The Diffusion Process of Mobile Telephony in Europe

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Sir Richard Stone (1913-1991) Premio Nobel de economía 1984, colaborador de J.M. Keynes durante la guerra, ha aportado a la economía los principios de la cuantificación rigurosa, desarrollando la contabilidad nacional y social, y ha sido pionero en el campo de la modelización macro y meso económica y de su utilización para la exploración y previsión de la evolución de la economía.

El Fondo de Investigación e Innovación Richard Stone (FIIRS) ha sido constituido para potenciar la actividad investigadora básica y aplicada y la difusión académica de sus resultados y facilitar así el pleno desarrollo de las carreras investigadoras en el Instituto L.R. Klein - Centro Stone.

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Abstract

The main goal of this paper is to describe the penetration process of mobile telephony across European countries in the eightieth and ninetieth decades. To achieve this goal we have revised different alternative specifications for the well-known diffusion models, starting with a simple logistic curve and moving through more flexible formulations like Nelder's model.

Since there is no "physical" or "technical" limit a priori which would mark the ceiling of penetration, needed in the estimation process, we have explored different ways to state that ceiling, including the Bass proposal to derive it from observed data.

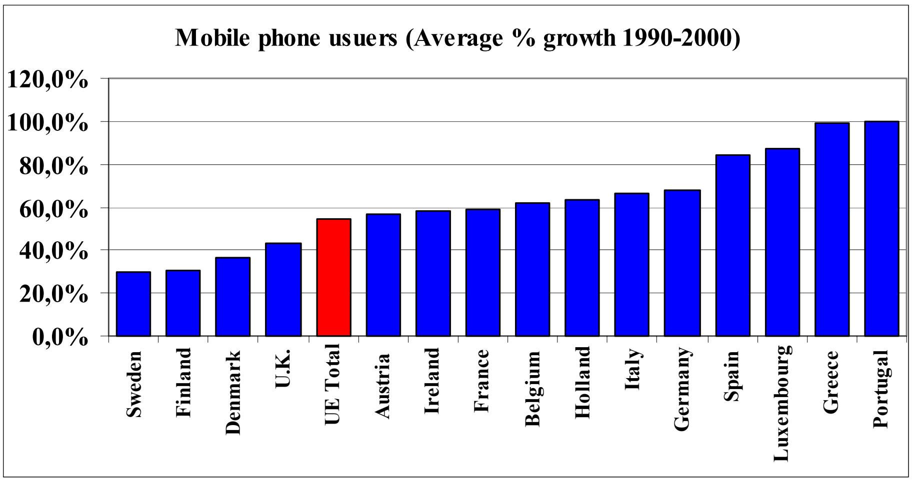
The main findings from the diffusion processes as estimated, apart from their asymmetry with respect to the inflexion point, is that they are very uniform across countries as regards the time when they reach that inflexion point (around 2000), implying that growth in mobile telephony markets started to tail off after that point.

Keywords: Diffusion Models, Mobile telephony**.**
JEL Code: C13; L96

1.- Introduction

The introduction of mobile telephony has accelerated spectacularly in most EU countries in recent years, having attained a 54% compound annual growth rate (CAGR) between 1990 and 2000, and exceeding over 100% growth per year in some cases, e.g. Portugal.

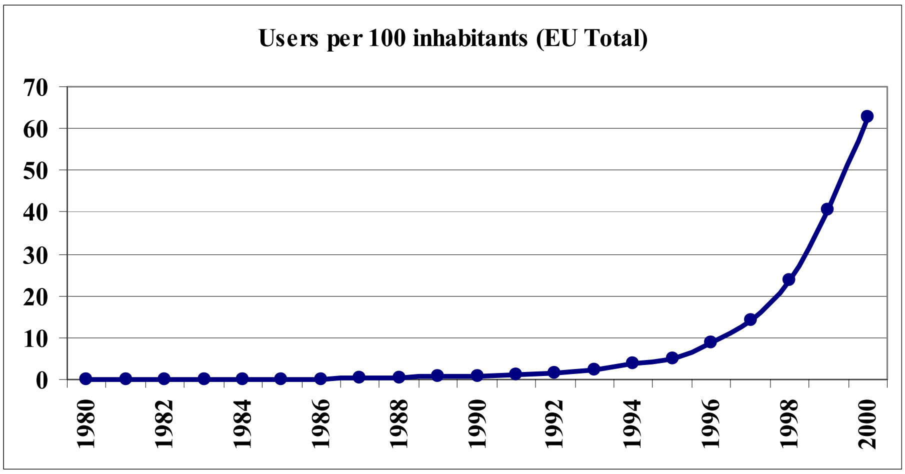
Figure 1



Source: Authors. Original data from Eurostat. New Cronos.

These rapid rates of growth, which are quite infrequent in the economic phenomena which are normally studied, appear to respond to a process of "contagion" in which the number of users increases in proportion to growth in the total number of users.

Figure 2



Source: Authors. Original data from Eurostat. New Cronos.

In fact, the number of mobile phone users per 100 inhabitants (i.e. the percentage of the population which has "caught the bug") increases exponentially: from under 10% with a mobile phone in 1995, the figure surged to 70% of Europeans in 2000 (Figure 2).

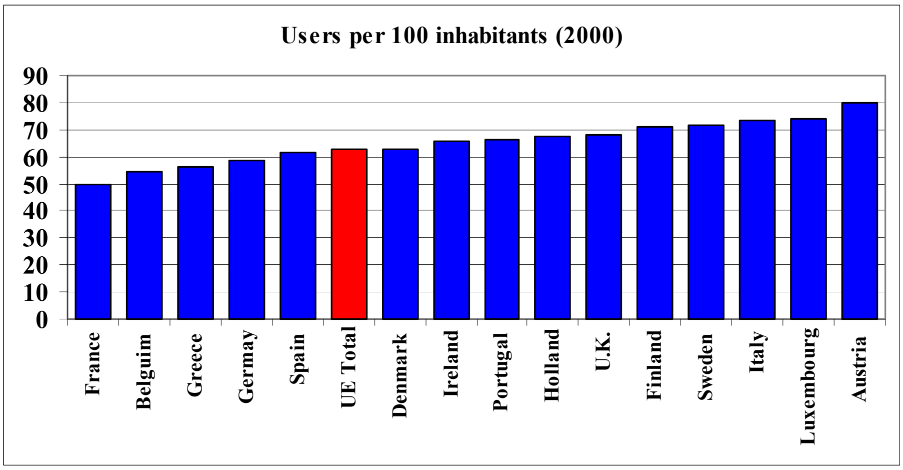
If we view the penetration of mobile telephony as an epidemiological phenomenon, it is reasonable to assume that the upper limit will be 100% of the population.

However, although the upper limit is initially set at 100%, other factors can modify it:

- Firstly, it is impossible to assume that the entire population are potential users; we must at least eliminate those under a certain age.
- There is also the possibility that a certain percentage of the population will have more than one handset (one for personal use, one for professional use).
- Moreover, replacement purchases may not be correctly reflected in the statistics.

Some analyses in this area suggest that the ceiling level is 80-90 per 100 inhabitants, which would mean that some countries (e.g. Austria) reached their ceiling in 2000 (Figure 3).

Figure 3



Source: Authors. Original data from Eurostat. New Cronos.

2.- Basic diffusion models

Having considered the phenomenon of mobile telephony penetration as a diffusion or contagion process, we will try to analyse the main methodologies proposed in the literature for modelling the phenomenon.

According to Mahajan and Peterson (1985), the general diffusion process can be represented by an equation of the following type:

$$\Delta Y_t = \varphi(t)\,(\delta - Y_t) \qquad [1]$$

where Y_t is the total number of users in a time period t, δ is the total number of potential users, and $\varphi(t)$ is a time function, generally referred to as the diffusion coefficient.

The time function that is defined by the diffusion coefficient can include both exogenous and endogenous factors associated with the status of volume of penetration at a given time, Y_t.

According to Vicéns (1990), the simplest models of the diffusion coefficient can be classified as shown in Table 1.

Table 1

General models of diffusion	
Model	**Diffusion coefficient**
External influence	$\varphi(t) = \alpha$
Internal influence	$\varphi(t) = \beta Y_t$
Combined influence	$\varphi(t) = \alpha + \beta Y_t$

In the first case, the externally-influenced diffusion model, the variation in the number of users (which can be identified with the sales of the product at a time t) depends only on the number of potential users who do not yet have the product (δ - Y_t.).

However, considering the nature of the product at issue, mobile phones, it is logical to assume some form of internal influence which, therefore, depends on the number of existing users since the usefulness of the product increases as the user base widens; accordingly, our analysis should focus on models of external or combined influence.

In the simplest approach, the internal diffusion model is expressed by an equation of the following type:

$$\Delta Y_t = \beta \, Y_t \, (\delta - Y_t) \qquad [2]$$

By integration, we get an associated function or accumulated trend of the type:

$$Y_t = \frac{\delta}{1 + \dfrac{\delta - y_0}{y_0} e^{-\beta \delta (t - t_o)}} \qquad [3]$$

where y_0 and t_0 are the initial conditions of the process.

Abstracting from the initial conditions of the process, this basic model of internal influence develops along a standard logistic curve with the well-known expression:

$$Y_t = \frac{\delta}{1 + e^{-(\alpha + \beta t)}} \qquad [4]$$

which, in differential form, is as follows:

$$\Delta Y_t = \frac{\beta}{\delta} Y_t \, (\delta - Y_t) \qquad [5]$$

Starting with the simplest expression of the internal influence diffusion models, which corresponds to an aggregate evolution of a logistic curve, we can make a first approach to the diffusion of mobile phones in Europe by estimating expression [2] econometrically using, as endogenous variable, the variation in the number of people with mobile phones and, as explanatory variable, the product of that same percentage and the difference between the maximum value of δ, set beforehand, and (again) the percentage of users.

$$TP_{i,t} - TP_{i,t-1} = \beta * TP_{i,t} * \left(\delta - TP_{i,t}\right) \qquad [7]$$

$$\Delta TP_{i,t} = \beta * XTP_{i,t} \qquad [8]$$

where $TP_{i,t}$ is the percentage of mobile phone users in country i at time t, $\Delta TP_{i,t}$ is the variation of that percentage and $XTP_{i,t}$ is the product of the percentage of the population that uses mobile phones and the percentage which does not (potential users).

Although we will revisit the problem of determining the ceiling values, in this first approach we set a maximum of 0.9, assuming that part of the under-age population is offset by users with more than one handset.

Table 2 shows the basic results of the estimates performed for the entire period available in each of the European Union countries, plus the aggregate total, sorted in order of diffusion coefficient β.

Most of the regressions in Table 2 show quite a good overall fit (apart from Luxembourg) and the coefficients are significant in all cases, showing diffusion coefficients (i.e. speed of fit of the process) ranging from 0.541 (Sweden) to 1.320 (Austria).

Table 2

Estimates of the simple internal diffusion model

	Period	β	Student's T (β)	R^2	D.W.
Sweden	1981-2000	0.541	14.653	0.863	1.235
Denmark	1982-2000	0.562	11.447	0.811	2.125
Finland	1981-2000	0.570	14.856	0.871	1.003
Luxembourg	1985-2000	0.913	3.524	0.332	3.189
France	1986-2000	0.950	17.152	0.937	1.658
UK	1985-2000	0.962	11.476	0.859	0.743
Italy	1985-2000	0.981	39.733	0.986	2.731
Portugal	1989-2000	1.017	17.764	0.946	1.783
Total EU	**1981-2000**	**1.021**	**32.161**	**0.977**	**0.424**
Ireland	1985-2000	1.061	42.518	0.989	2.248
Greece	1994-2000	1.194	11.503	0.914	3.135
Belgium	1981-2000	1.195	49.825	0.991	1.450
Spain	1986-2000	1.225	11.558	0.877	2.582
Holland	1985-2000	1.254	25.052	0.969	2.090
Germany	1985-2000	1.303	11.245	0.868	1.213
Austria	1984-2000	1.320	28.457	0.975	0.210

Source: Authors.

However, if we analyse these estimates in more detail, we find that, with some exceptions, the Durbin-Watson statistic shows specification problems that manifest themselves in the form of high correlations in the residuals.

Mahajan and Peterson (1985) identified a number of problems arising with these simple diffusion models, many of which may be affecting our analysis and leading to the aforementioned specification problems:

- Simple models assume that the process of adaptation to a new product is fixed and they do not consider the possibility of changes or stages in this process. In the case at hand, there were changes in the technology (from analogue to GSM), which means that the products were not entirely uniform over time.
- The saturation level is taken to be constant and known or susceptible to estimation, no change in system size being admissible. In the case at hand, the saturation level can only be approximated as the real limit of potential users is unknown.
- They do not take account of repeat purchases, i.e. all handset sales are assumed to correspond to new subscribers, when they may actually be replacements.
- The coefficient of diffusion or internal influence, β, is assumed to be constant and not to vary with the level of the process or the passage of time.
- No account is taken of the possible influence of external variables such as advertising, promotion plans or the addition of new services (i.e. WAP).

Most of these problems are related to rigidities implicit in the evolution pathways of the simple diffusion models, and the literature contains numerous alternative proposals which seek to make these initial assumptions more flexible [e.g. Vicéns, 1999)].

3.- Estimating the maximum levels of diffusion

One of the first problems in estimating diffusion models lies in determining the maximum or saturation levels, which are generally assumed to be known or susceptible to estimation.

In the case at hand, determining that ceiling level (tentatively set at 0.9, i.e. 90 users per 100 inhabitants) presents a number of specific problems since, on the one hand, we cannot assume that the entire population are potential users and, on the other hand, there may be users with more than one subscription (e.g. for professional and personal use).

In a tentative first approach, we can assume that the maximum limit is given by an expression of the type:

$$\delta = 1 - q_a + 2 * \gamma \ (1 - q_a) \quad = (1 + 2\gamma) * (1 - q_a) \qquad [9]$$

where q_a is the percentage of the population below a certain age and γ is the percentage of the population that might have two subscriptions.

Determining the population below a certain age presents no major problems: we simply define the threshold age for becoming a potential user. Estimating the number of users that could have more than one subscription is more problematic and is always an exercise in speculation, to a greater or lesser extent.

In view of these difficulties, an alternative approach based on the available statistics can be considered.

For example, in the general expression of the simple internal influence model [2], we can re-write the expression of the difference as a function of penetration at any time t:

$$\Delta Y_t = \beta\, Y_t\, (\delta - Y_t) = \beta\, \delta\, Y_t \;\; - \beta\, Y_t^2 \qquad [10]$$

changing variable and using λ as the product of β and δ, we get a new expression of the type:

$$\Delta Y_t = \lambda\, Y_t \;\; - \beta\, Y_t^2 \qquad [11]$$

where, once λ and β are known, we can estimate the ceiling value δ as the simple ratio between the two:

$$\delta = \frac{\lambda}{\beta} \qquad [12]$$

in a first approach, we can try to delimit the value of the unknown coefficients, λ and β, by a regression model in which the difference in the level of penetration is made dependent on that level and on its square through a formula of the type:

$$\Delta Y_t = \alpha_1\, Y_t \;\; + \alpha_2 Y_t^2 + \varepsilon_t \qquad [13]$$

calculating the saturation, δ, as the ratio between the estimated coefficients:

$$\hat{\delta} = \frac{\hat{\alpha}_1}{-\hat{\alpha}_2} \qquad [14]$$

However, if we consider that the two regression coefficients in this model are highly correlated, the results contain a bias which leads to conclusions that are not significant, as shown in the table below, which presents the estimated results for each of the coefficients and the saturation level deduced in each case.

Table 3

Direct estimates of the maximum penetration			
Country	$\hat{\alpha}_1$	$\hat{\alpha}_2$	δ
Germany	0.335	2.480	-
Austria	1.056	-0.988	1.069
Belgium	0.891	-0.463	1.926
Denmark	0.402	-0.262	1.531
Spain	1.117	-1.274	0.877
Finland	0.621	-0.805	0.772
France	1.222	-2.246	0.544
Greece	0.768	0.007	--
Holland	1.220	-1.520	0.803
Ireland	0.995	-1.177	0.846
Italy	0.840	-0.872	0.963
Luxembourg	1.985	-3.397	0.584
Portugal	1.080	-1.459	0.740
UK	0.644	-0.283	2.273
Sweden	0.356	-0.208	1.707
Total EU	**0.735**	**-0.422**	**1.740**
Source: Authors.			

One possibility for avoiding the problem of correlation between regression coefficients would be to divide both sides of equation [13] by Yt and estimate an alternative model of the type:

$$\frac{\Delta Y_t}{Y_t} = \alpha_1 + \alpha_2 Y_t + \eta_t \qquad [15]$$

However, the new random perturbance creates a non-constant variance and we must perform an estimate by Weighted Least Squares using Y_t as the weighting variable, which leads back to the same results as in the previous table.

Bass (1969) proposed a mixed influence diffusion model which allows for direct estimation of both the diffusion coefficients and the saturation level, based on an expression of the following type:

$$\Delta Y_{t+1} = (\alpha + \frac{\beta}{\delta} Y_t)\,(\delta - Y_t) \qquad [16]$$

operating on the foregoing expression as before, we get:

$$\Delta Y_{t+1} = \alpha\delta - \alpha Y_t + \frac{\beta}{\delta} Y_t \delta - \frac{\beta}{\delta} Y_t Y_t = \alpha\delta + (\beta - \alpha) Y_t - \frac{\beta}{\delta} Y_t^2 = aY_t^2 + bY_t + c \qquad [17]$$

Bass proposes estimating the foregoing expression by Ordinary Least Squares and determining the saturation level δ by setting the increment of the variable to 0, i.e. when:

$$aY_t^2 + bY_t + c = 0 \qquad [18]$$

so that the saturation level δ is obtained by finding the roots of a second-order polynomial:

$$\delta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad [19]$$

Again, this method leads to unacceptable results, as shown in Table 3: the saturation limit is well above 100% of the population in some cases.

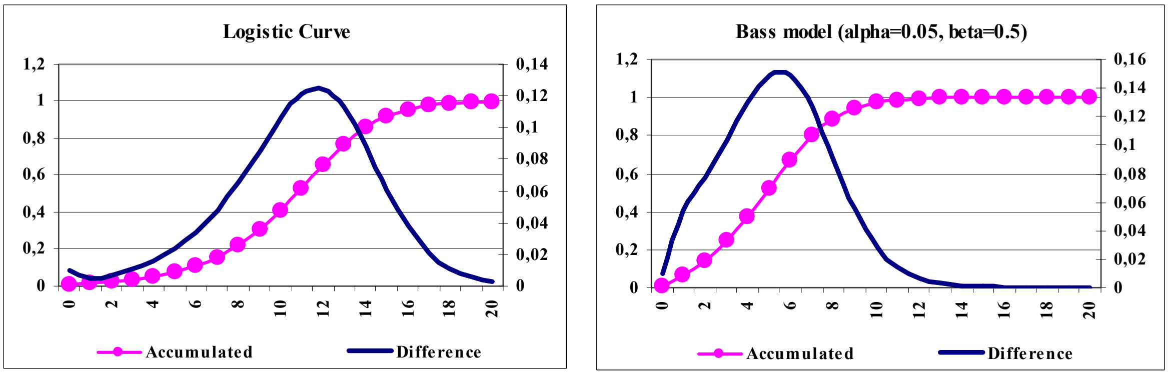
Table 4

Direct estimates of the saturation level Bass model				
	a	**b**	**c**	**δ**
Germany	2.579	0.301	0.002	-
Austria	-1.157	1.156	-0.008	0.992
Belgium	-0.490	0.901	-0.001	1.838
Denmark	-0.236	0.387	0.001	1.642
Spain	-1.320	1.139	-0.002	0.861
Finland	-0.940	0.719	-0.011	0.749
France	-2.385	1.278	-0.004	0.533
Greece	0.465	0.597	0.011	-
Holland	-1.707	1.316	-0.007	0.765
Ireland	-1.267	1.043	-0.004	0.819
Italy	-0.877	0.842	0.000	0.961
Luxembourg	-3.593	2.106	-0.010	0.581
Portugal	-1.409	1.052	0.003	0.749
UK	-0.457	0.736	-0.008	1.602
Sweden	-0.194	0.346	0.001	1.791
Total EU	**-0.490**	**0.766**	**-0.002**	**1.561**
Source: Authors.				

Since these results bear little resemblance to those which were expected *a priori*, we must ascertain the reasons for the anomaly.

Analysing the intrinsic properties of the models used for direct estimation of the saturation levels, we find that both the logistic model (used in the first approach) and the Bass model (used in the second) present inflexion points at around 50% of the ceiling values, in the first case, and between 0% and 50%, in the second case (Figure 4).

Figure 4



Source: Authors.

In other words, both models assume that the differentiated variable (i.e. addition of new users) peaks when the accumulated variable is at most 50% of the ceiling value.

However, if we analyse actual trends in the addition of mobile telephony users during the period in question (Annex 1), we find that, apart from Finland, no such maxima were reached. This implies, based on the models being used, that saturation will be attained at levels which are more than double the values registered in the last year, i.e. clearly higher than what can be considered as "admissible".

In view of the evidence, we could reconsider the estimate for the ceiling or saturation level, using some of the alternative models offered by the literature, which present inflexion points above 0.5 (like the actual data).

Vicéns (1992) published a table of the main diffusion models with their basic features as regards inflexion point and symmetry. The table lists the following alternative models which admit inflexion points above 50% of the saturation value: Bertalanffy (1957), Nelder, reported in McGowan (1986) and Flog (1988).

Table 5

Alternative specifications with variable inflexion point				
Model	Accumulated (1)	Differences (1)	Inflexion point	Symmetry
Bertalanffy (1957)	$Y_t = \left(1 - e^{-(\alpha+\beta\, t)}\right)^{1/(1-\theta)}$	$\Delta Y_t = \dfrac{\beta}{1-\theta} Y^{\theta}\left(1 - Y^{1-\theta}\right)$	0-1	Yes or No
NELDER (1986)	$Y_t = \dfrac{1}{\left(1 + \theta\, e^{-(\alpha+\beta\, t)}\right)^{1/\theta}}$	$\Delta Y_t = \beta\, Y\left(1 - Y^{\theta}\right)$	0-1	Yes or No
FLOG (1988)	$Y_t = \dfrac{1}{\left(1 + e^{-(\alpha+\beta\, t(\mu,k)}\right)}$ For: $\mu \neq 0;\ k = 0;\ t(\mu,k) = \dfrac{e^{\mu\, t} - 1}{\mu}$	$\Delta Y_t = \beta\left[(1 + k\, t)^{1/k}\right]^{\mu-k}$	0-1	Yes or No
(1) Expressions for normalized variables with ceiling equal to 1. Source: Vicéns (1992).				

Using the Nelder model because of the simplicity of its approach, we can specify the equation in differences, this time using the expression with a generic upper limit δ, as:

$$\Delta Y_t = \beta\, Y_t\, (\delta - Y_t^{\theta}) \quad [20]$$

again, developing expression [20] like the previous ones, we can redefine it as:

$$\Delta Y_t = \beta\delta\, Y_t - \beta\, Y_t Y_t^{\theta} = \beta\delta\, Y_t - \beta\, Y_t^{\theta+1} \qquad [21]$$

again replacing the product $\beta\ \delta$ with a new variable λ, we can perform an indirect estimate by using an expression of the following type (this time, estimated by non-linear methods):

$$\Delta Y_t = \lambda\ Y_t - \beta\, Y_t^{\theta+1} \quad [22]$$

Table 6 presents the estimated values for each of the three parameters in expression [22] and the calculation of the upper limit, δ, that is derived from them for each country.

Apart from the special case of Greece (which provides anomalous results, as in the two previous estimates), the values look considerably more congruent and it could be accepted, without forfeiting generality, that the maximum limit of penetration of mobile telephony is over 90%, and that the reference obtained by finding the average values for the European Union as a whole is around 0.96.

Table 6

Direct estimates of the maximum penetration levels Nelder model				
	β	λ	θ	$\delta=\lambda/-\beta$
Germany	-41.481	42.928	-0.008	1.035
Austria	34.688	-34.250	-0.006	0.987
Belgium	9.022	-8.360	-0.008	0.927
Denmark	4.884	-4.664	-0.016	0.955
Spain	29.287	-28.854	-0.008	0.985
Finland	30.572	-30.530	-0.007	0.999
France	49.510	-49.442	-0.008	0.999
Greece	0.008	0.736	-0.933	-
Holland	43.503	-43.147	-0.006	0.992
Ireland	33.966	-33.670	-0.007	0.991
Italy	25.097	-24.851	-0.009	0.990
Luxembourg	80.028	-80.310	-0.010	1.004
Portugal	-2.595	2.615	0.212	1.008
UK	16.103	-15.581	-0.001	0.968
Sweden	5.737	-5.530	-0.011	0.964
Total EU	**11.472**	**-10.950**	**-0.005**	**0.955**
Source: Authors.				

If we accept the 0.96 figure as valid, once we have established the threshold age for potential users, we can estimate the implicit value of the population that is likely to have two handsets (γ) by solving for γ in expression [9].

$$\gamma = \frac{\dfrac{\delta}{(1-q_a)} - 1}{2} \qquad [23]$$

Table 7 shows the percentage of population that is excluded, q_a, for different threshold ages and the values which should be attained by the multiple subscription coefficient γ, taking 0.96 as the valid ceiling.

Table 7

Estimates of excluded population and double subscription rates (Data for the average of the period 1990-2000)		
Threshold age	$\mathbf{q_a}$	$\boldsymbol{\gamma}$**.**
5 years	0.06	0.01
9 years	0.12	0.04
14 years	0.18	0.08
19 years	0.24	0.13
Source: Authors.		

Taking a threshold age of 14 (which is quite plausible given the current diffusion of mobile phones among teenagers), we must accept that 8% of the population could have two subscriptions.

4.- Estimating diffusion models

Having determined the maximum diffusion value, we can estimate the models for each country by first using the simplest specification (i.e. the logistic model) shown in expressions [7] and [8].

The results of estimating this model are shown in Table 7: they are very similar to those in Table 2, in which a diffusion limit of 0.9 was established.

Table 8

Results of estimating the simple model with δ=0.96					
	Period	β	Student's T (β)	R^2	D.W.
Sweden	1981-2000	0.486	15.814	0.881	1.376
Finland	1981-2000	0.496	13.320	0.842	0.951
Denmark	1982-2000	0.508	11.694	0.818	2.202
Luxembourg	1985-2000	0.785	3.391	0.308	3.150
France	1986-2000	0.863	16.234	0.930	1.632
UK	1985-2000	0.874	11.943	0.869	0.824
Italy	1985-2000	0.876	41.832	0.988	2.696
Portugal	1989-2000	0.911	16.692	0.939	1.598
Total EU	**1981-2000**	**0.928**	**35.955**	**0.982**	**0.412**
Ireland	1985-2000	0.956	38.983	0.987	2.383
Greece	1994-2000	1.094	11.820	0.919	3.192
Belgium	1981-2000	1.095	55.674	0.993	1.481
Spain	1986-2000	1.113	11.527	0.876	2.649
Holland	1985-2000	1.129	23.826	0.966	2.230
Austria	1984-2000	1.181	31.583	0.979	0.231
Germany	1985-2000	1.201	11.598	0.876	1.217
Source: Authors.					

Comparing the newly-estimated figures with those in table 2, we find that they are very similar in general terms, with imitation coefficients ranging from 0.49 (Sweden) to 1.20 (Germany), all coefficients being statistically significant and there being a certain relative improvement in the general fit.

However, the specification problems outlined in the first estimate are still present and, although many of the D.W. statistics have improved slightly, the aforementioned problems do not appear to have been overcome, suggesting that they are not directly related to the value of the asymptotic upper limit δ.

Analysing the results of those estimates in greater detail, we observe two phenomena which recur in most of the equations and are reflected in the figures on the following pages:

- The models generally overestimate in the central part of the sample and underestimate in the extremes, suggesting that the estimated curve is smoother than actual observation (Figure 5).
- The estimated coefficients are not stable over the sample; rather, they rise as we approach the present day (Figure 6). This structural change was confirmed in all cases with recursive residual contrasts of the CUSUM-SQ type.

The R in **RET** series refers to the residues of estimation of the simple diffusion model with δ=0.96, and the letters **TP** make reference to the total subscribers as a percentage of population.

The central part of the name reflects the countries in which the estimate was made:

EU	European Union total	LU	Luxembourg
BE	Belgium	NL	Holland
DK	Denmark	AU	Austria
GR	Greece	PO	Portugal
ES	Spain	FIN	Finland
FR	France	SE	Sweden
IR	Ireland	UK	UK
IT	Italy	DE	Germany

Figure 5

Estimation errors in simple diffusion models (δ=0.96)

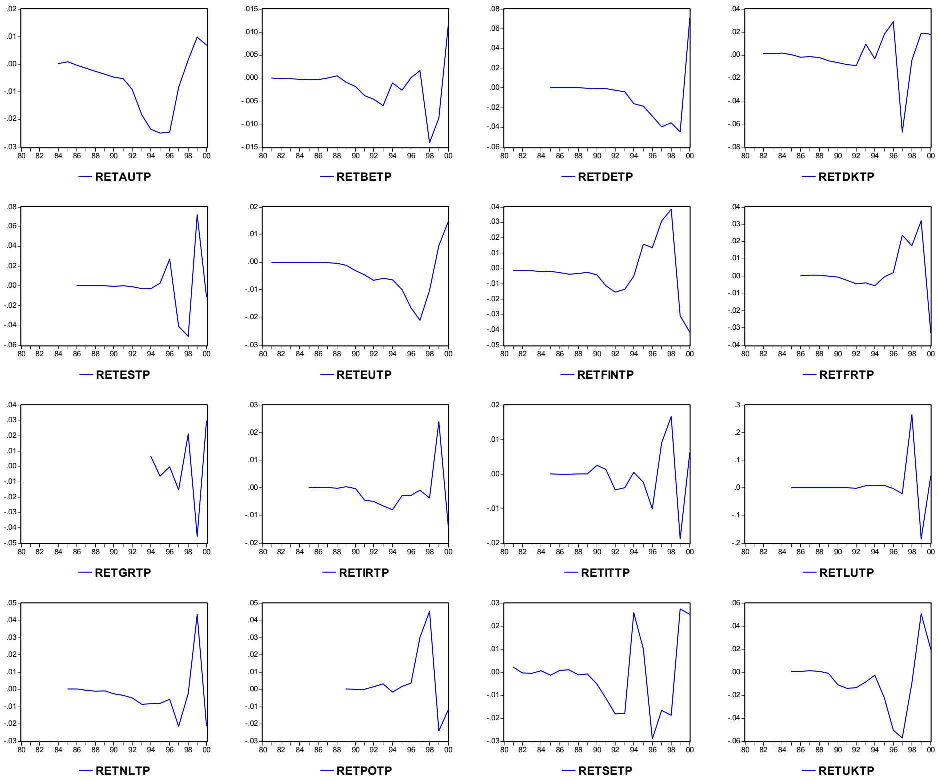
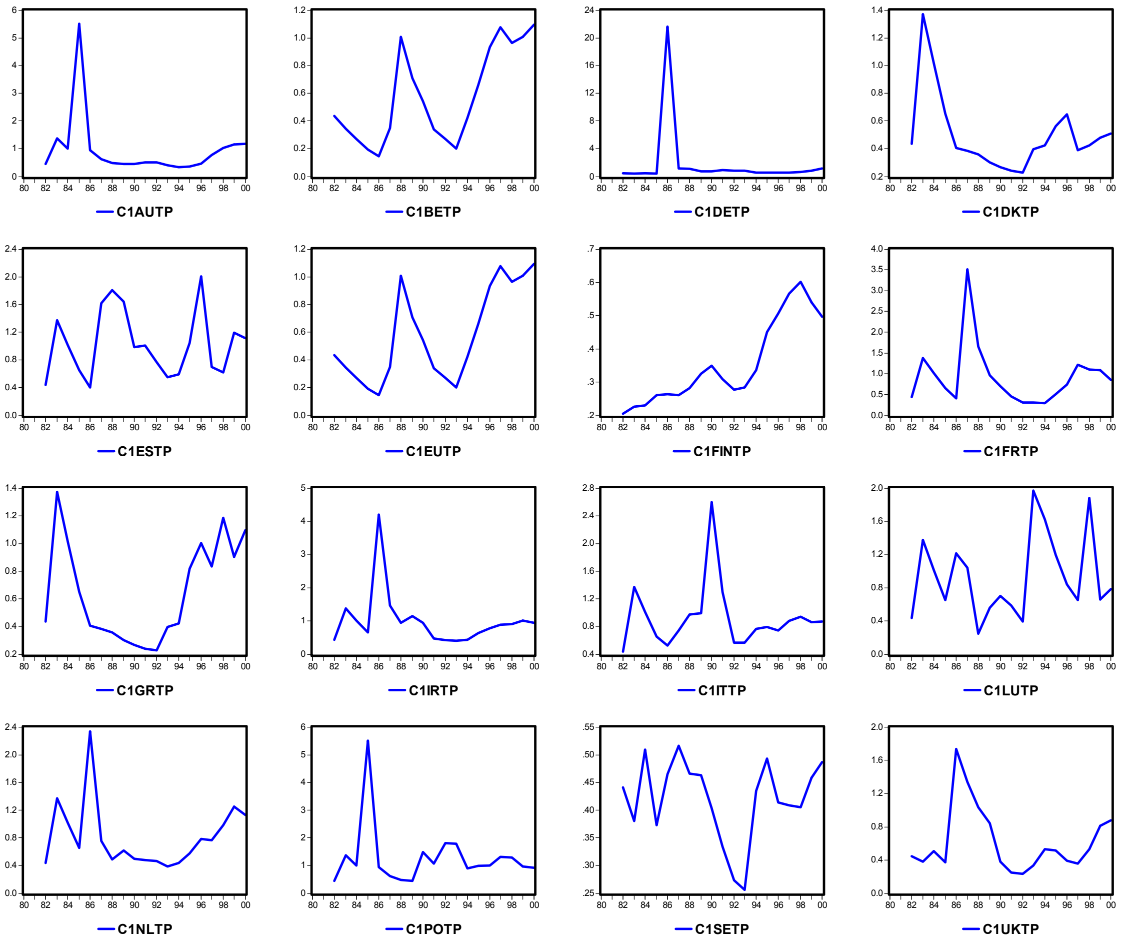


Figure 6

β coefficients estimated recursively



Source: Authors.

In **C1** at the beginning of the name refers to the β estimated recursively for the simple diffusion model with (δ=0.96), and the letters **TP** make reference to the total subscribers as a percentage of population.

The central part of the name reflects the countries in which the estimate was made:

EU	Total European Union	LU	Luxembourg
BE	Belgium	NL	Holland
DK	Denmark	AU	Austria
GR	Greece	PO	Portugal
ES	Spain	FIN	Finland
FR	France	SE	Sweden
IR	Ireland	UK	UK
IT	Italy	DE	Germany

Considering also the matter discussed with regard to calculating the limits of the diffusion process, i.e. the shift of the inflexion points to values over the 0.5 presented by the simple logistic model, then it appears necessary to re-specify the model towards a new formulation with the following characteristics:

- Increase the slope at the end of the sample.
- Flexibility in the contagion coefficients β.
- Shift the inflexion point closer to the ceiling δ.

Once again, the literature contains three alternative approaches that appear to conform to these criteria: precisely the ones set out in Table 4 (Bertalanffy, Nelder and Flog).

As stated earlier, the Nelder model is preferable over the other two because of the simplicity of approach and the better conditions of estimation [cf. Nelder (1992)].

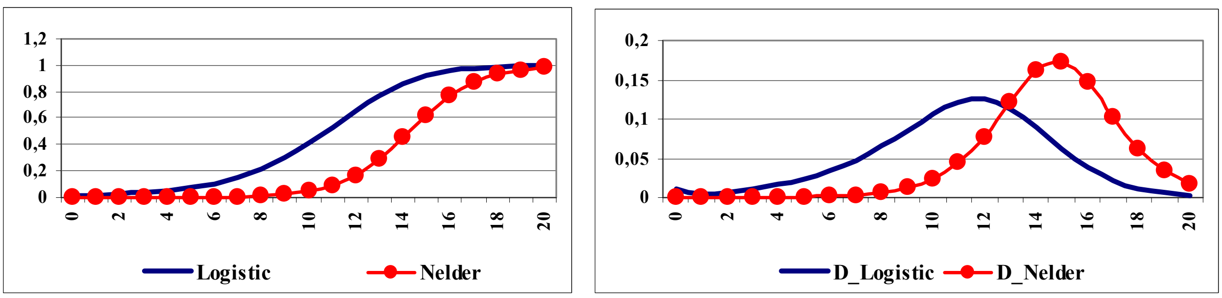
Figures 7 and 8 which illustrate the distinguishing features of the Nelder model when compared with the normal logistic model (Figure 8), showing the cumulative functions on the left and the difference functions on the right; clearly, both profiles fit the actual data better when the Nelder model is used (Figure 9).

$$Y_t = \frac{1}{\left(1+\theta e^{-(\alpha+\beta t)}\right)^{1/\theta}} \qquad [24]$$

$$\Delta Y_t = \beta Y\left(\delta - Y^{\theta}\right) \qquad [25]$$

Figure 7

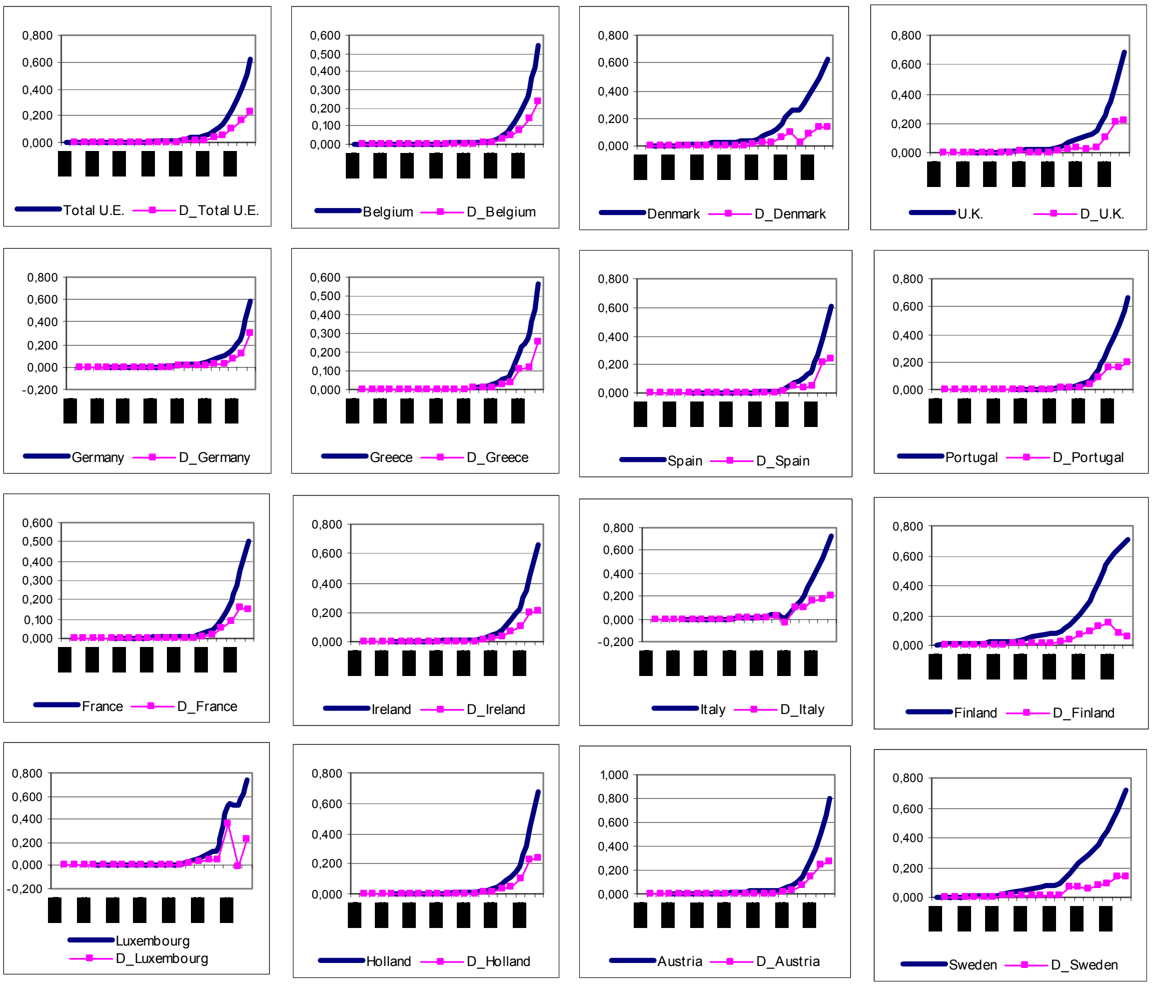
Comparison of the evolution profiles under the two alternative models



Source: Authors. (Nelder parameters θ = 0,7; α=-10; β= 0,7)

Figure 8

Evolution in the diffusion process of mobile telephony



Variables preceded by **D_** reflect the evolution in differences.
Source: Authors.

Using the general statement of the Nelder model (expressions [24] and [25]), we can consider two alternative estimation strategies:

1. Perform the estimate on the model in cumulative terms by applying non-linear estimation proceedings on the (previously-normalised) variable.
2 Perform the estimate on the model in differences, again applying non-linear estimation proceedings.

Where as the first approach raises greater problems of estimation, since the specification is more complex, the second has drawbacks for making predictions since it is affected by past variables of the diffusion process. For that reason, we elected to use the first specification, employing a general expression of the type:

$$QY_t=1/((1+c(2)*\exp(-1*(c(3)+c(1)*T)))^\wedge c(2)) \qquad [26]$$

where QY_t is the percentage of the population with a mobile phone subscription in each period t divided by the pre-set saturation value (0.96), C(2) is the coefficient θ, C(1) the coefficient β and C(3) the coefficient α, which must be estimated, and T is a time series with the value of 1 in 1980.

As in the previous cases, the estimates were made using Eviews 4.0 which, for non-linear specifications, uses the procedure described by Davidson and MacKinnon (1993).

The results of estimating the Nelder model shown in Table 9 are significantly better than those offered by the simple logistic model; they attain high levels of fit with coefficients that are significant in most cases, and the values of the Durbin Watson statistic are also closer to the tabulated values (with some exceptions).

Table 9

Results of estimating the Nelder model (δ=0.96)

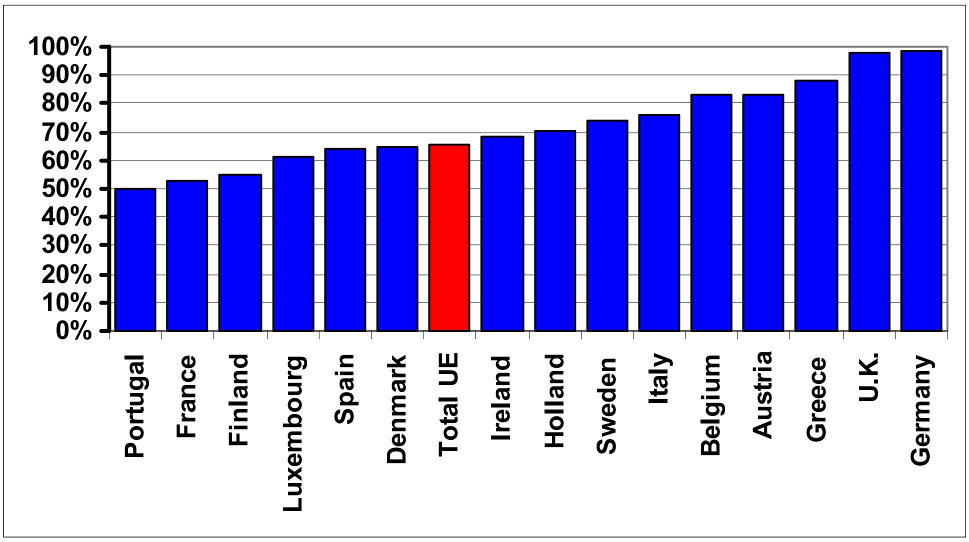
	C(1) =β	T_β	C(2) = θ	T_θ	C(3)=α	T_α	R^2	D.W.
Germany	9.860	0.002	0.061	0.002	-218.264	-0.002	0.994	1.265
Austria	2.121	9.325	0.295	7.267	-45.608	-9.044	0.999	0.374
Belgium	1.739	12.808	0.356	10.957	-38.912	-12.478	1.000	0.955
Denmark	0.718	4.983	0.456	3.802	-16.312	-4.478	0.995	1.692
Spain	1.627	3.712	0.402	2.894	-35.770	-3.549	0.996	2.727
Finland	0.542	7.729	0.740	4.143	-10.813	-5.622	0.995	0.898
France	0.622	12.637	1.534	4.877	-11.982	-8.146	0.999	1.563
Greece	2.378	1.502	0.249	1.390	-53.338	-1.495	0.998	3.039
Holland	1.428	9.740	0.487	6.688	-30.751	-9.025	0.999	1.759
Ireland	1.159	17.205	0.532	11.594	-25.001	-15.586	1.000	1.696
Italy	1.076	20.495	0.530	13.224	-22.855	-18.248	1.000	2.975
Luxembourg	0.633	3.306	2.194	0.739	-10.485	-1.594	0.971	3.344
Portugal	0.773	15.295	1.138	6.317	-15.156	-11.015	0.999	2.972
UK	5.845	0.076	0.076	0.076	-129.906	-0.077	0.994	1.303
Sweden	0.852	7.103	0.331	5.806	-19.389	-6.680	0.998	1.345
Total EU	**1.955**	**8.344**	**0.256**	**7.484**	**-43.859**	**-8.253**	**1.000**	**1.068**

Source: Authors.

Figure 9 shows the values (in terms of the percentage of the saturation level) at which each country attains the inflexion point; those points are, in fact, above the 50% average value assumed by the classical logistic model.

Figure 9

% of the saturation level at which the inflexion point is reached



Source: Authors.

5.- Analysis of the diffusion processes

Having estimated the process of diffusion of mobile telephony in Europe, we obtain certain basic features or "stylized facts" about these processes by analysing the performance in the last few years and the predictions obtained from the models as estimated.

The first prediction is that most mobile telephony markets in Europe attained their maximum potential growth, in terms of new users, around 2000; accordingly, the growth rates in those markets began to decline significantly after that point, as shown in Table 10, which shows the growth derived from the diffusion process for the period 1996-2000, the estimates for the period 2001-2005, and the average population growth.

Table 10

Growth rates in mobile telephony markets			
	Diffusion effect (average growth %)		Population growth (%)
	1996-2000	2001-2005	
Germany	67.6	12.5	0.0
Austria	76.7	4.0	0.1
Belgium	88.4	13.2	0.2
Denmark	32.9	8.5	0.4
Spain	100.8	10.1	0.1
Finland	29.3	5.9	0.3
France	88.4	13.6	0.4
Greece	86.1	12.6	0.1
Holland	81.7	7.8	0.6
Ireland	72.4	8.3	1.2
Italy	61.4	5.7	0.1
Luxembourg	76.7	5.1	0.0
Portugal	83.7	7.6	0.2
UK	48.9	8.1	0.4
Sweden	25.6	6.0	0.1
Total EU	**61.2**	**9.6**	**0.2**
Source: Authors.			

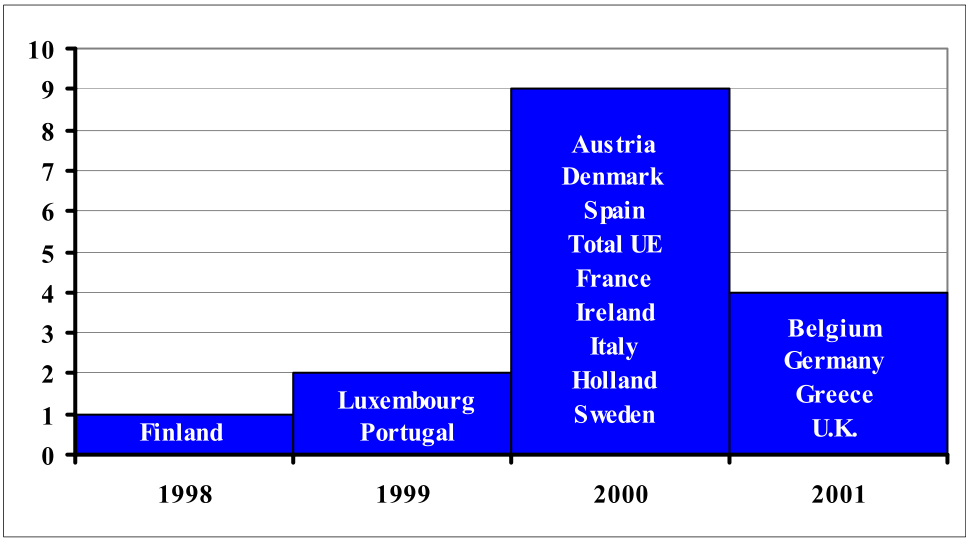
Another noteworthy feature is that the diffusion processes are homogeneous in that, regardless of when mobile telephony was introduced (early 1980s in Sweden, Belgium and Finland, early 1990s in Portugal), the inflexion points are clustered around 2000 (Fig. 10).

Considering that the inflexion point of the diffusion processes is the point where the slope of the first derivative peaks, and interpreting this as the rate of market growth, this confirms the hypothesis stated above that growth in the mobile telephony markets has been more contained since 2000.

The fact that a country has attained its inflexion point does not mean that it has no growth potential; this will only happen at the saturation point (when the slope of the first derivative of the diffusion process will be zero and the expected growth from the addition of new users will also be zero).

Figure 10

Distribution of inflexion points
(No. of countries attaining the inflexion point in each year)

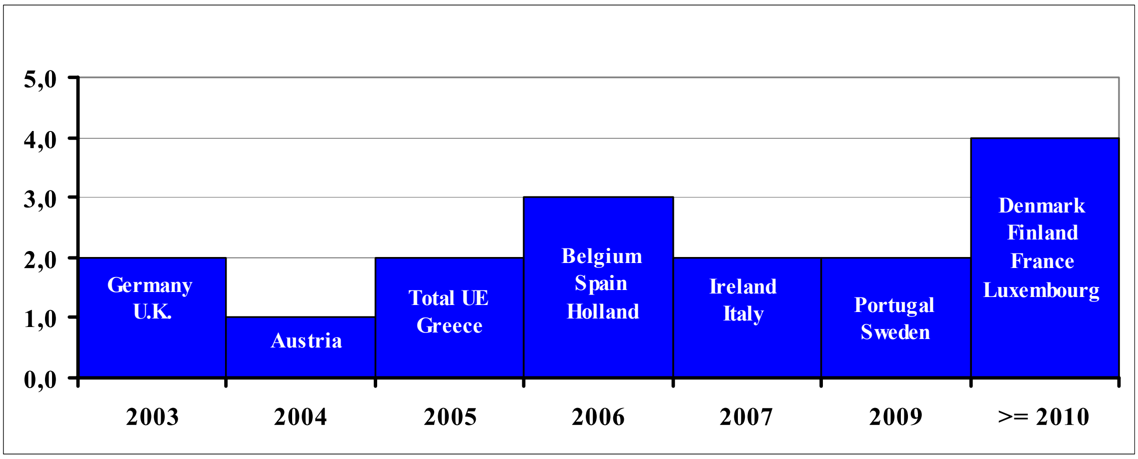


Source: Authors.

The saturation points are more dispersed than the inflexion points (Fig. 11), as they are located in the range 2003-2010, depending on the velocity attained by the diffusion processes in the final periods.

Figure 11

Distribution of the saturation points
(No. of countries attaining the saturation point in each year)



Source: Authors.

Table 11 summarises the results of analysing the processes of diffusion of mobile telephony in Europe, showing, in each case, the year the process commenced in each country, the period in which the inflexion point (maximum growth rate) is attained and the period in which the process reaches saturation (zero growth rate).

The table also shows the lengths of the various processes, distinguishing between the introduction period (up to the inflexion point), the saturation period (from the inflexion point up to saturation) and the total.

Table 11 suggests that the complete process of diffusion of mobile telephony lasts 23 years, on average, and is considerably shorter in the late adopters (e.g. Greece).

The figures also show that the processes are asymmetric, the introduction period being significantly longer than the saturation period.

Table 11

Basic features of the processes of diffusion of mobile telephony

	Process commenced (1)	Inflexion point (2)	Saturation point (3)	Introduction phase (2)-(1)	Saturation phase (3)-(2)	Total (3)-(1)
Austria	1983	2000	2004	17	4	21
Belgium	1980	2001	2006	21	5	26
Germany	1984	2001	2003	17	2	19
Denmark	1981	2000	2010	19	10	29
Spain	1985	2000	2006	15	6	21
Finland	1980	1998	2010	18	12	30
France	1985	2000	2010	15	10	25
Greece	1993	2001	2005	8	4	12
Ireland	1984	2000	2007	16	7	23
Italy	1984	2000	2007	16	7	23
Luxembourg	1984	1999	2010	15	11	26
Holland	1984	2000	2006	16	6	22
Portugal	1988	1999	2009	11	10	21
Sweden	1980	2000	2009	20	9	29
UK	1984	2001	2003	17	2	19
Total EU	**1980**	**2000**	**2005**	**20**	**5**	**25**

Source: Authors.

6.- Summary and conclusions

This paper has addressed the capacity of diffusion models to adequately represent the process of introduction of a new technology, such as mobile telephony, into the European Union countries as a whole.

As is normal in this type of application, the initial difficulty lies in estimating the saturation level of the process since there is no "physical" or "technical" limit a priori which would mark the ceiling of penetration, not even in terms of population, since it is possible for an individual to have more than one of the products being discussed.

Several alternatives were explored for quantitatively delimiting the saturation level using actual observations; it was finally estimated at 96% of the population, implying that 8% of the population aged over 14 (defined as potential users) could have at least two handsets.

After estimating the saturation ceiling, it was found that normal diffusion processes, based on the logistic model, do not adequately fit the observed data, which appear to present an inflexion point at over 50% of the ceiling, whereas the classical symmetrical models put the inflexion point at 50%.

To deal with the observed asymmetry in the processes, it was decided to use the Nelder model, which allows the inflexion point to fall anywhere between 0 and 100% saturation; the inflexion point was found to lie at 70% saturation, on average, though ranging from 50% (Portugal) to 99% (Germany).

The degree of fit of the models finally chosen was quite good and, with some exceptions, there were no signs of a lack of specification of the model or of serious breaches of the basic assumptions.

The main conclusion from the diffusion processes as estimated, apart from their asymmetry, is that they are very uniform as regards the time when they reach the inflexion point (around 2000), implying that growth in mobile telephony markets started to tail off after that point.

This uniformity is not maintained as regards the saturation point, since some countries will reach saturation in the next two years whereas others could continue to grow until the end of the decade, albeit at steadily decreasing rates.

The analysis suffers from a number of limitations and, therefore, has a number of points for improvement.

The first limitation lies in the initial data since, as they are expressed in absolute terms, a subscriber who changes number counts as two subscribers.

The relative volatility of the quantitative results obtained in estimating the saturation ceilings under a number of alternative assumptions is a warning about the reliability of the results; although they are not excessively dependent on the saturation value, this factor could nuance the specific results for individual countries.

Finally, it is possible to apply alternative models, and even techniques, for estimation out of the many presented in the literature[1], which might give us a certain range of confidence as to the results presented here.

[1] Cf. Skiadas, 1986, for example.

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