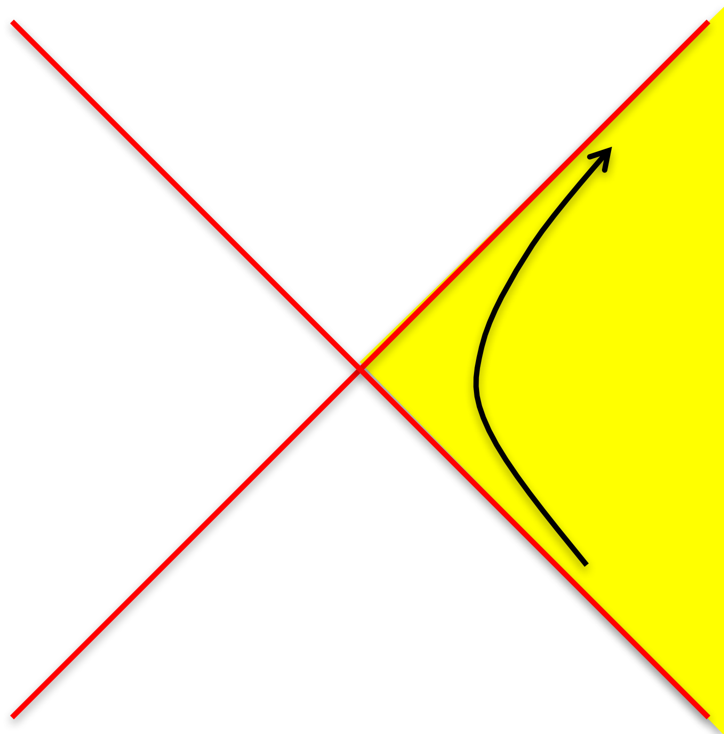
$$\beta = \frac{1}{T} = 2\pi r = \frac{2\pi}{a}$$

Thermische Unruhe

Ordinary accelerations are very small, g= 9.8 m/s^2 → β = 1 light year

Bisognano Weichman, Unruh

Entanglement & temperature

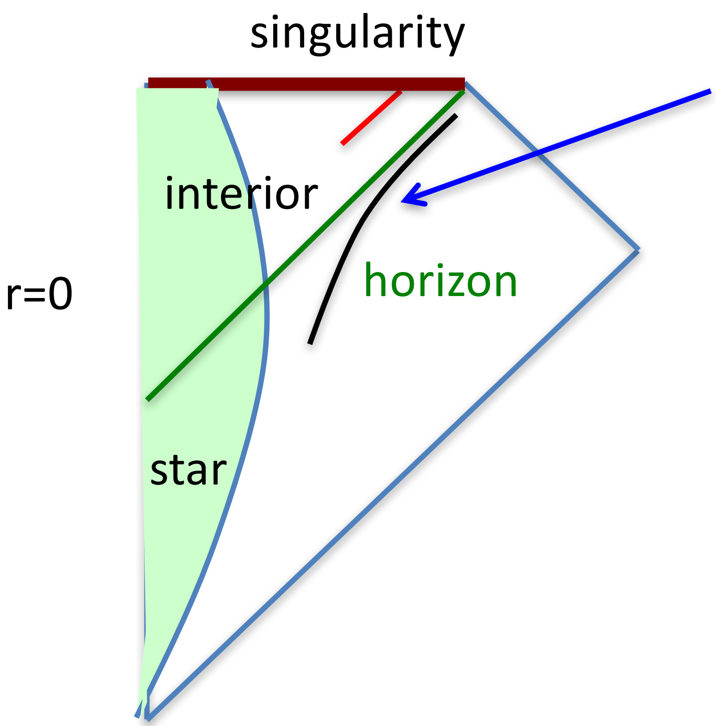


Horizon: accelerated observer only has access to the right wedge.

If we only make observations on the right wedge → do not see the whole system → get a mixed state (finite temperature).

General prediction, only special relativity + quantum mechanics + locality
Vacuum is highly entangled !

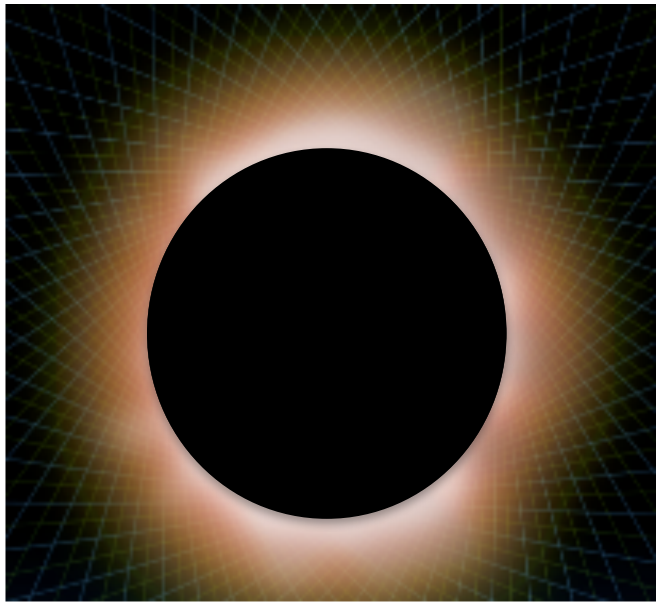
Black hole case



Equivalence principle: region near the horizon is similar to flat space.

If we stay outside → accelerated observer→ temperature.

Black hole from collapse



Black holes have a temperature

Do they obey the laws of thermodynamics ?

Black hole entropy

$$T \sim \frac{\hbar}{r_H}$$

Special relativity near the black hole horizon

$$r_H \leftrightarrow M$$

Einstein equations

$$dM = TdS$$

1st Law of thermodynamics

$$S = \frac{(\text{Area})}{4\hbar G_N}$$

Black hole entropy

Bekenstein, Hawking

2nd Law → area increase from Einstein equations and positive null energy condition. Hawking

Including the quantum effects

$$S_{BH} = \frac{(\text{Area})}{4G_N} + S_{\text{entanglement}}$$

Entanglement entropy of quantum fields across the black hole horizon

Has been understood better in quantum field theory
2nd Law extended to include this term.

Wall 2011

Bekenstein bound → automatic in relativistic quantum field theory.

Casini 2008

Focusing theorems and better understanding of the positivity of energy, and new ``area'' increase statements.

Bousso's talk
Bousso, Englehardt, Wall, Faulkner, …

Bekenstein – Casini bound

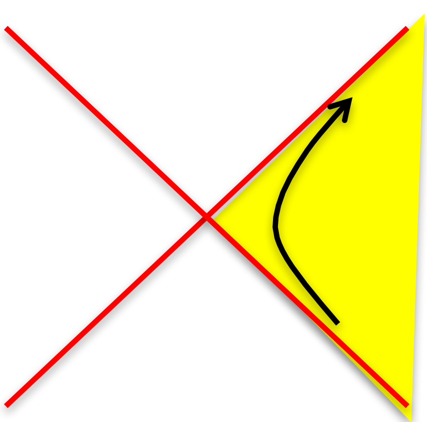
$$S \leq 2\pi E R$$

Bekenstein

When is this true ? Is it true ? Does it impose a constraint on QFT ?

$$\Delta S \leq 2\pi \Delta E_{\mathrm{Rindler}}$$

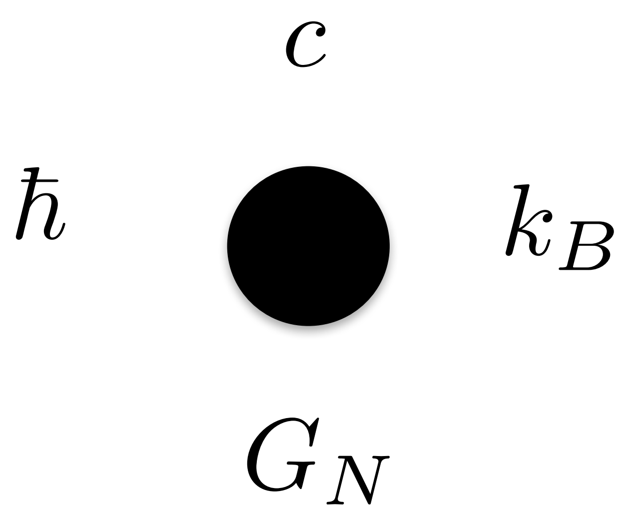
Casini 2008



It is always true in any relativistic QFT.

2^{nd} Law always satisfied.

General relativity and thermodynamics



General relativity and thermodynamics

- Black hole seen from the outside = thermal system with finite entropy.
- Is there an exact description where information is preserved ?
- Yes…

Gauge/Gravity Duality

(or gauge/string duality, AdS/CFT, holography)

Theories of quantum interacting particles

(very strongly interacting)

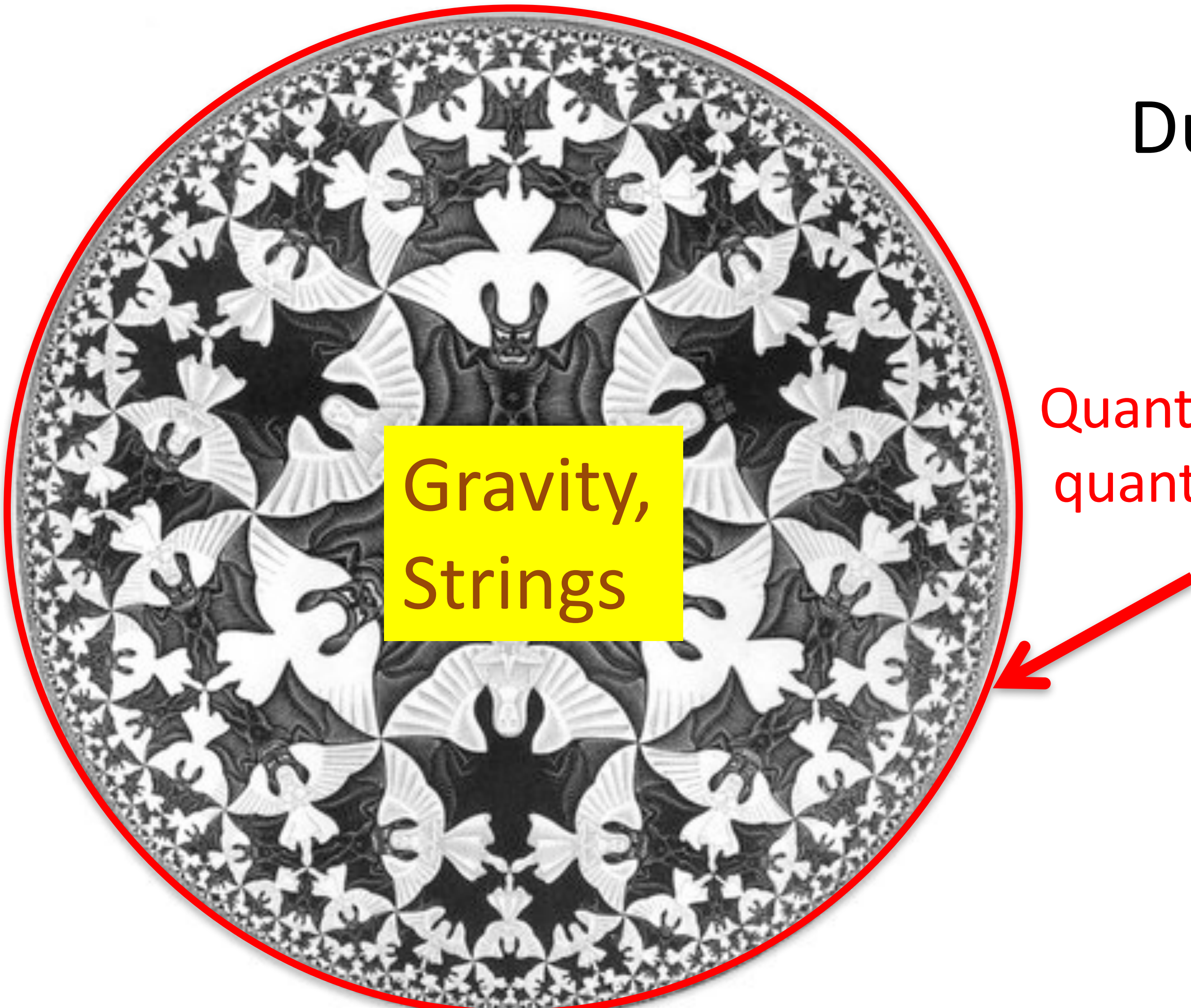
Quantum dynamical
Space-time
(General relativity)
string theory

JM 97

Witten, Gubser, Polyakov, Klebanov

….

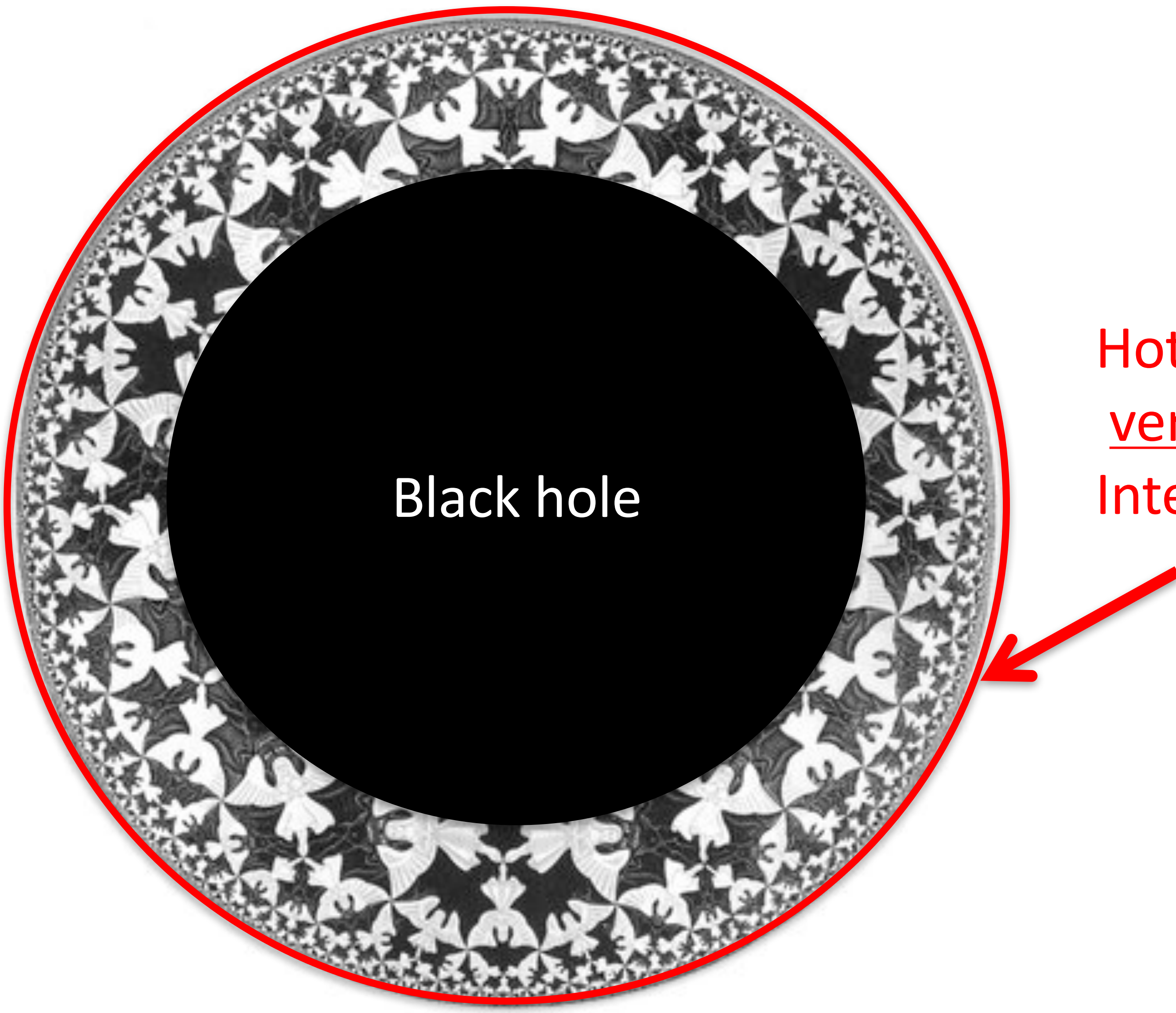
Gravity in asymptotically Anti de Sitter Space



Duality

Quantum interacting particles
quantum field theory

Black holes in a gravity box



Hot fluid made out of
very strongly
Interacting particles.

Lessons for black holes

- Black holes as seen from outside (from infinity) are like an ordinary quantum system.
- Black hole entropy = ordinary entropy of the quantum system.
- Absorption into the black hole = thermalization
- Chaos → near horizon gravity

Shenker Stanford
Kitaev , 2013-2015

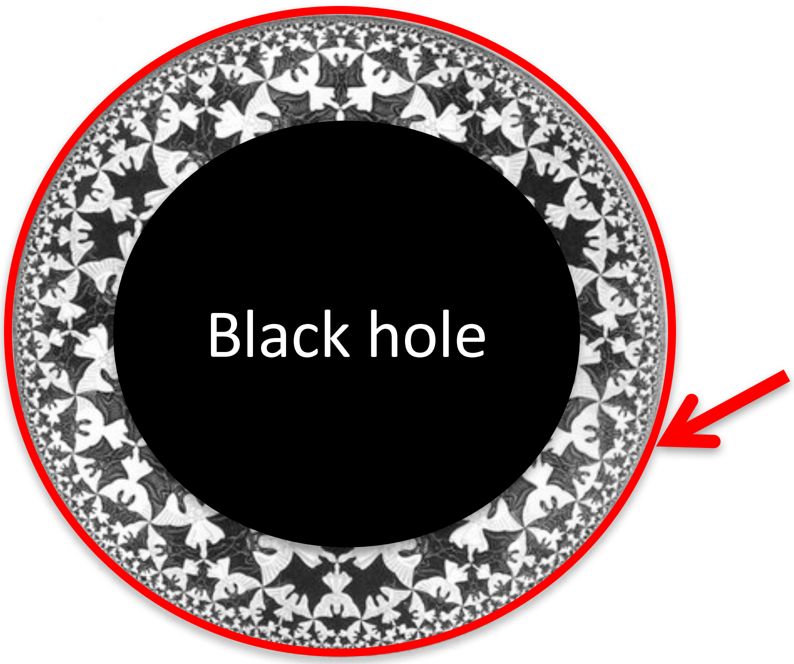
- Interior ?

Mathur, Almheiri, Marolf, Polchinski, Sully

In the meantime

Black holes as a source of information

Black holes as toy models



Used to model strongly interacting systems in high energy physics or condensed matter physics.

Hot fluid made out of strongly Interacting particles.

Key insights into the theory of hydrodynamics with anomalies.

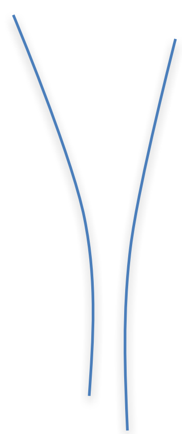
Damour, Herzog, Son, Kovtun, Starinets, Bhattacharyya, Hubeny, Loganayagam, Mandal, Minwalla, Morita, Rangamani, Reall, Bredberg, Keeler, Lysov, Strominger…

Let us go back to chaos

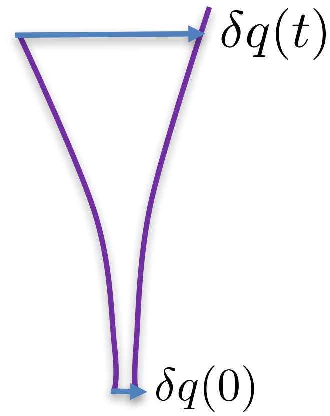
Chaos → divergence of nearby trajectories

Thermal system → average over all trajectories

Growth → Where you are after the perturbation vs. where you would have been.



Classical



$$\delta q(t) \propto e^{\lambda t}$$

$$\frac{\delta q(t)}{\delta q(0)} = \{q(t), p(0)\} \ , \qquad \{q(t), p(0)\}^2$$

Quantum

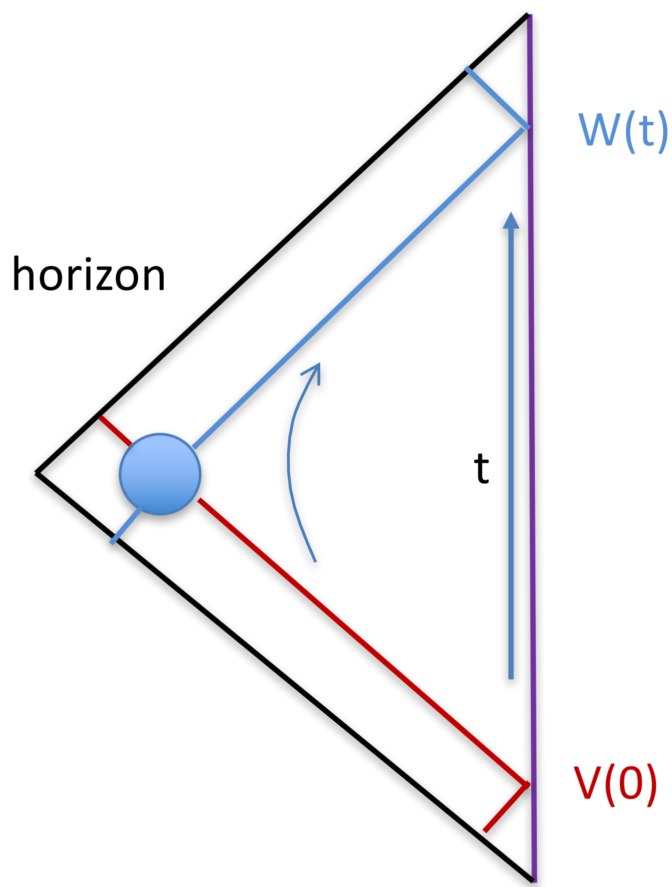
$$[Q(t), P(0)]^2$$

Quantum General:

$$\langle [W(t), V(0)]^2 \rangle_\beta \propto \frac{1}{N} e^{\lambda t}$$

W, V are two ``simple'' (initially commuting) observables.
Imagine we have a large N system. This is the definition of the quantum Liapunov exponent

For quantum systems that have a gravity dual



Commutator → involves the scattering amplitude between these two excitations.

Leading order → graviton exchange

Large t → large boost between the two particles.

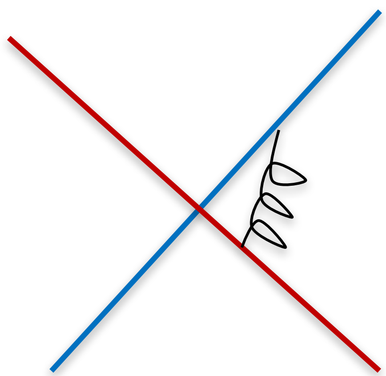
Gravitational interaction has spin 2, Shapiro time delay proportional to energy.

Energy goes as e^t

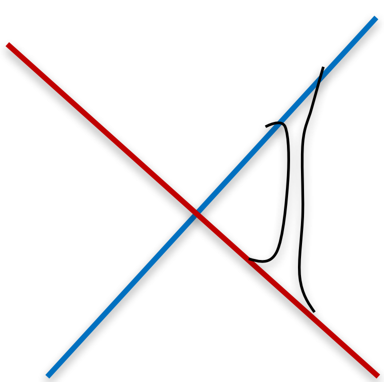
$$\langle [W(t), V(0)]^2 \rangle_\beta \propto \frac{1}{N} e^{\frac{2\pi}{\beta} t}$$

$$\lambda = \frac{2\pi}{\beta} = 2\pi T$$

Can it be different ?



Graviton → phase shift : $\delta(s) \sim G_N s \longrightarrow \lambda = \frac{2\pi}{\beta}$



String → phase shift

$$\delta(s) \sim G_N s^{1+\alpha' t} \longrightarrow \lambda = \frac{2\pi}{\beta}(1 - \frac{l_s^2}{R^2})$$

Typical size of string
(of graviton in string theory)

Radius of curvature of black hole

s, t = Mandelstam invariants

It can be less…

More ?

In flat space a phase shift has to scale with a power of s less than one in order to have a causal theory

Maybe there is a bound…

Black holes as the most chaotic systems

$$\lambda \leq \frac{2\pi}{\beta} = 2\pi T$$

Sekino Susskind

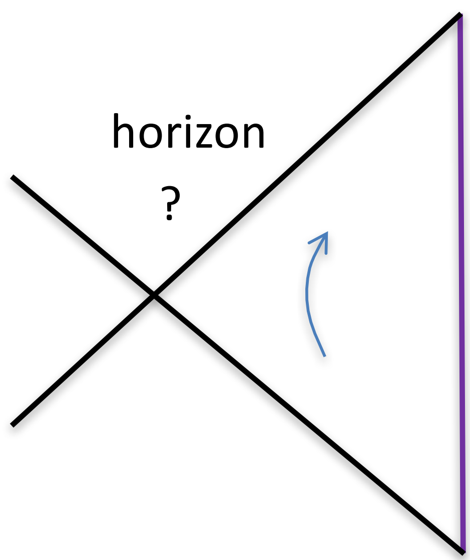
JM, Shenker, Stanford

For <u>any</u> large N (small hbar) quantum system.

(Strings connect weakly coupled to strongly coupled systems)

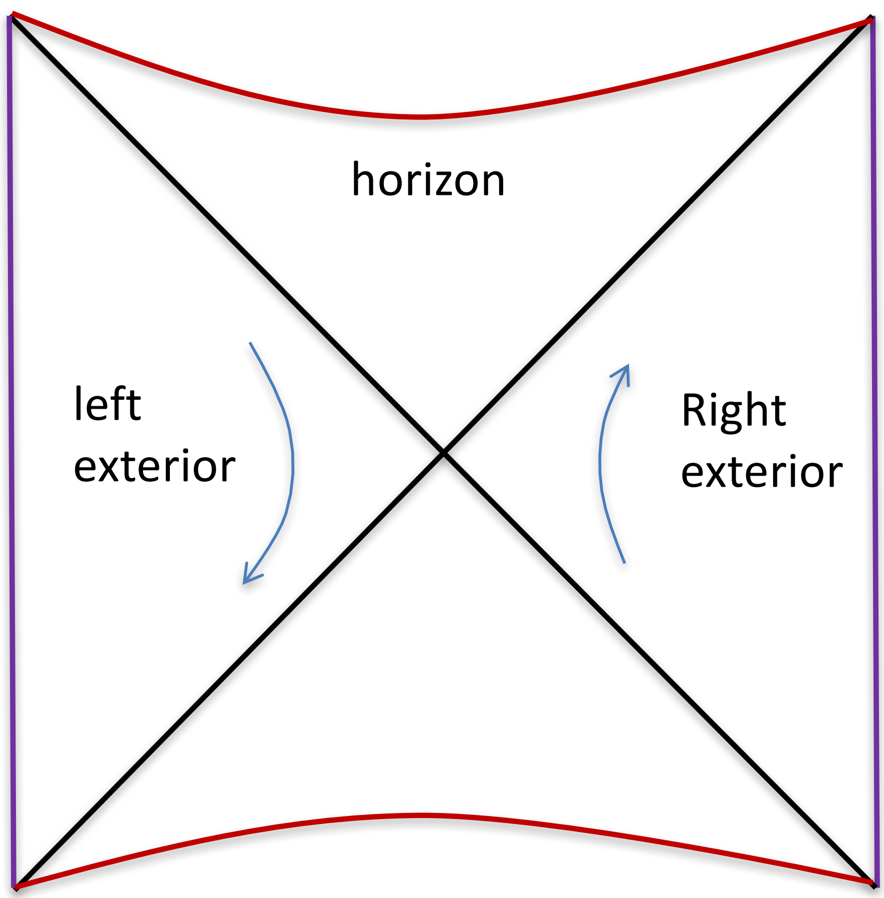
How do we get order from chaos ?

How do we get the vacuum from chaos, or from a chaotic quantum system?



Example: hydrodynamics, we get something simple for some interactions, but it is more complicated with very small interactions (Boltzman equation).

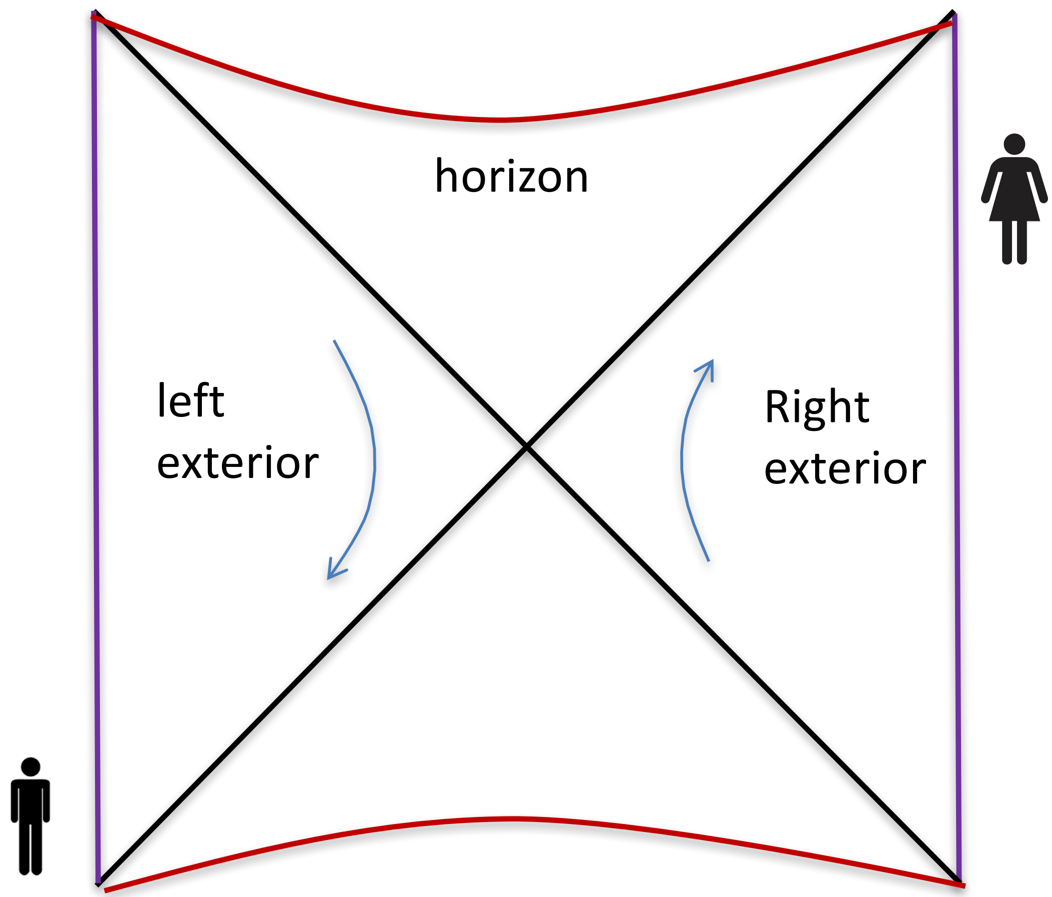
The full Schwarzschild wormhole

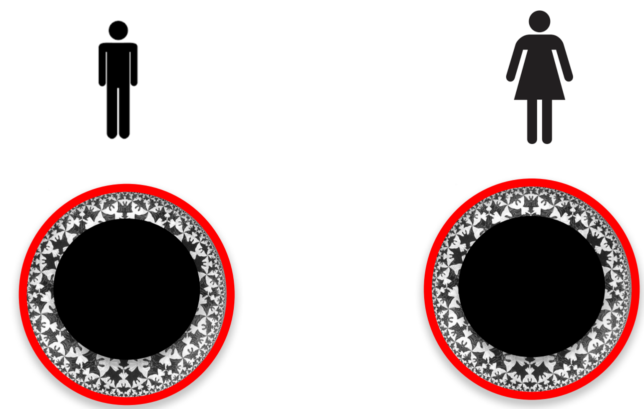


No need to postulate any exotic matter

No matter at all !

View it as an entangled state

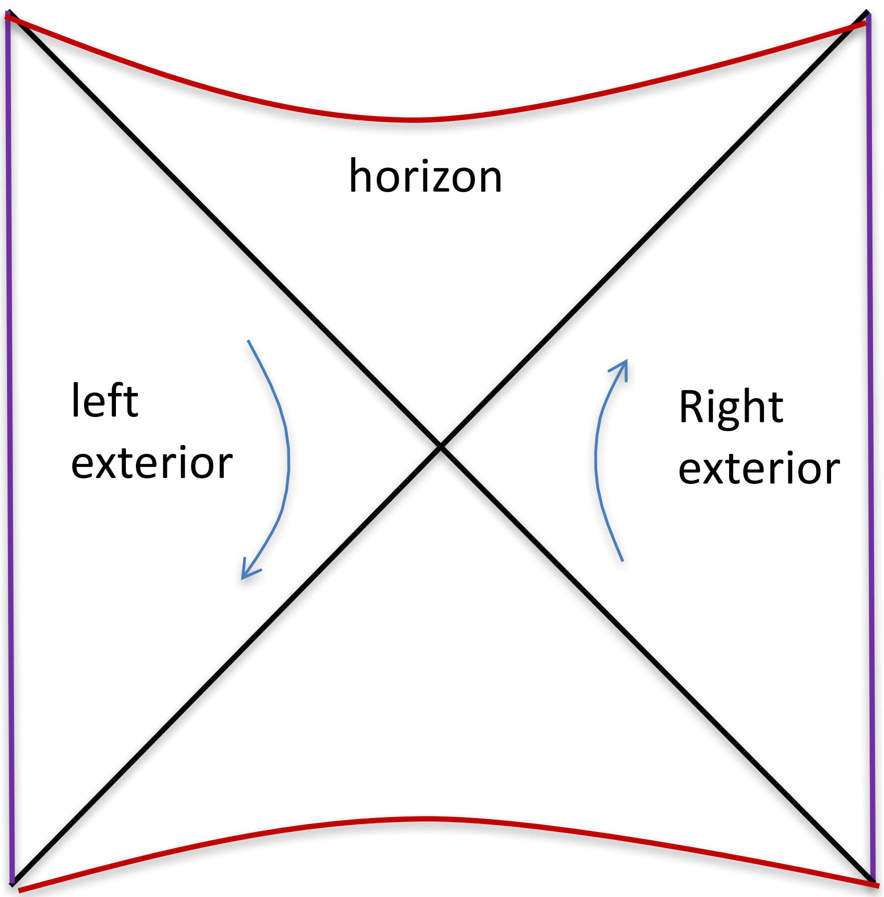




$$|TFD\rangle = \sum_n e^{-\beta E_n/2} |\bar{E}_n\rangle_L |E_n\rangle_R$$

Israel 70's
JM 00's

Symmetry



What is this funny
"time translation symmetry" ?

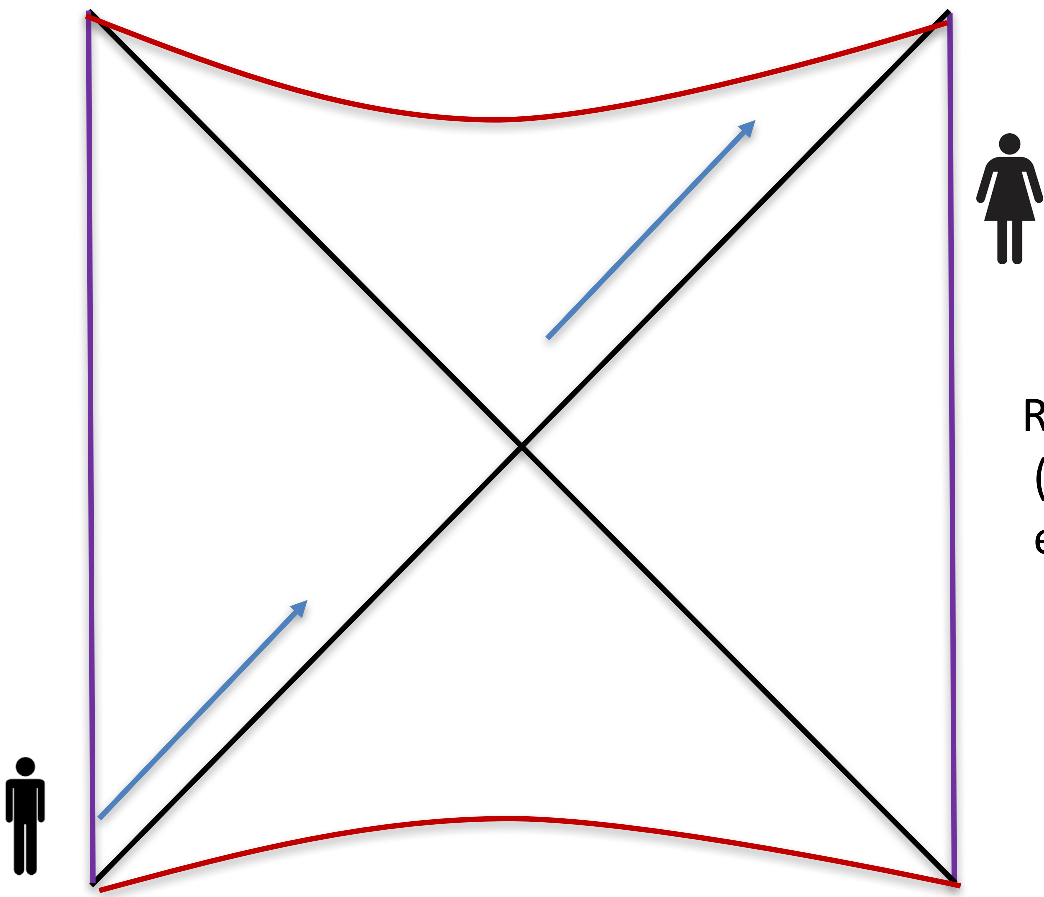
$$U = e^{it(H_R - H_L)}$$

Exact symmetry
Exact boost symmetry!

This symmetry should be exact
for any theory of quantum gravity

$$|TFD\rangle = \sum_n e^{-\beta E_n/2} |\bar{E}_n\rangle_L |E_n\rangle_R$$

True causal separation



If Bob sends a signal , then
Alice cannot receive it.

These wormholes are not traversable

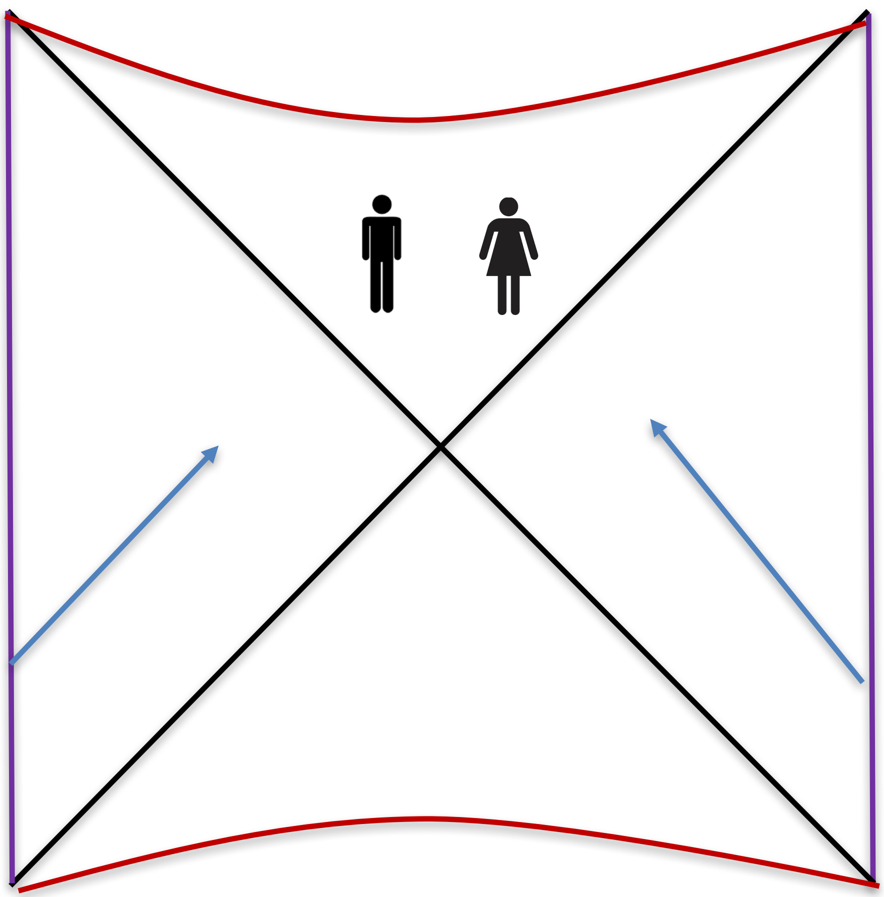
Relay on Integrated null energy condition
(Now a theorem, proven using
entanglement methods) Balakrishnan, Faulkner, Khandker, Wang

Not good for science fiction.

Good for science!

$$|TFD\rangle = \sum_n e^{-\beta E_n/2} |\bar{E}_n\rangle_L |E_n\rangle_R$$

Interior is common

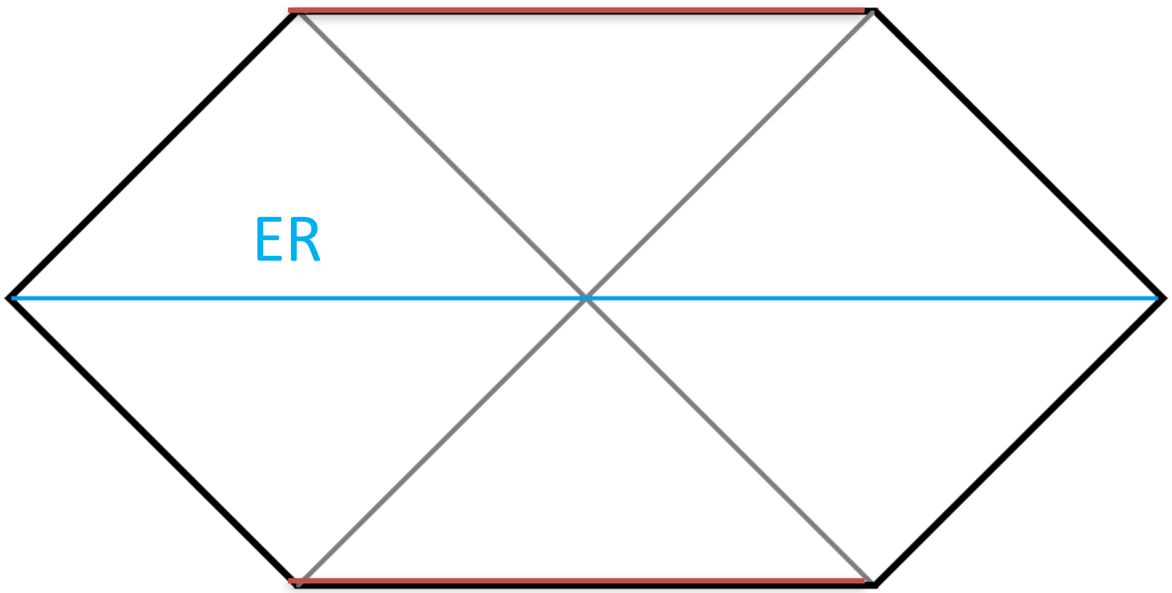


If they jump in,
they can meet in the interior !

But they cannot tell anyone.

$$|TFD\rangle = \sum_n e^{-\beta E_n/2} |\bar{E}_n\rangle_L |E_n\rangle_R$$

Spacetime connectivity from entanglement



ER = EPR

Van Raamsdonk
Verlinde^2
Papadodimas Raju

JM Susskind

Entanglement and geometry

Local boundary quantum bits are highly interacting and very entangled

Ryu, Takayanagi, Hubeny, Rangamani

$$S(R) = \frac{A_{\min}}{4G_N}$$

Generalization of the Black hole entropy formula.

Interesting connections to quantum information theory

Quantum error correction — Almheiri, Dong, Harlow, Preskill, Yoshida, Pastawski

Complexity theory — Harlow, Hayden, Brown, Susskind

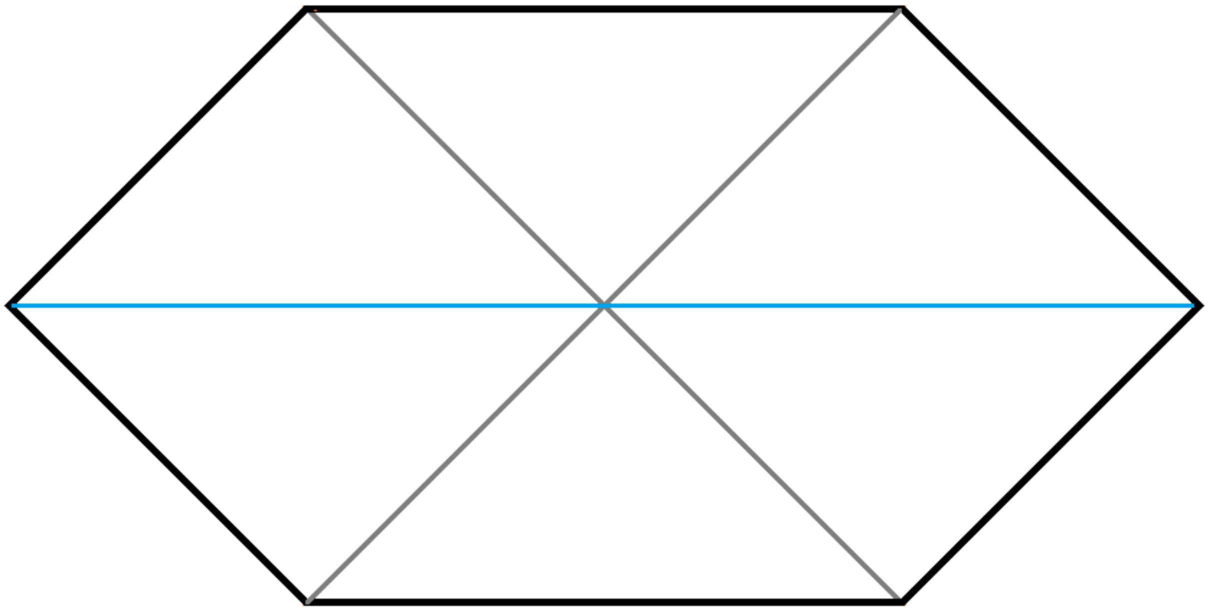
Conclusions

- Black holes are extreme objects: most compact, most efficient energy conversion, most entropic, most chaotic,…
- Most confusing...
- The process of unravelling these confusions has lead to better understand of gravity, quantum systems, string theory and their interconnections.
- Black holes are not only in the cosmos, but can also be present in the lab.
- And there are still very important confusions and open problems: Interior and singularity ?

Thank you!

Extra slides

Full Schwarzschild solution

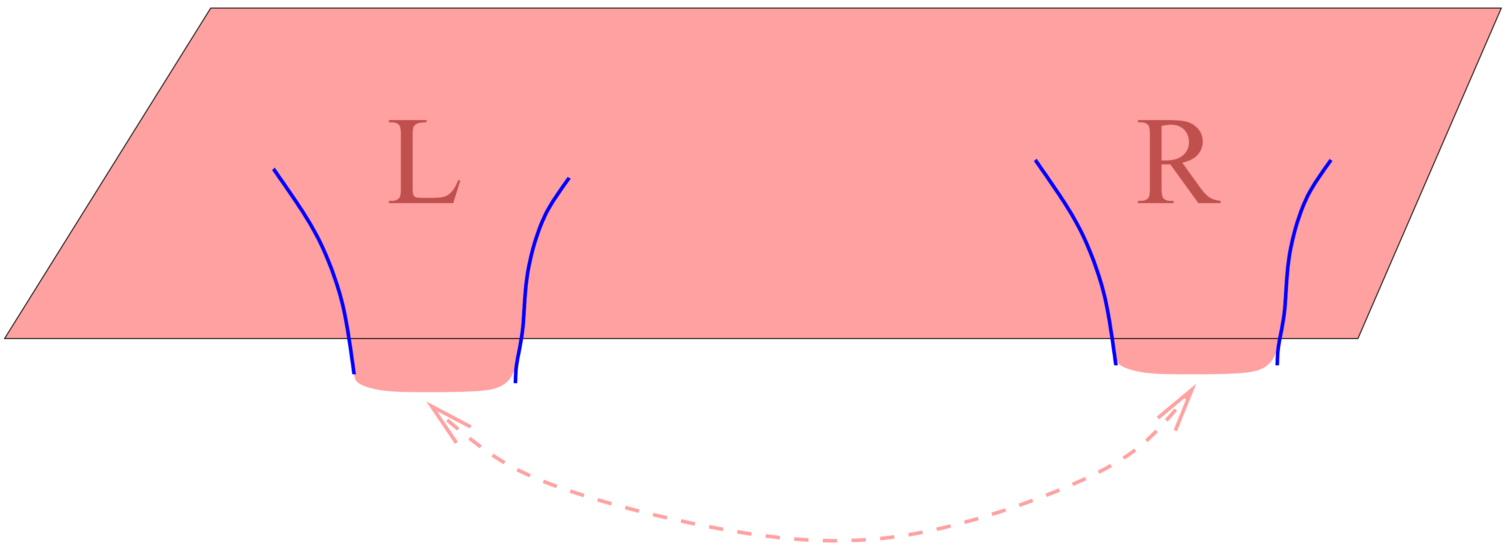


Eddington, Lemaitre,
Einstein, Rosen,
Finkelstein
Kruskal

Simplest spherically symmetric solution of pure Einstein gravity
(with no matter)

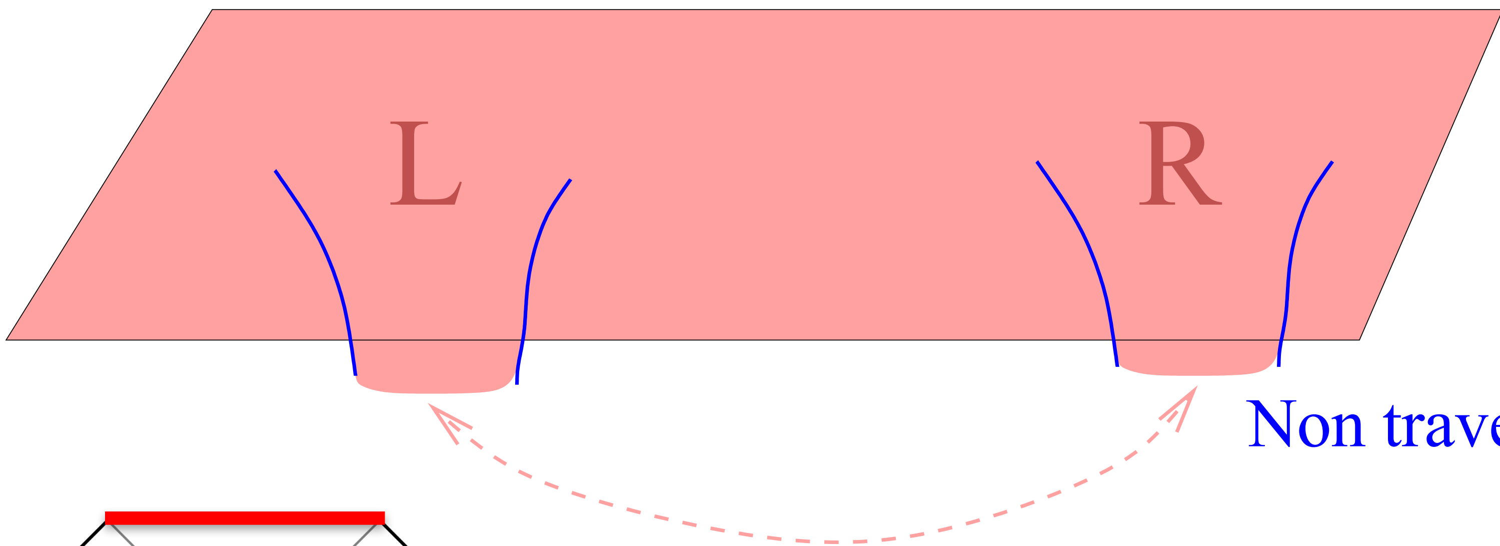
Wormhole interpretation.

Wormhole interpretation.



Note: If you find two black holes in nature, produced by gravitational collapse, they will not be described by this geometry

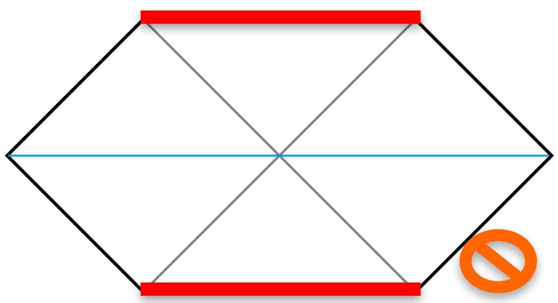
No faster than light travel



Non travesable

No signals

No causality violation



Fuller, Wheeler, Friedman, Schleich, Witt, Galloway, Wooglar

In the exact theory,
each black hole is described by a set of microstates from the outside

Wormhole is an entangled state

$$|\Psi\rangle = \sum_n e^{-\beta E_n/2} |\bar{E}_n\rangle_L \times |E_n\rangle_R$$

EPR

Israel
JM

Geometric connection
from entanglement. ER = EPR

Susskind JM

Stanford, Shenker, Roberts, Susskind

MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

EPR

JULY 1, 1935 PHYSICAL REVIEW VOLUME 48

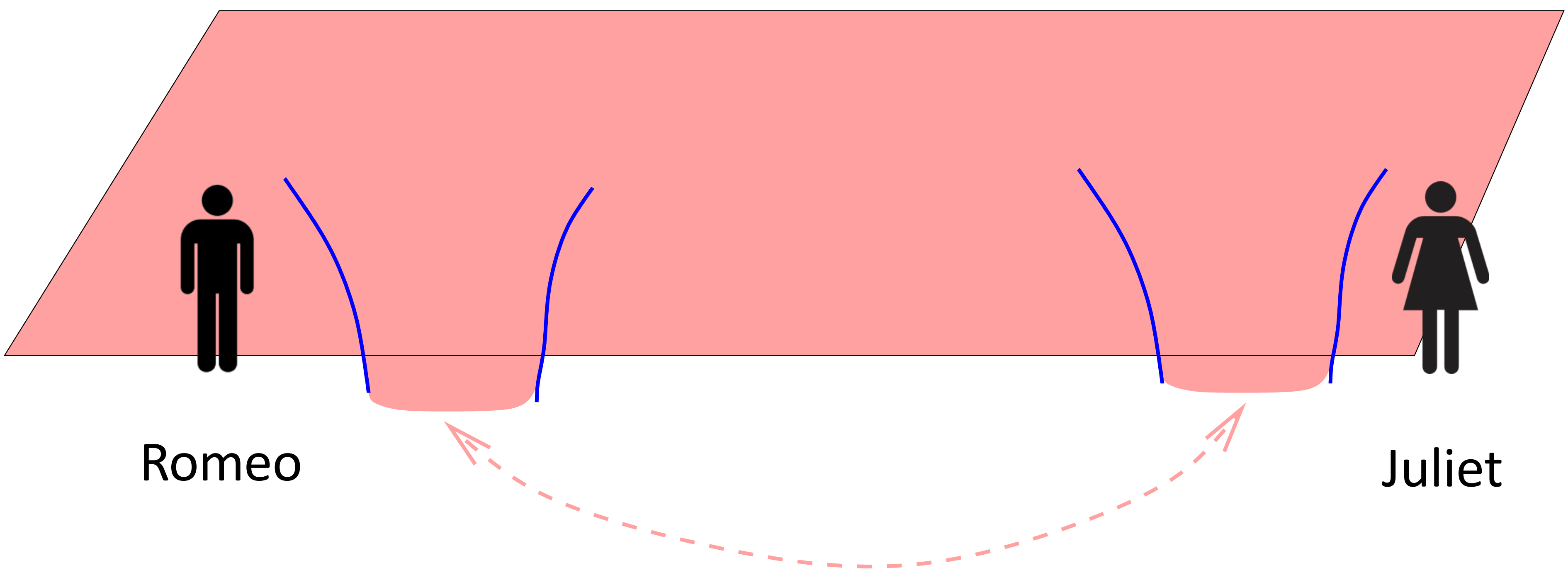
The Particle Problem in the General Theory of Relativity

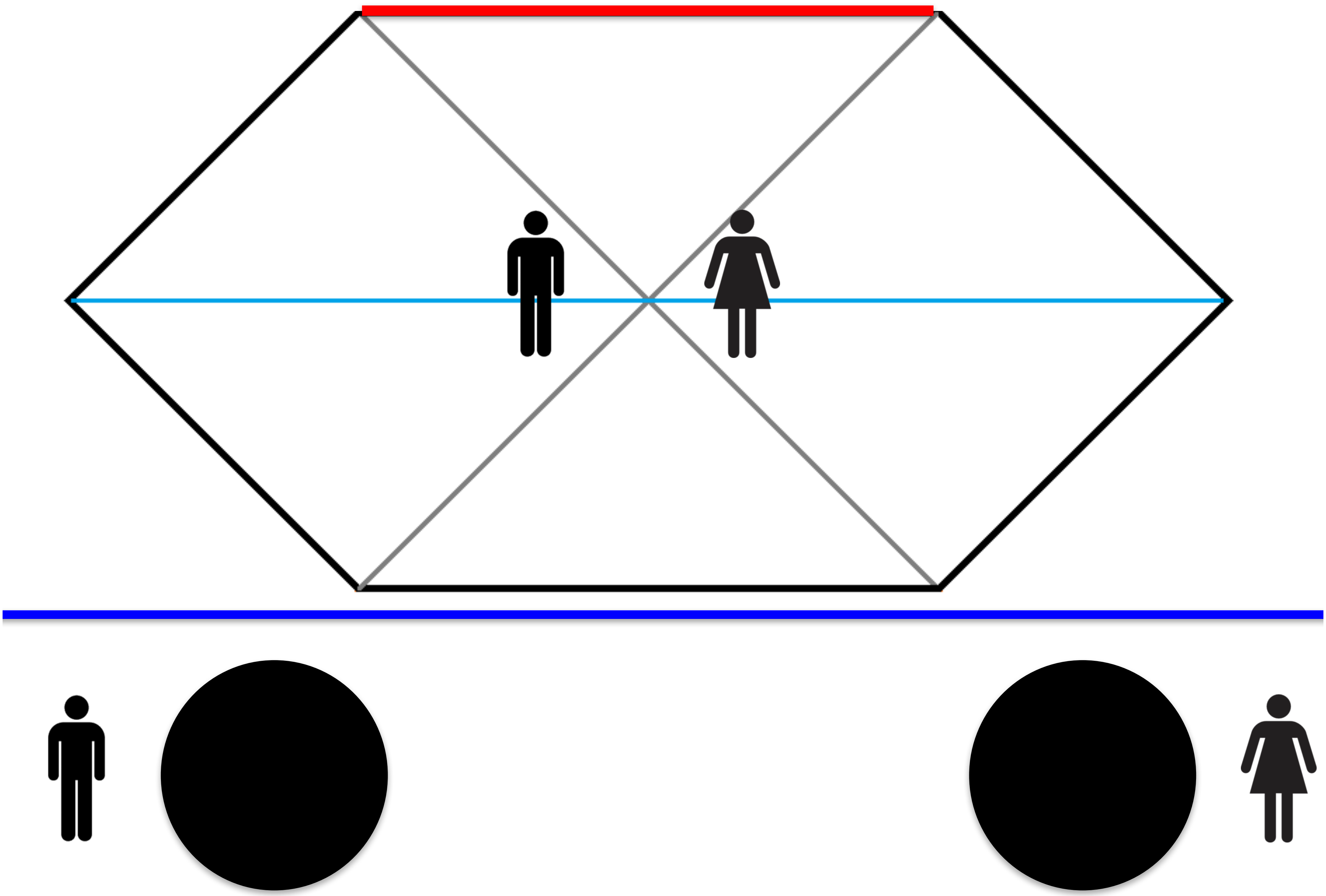
A. EINSTEIN AND N. ROSEN, *Institute for Advanced Study, Princeton*

(Received May 8, 1935)

ER

A forbidden meeting





String theory

- String theory started out defined as a perturbative expansion.
- String theory contains interesting solitons: D-branes.

Polchinski

- Using D-branes one can ``count'' the number of states of extremal charged black holes in certain superstring theories.

Strominger Vafa

- D-branes inspired some non-perturbative definitions of the theory in some cases.

Matrix theory: Banks, Fischler, Shenker, Susskind
Gauge/gravity duality: JM, Gubser, Klebanov, Polyakov, Witten

Entanglement and geometry

- The entanglement pattern present in the state of the boundary theory can translate into geometrical features of the interior.

Van Raamsdonk

- Spacetime is closely connected to the entanglement properties of the fundamental degrees of freedom.

- Slogan: Entanglement is the glue that holds spacetime together…

- Spacetime is the hydrodynamics of entanglement.

Questions

- Black holes look like ordinary thermal systems if we look at them from the outside. We even have some conjectured exact descriptions.
- How do we describe the interior within the same framework that we describe the exterior ?

Modern version of the information paradox;
Mathur, Almheiri, Marolf, Polchinski, Stanford, Sully,..

- Once we figure it out: what is the singularity ?
- What lessons do we learn for cosmology ?