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Qualitative Comparative Analysis (QCA) and Fuzzy Sets

Applications and Perspectives for a Mixed Methods Strategy

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1 Set Theory and Qualitative Comparative Analysis

Over the last 25 years, ‘Qualitative Comparative Analysis’ (QCA) has become an important addition to the methodological canon of the social sciences. Most of the seminal publications and textbooks on comparative methodology at least mention QCA (Brady and Collier 2004; Della Porta and Keating 2008; George and Bennett 2005; Gerring 2012; King et al. 1994; Mahoney and Rueschemeyer 2003). Well-selling textbooks have been published in French (DeMeur and Rihoux 2002), German (Schneider and Wagemann 2007) and soon English (Schneider and Wagemann 2012). Publications in which the authors apply ‘Qualitative Comparative Analysis’ (QCA) can be found frequently. The most important European Ph.D. programs and summer schools provide regular workshops and courses on QCA. Finally, the acronym can be easily detected in the timetables of large international conferences.

The reasons for this increasing diffusion of QCA are manifold. One frequently quoted motivation for its application is related to the numbers of cases under investigation. On the one hand, QCA is praised as a method for a small number of cases – although an application of QCA to too small a number of cases can lead to notable problems (Wagemann 2008: 252). It would be much more correct to say that an advantage of QCA is that it can be applied to a mid-sized number of cases (Ragin 2000: 35ff.). With this potential, QCA fills an important gap in social science methodology. It seems that, up to the 1980s, the scholars of comparative politics just had the choice between ‘orthodox’ forms of macro-comparisons of a very small number of cases (usually not more than four) and the use of mass surveys, providing micro level information on large numbers of individuals. QCA offered a reasonable alternative for research questions investigating about between ten and 50 cases. A further advantage is often seen in QCA being a kind of ‘third way’ between the epistemologies of qualitative and quantitative methods. Indeed, the book with which the American social scientist Charles C. Ragin introduced QCA to the wider public (Ragin 1987) had the sub-title ‘Moving beyond qualitative and quantitative strategies’, saying, thus, that QCA would overcome the schism which had become dominant in the methodological discussion of those years. Admittedly, referring to QCA as
'Qualitative Comparative Analysis' becomes paradoxical at this point, since, with this choice, the QCA applicants precisely do not seem to go beyond the distinction between qualitative and quantitative methods, but to categorize QCA clearly within the qualitative tradition. However, the name and the label QCA were not yet directly mentioned in this first book on QCA (Ragin 1987), but emerged during the discussion evolving in the aftermath. Indeed, in his second book on the topic, Ragin seems to prefer the adjective ‘case-oriented’ (Ragin 2000: 23; see also Ragin 2004), although he also explicitly underlines that QCA belongs to the qualitative methodological tradition (Ragin 2000: 13). Finally, a third reason which is often given for the use of QCA is that, in its standardized approach to comparison, it would be a strong response to the frequent critique on comparative methods following which they would be neither standardized well enough nor scientifically reliable in order to compete with standard statistical techniques (which is also a bit the bottom line of King et al. 1994).

Whereas all these are certainly good reasons why QCA has to be considered when speaking about comparative methodology, they are themselves consequences from an underlying characteristic of QCA which shall be discussed in this contribution, namely, of QCA being rooted in set theory. Set theory is a mathematic sub-discipline and shares many aspects with Boolean algebra (and its further developments, such as fuzzy algebra, see Klir et al. 1997 on this) and formal logic. Being an algebra allows for a standardization of QCA which can complement the highly standardized statistical methods; on the other hand, not being a linear form of algebra, set theory might enable researchers to work on patterns which are typical for case-oriented (or also ‘qualitative’) research. In the following, QCA and its most important aspects will be briefly introduced (part 2). In the central part of the chapter, some important properties of QCA are presented, above all with regard to the use of set theory (part 3). The last part names some applications of QCA, proposes an agenda for the future development of it, and makes an outlook in how far st theoretic methods can be used in order to amplify the variety of available social science methods (part 4).

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1 He also uses the adjective ‘diversity-oriented’ (Ragin 2000: 19) which makes clear that QCA predominantly looks at the differences between cases and not at their similarities (more on this, see fn. 20).
2 A Short Presentation of QCA²

First of all, using the acronym QCA as such, without any further specification, is not very useful. There are various versions of QCA. Although the original version (Ragin 1987) is often just presented as QCA (without any further specification), it would be more accurate to refer to this first version as ‘Crisp Set QCA’ (csQCA). In this way, the 1987 version (which has not been replaced by the subsequent developments, but is instead still valid and important) can be differentiated from the fuzzy set version of 2000 (fsQCA) (Ragin 2000). Furthermore, ‘Multi Value QCA’ (mvQCA) is often presented as a separate form (Cronqvist and Berg-Schlosser 2009).³ All these versions share the common interest to find sufficient and necessary conditions for a given outcome in causally complex settings. Such a reasoning is very frequent in the social sciences.⁴ However, QCA goes beyond the pure assessment of sufficiency and necessity and also extends to more sophisticated forms of causal complexity. In order to illustrate this point, let us imagine the following hypothetical result of a QCA analysis:⁵

\[ AB + \sim AC \rightarrow Y. \]

This is the result of an analysis of sufficiency (indicated by the arrow \( \rightarrow \)). We see that three conditions A, B and C have been identified which are assumed to have a somewhat causal role in explaining the outcome Y. The result provides us with two alternatives (indicated through the plus

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² For more details, please refer to some important contributions to the topic (Ragin 1987; 2000; 2008; Rihou and Ragin 2009) and to the various available textbooks (DeMeur and Rihou 2002; Schneider and Wagemann 2007; Schneider and Wagemann 2012), or to other short overviews (such as Wagemann and Schneider 2010).

³ mvQCA will not be treated in much details in this paper, since it is accused of not being entirely rooted in set theory (Vink and Van Vliet 2009). Furthermore, as Schneider and Wagemann (2012) show, there is no technical reason why a csQCA run in a dummy form (where all but one category of a multi-value concept represent separate crisp sets) would be inferior to a mvQCA (for more critique on mvQCA, see Schneider and Wagemann 2007: 262ff., Vink and Van Vliet 2009, and Wagemann and Schneider 2010: 388f.).

⁴ For a list of important political science contributions of the last 20 years which can be reformulated in terms of necessary conditions, see Goertz (2003: 76ff.). Since it is logically easier to find sufficient conditions (for reasons for this, see Schneider and Wagemann 2007: 58ff., 65), we can assume that hypotheses dealing with sufficiency arguments are also very diffused.

⁵ For technical details, see the textbooks indicated above (fn. 2).
sign)\(^6\) for the explanation of Y. The alternative ‘AB’ tells us that the simultaneous presence of the conditions A and B is a sufficient condition for Y.\(^7\) The alternative statement to this is that the absence of A (⇐ ~A),\(^8\) combined with the presence of C (thus, the combination ~AC), is also sufficient for Y. The solution can be read as ‘A and B or non-A and C are sufficient for Y’. Note that neither A nor B nor C (and neither non-A, non-B, or non-C) are sufficient conditions for Y. If A were a sufficient condition, then it would not need to be combined with B in order to imply the presence of Y. A, B and C (and also ~A) are so-called INUS conditions (INUS = ‘insufficient but necessary part of a condition which is itself unnecessary but sufficient for the result’) (Goertz 2003: 68) which – standing alone – are neither sufficient (nor necessary) conditions. Indeed, it is central in QCA not only to examine the individual conditions, but also the various combinations thereof, whether they are sufficient for the outcome.

Necessary conditions have been undervalued for a long time in QCA,\(^9\) but more recent publications draw our attention to them. When analyzing necessary conditions, the logic must be inverse: it does not make sense to analyze combinations of conditions for necessity, because if a condition is not necessary, then any combination including this condition will not be necessary, either. However, in the case of an analysis of necessity, it is possible to refer to ‘OR’ combinations, i.e. so-called ‘functional equivalents’ (Schneider and Wagemann 2007: 62f.). Instead of INUS conditions, this procedure will reveal the so-called SUIN conditions which describe a “sufficient, but unnecessary part of a factor that is insufficient, but necessary for the result” (Mahoney et al. 2009: 126). For example, in the hypothetical result of an analysis of necessity (indicated through the inverse arrow ←), we could get:

\[(D+E) \ast F ← Y.\]

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\(^6\) The plus sign is thus interpreted in a different way in Boolean and linear algebra. Why it means “and” in linear algebra, it means “or” in Boolean algebra.

\(^7\) The verbal statement that AB would ‘cause’ Y should be avoided (see below, fn. 13).

\(^8\) The absence of a condition or an outcome is most usually indicated with a tilde (~). More rarely, a small letter (a) or other signs (~A) are used.

\(^9\) A reason for this lies in the fact that the algorithm for detecting necessary conditions is not so sophisticated than the one for sufficient conditions. Thus, both in teaching and in research, necessary conditions become a kind of second-order interest.
While F is a truly necessary condition in this hypothetical example, it is also necessary that “D or E” is present – in other words, these are alternatively necessary conditions and, thus, SUIN conditions.

Whereas this general interest in sufficient and necessary conditions is the same for all variants of QCA (csQCA, fsQCA, to a lesser extent for mvQCA), there is an important difference with regard to the kind of data which is analyzed. In csQCA, the data matrix can only contain dichotomous values (the conditions and the outcome are either present or absent, that is, they have to be defined ‘1’ or ‘0’). In fsQCA, other values between 0 and 1 are also permitted, creating so-called ‘fuzzy sets’.10 Fuzzy sets were introduced in computer sciences more than 40 years ago (Zadeh 1965; 1968) and expand the classical perception of what a set is, following which an element would either be in a set or not in a set (Klir et al. 1997: 48). If the borders of a set become ‘fuzzy’ (Klir et al. 1997: 73ff.), then the single elements can also be contained only partially. In this case, ‘0’ means that the element is clearly not in the set; ‘1’ that the element is clearly in the set; ‘0.5’ that it cannot be decided if the element is in the set or not; whereas all the other values between 0 and 1 indicate the degree of membership in the set.11 Successively, the truth tables (no matter if filled with dichotomies or fuzzy values) are transformed in logical equations (‘solutions’), applying Boolean algebra (for csQCA) and fuzzy algebra (for fsQCA). mvQCA allows for the use of multinomial conditions (but not of multinomial outcomes – one of its major shortcomings), such as political parties, nationality, etc. (Cronqvist and Berg-Schlosser 2009), i.e., phenomena with more than two categories which do not follow a rank order.

For the evaluation of a QCA analysis, two parameters were developed, namely, consistency and coverage (Ragin 2008: 44ff.). In this way, the earlier critique that methods based on sufficient and necessary conditions (such as QCA) would be too deterministic (Goldthorpe 1997: 4f.; Mahoney 2000: 391f.) was weakened, since the consistency parameter also allows for not-deterministic results, and the coverage parameter indicates how much of the outcome can be explained.

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10 This also means that a crisp set, where only the values of 0 and 1 are permitted, is nothing else than a special version of a fuzzy set. Thus, it is no surprise that a crisp set analysis and a fuzzy set analysis at a certain point follow the same algorithm. This also means that the distinction between csQCA and fsQCA is artificial and not very helpful.

11 It will be discussed further below (section 3.3) how these values can be attributed to the single cases.
3 The Contributions of Set Theory to QCA

In this central section of this paper, the contribution of set theory to QCA techniques is assessed. First, it is shown how set theory helps in the analysis of sufficiency and necessity (3.1); then, which implications this has on causality (3.2); in the following, how set theory and concept formation are connected in QCA (3.3); and, finally, how set theory becomes manifest in the constructions of cases as configurations of properties and how this can be applied in the solution of problems of limited empirical diversity (3.4).

3.1 Set Theory and the Analysis of Sufficiency and Necessity

As mentioned, the central goal of a QCA is assess the sufficiency and necessity of conditions and to work out INUS and SUIN conditions. This sub-section is aimed at clarifying what this central interest of QCA has to do with set theory.

First of all, it has to be mentioned that necessity and sufficiency relations and therefore QCA are suitable for very specific hypotheses, namely, “if…then…” hypotheses. This has an interesting implication, since these hypotheses are more limited in their range, but point at the same time very precisely to a given argument. If we compare a hypothesis such as ‘there is a positive correlation between economic development and the level of democracy’ – which could be a typical correlational hypothesis for statistical analysis – and ‘if a country is economically developed, then it is a democracy’ (in order words: ‘economic development is a sufficient condition for a democracy’) – a typical hypothesis for QCA – then these two do not seem to be so different at the first glance. However, there is a fundamental difference: the hypothesis ‘if a country is economically developed, then it is a democracy’ permits that there are also democracies without economic development, because economic development has been hypothesized as a sufficient, but not a necessary condition for democracy. This is a clear difference to standard statistical techniques which – in order to confirm the positive correlation between the two variables – would not permit the absence of economic

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12 The example is obviously banal. It is just used for reasons of illustration.
development in the case of a democracy. Rather, this situation would worsen the correlation, perhaps even notably. Furthermore, the hypothesis that economic development is a sufficient condition for democracy does not contain any information on the situation in which economic development (= the condition) is absent. By contrast, the correlational hypothesis gives us this information: if there is a positive correlation between economic development and democracy, then the cases with low values for democratic development should also display low values for democracy. The hypothesis which proposes a sufficiency of economic development for democracy instead does not tell us anything about what would happen in case that economic development is absent. As can be seen, the hypotheses used in QCA are more limited in their range, but are more precise at the same time. Additionally, we could also argue that these more narrow hypotheses correspond better to our view of the world which is much too complex as to allow for simple correlational hypotheses (Hall 2003).¹³

This way of formulating hypotheses is closely linked to set-theoretic thinking. An if-then-formulation corresponds to a subset-superset relationship in set theory. If we formulate that X is a sufficient condition for Y, then this equivalent to the more formal statement that X is a subset of Y and (in the consequence) that Y is a superset of X. By definition, this means that all elements which are contained in the set X are also contained in the set Y, but not all elements which are contained in the set Y are also contained in the set X.
As can be seen from the figure, X is a subset of Y. This figure immediately suggests two insights: first, the relation between the two sets is asymmetric. We can infer from X to Y, but not vice versa. Furthermore, we cannot say anything about the causal effect of \( \sim X \): the area outside of the circle for X (i.e., those cases which are not members of the set X and which are, therefore, members of \( \sim X \)) overlaps both with Y and \( \sim Y \). Second, Y is not fully accounted for by X. There must be other conditions (Z, A, B, etc.) which help filling up the circle of Y. Nevertheless, X is a perfectly sufficient condition for Y, because everywhere where we find X, we also find Y. Later (3.2), we will refer to the first phenomenon as asymmetric causality and to the second phenomenon as equifinality.

The circles also teach us two more lessons which can be easily formalized in QCA: first, let us compare a situation in which the inner circle (representing the sufficient condition) is much smaller than the outer circle (representing the outcome) with a situation where the inner circle comes close to the outer circle. If the inner circle is small, then this means that it only contains a small number of cases. The condition represented by such a small circle might just be applicable for a very small number of countries or cases. If we, e.g., argue that being a country located in the Alps which produces good chocolate and uses four official languages is a sufficient condition for being a democracy, then this is certainly true. There is just one country – Switzerland – which can be described by such a condition, and, indeed, it is a democracy. However, this is not a very powerful argument, since the condition is so limited in scope, i.e., the inner circle is so small. In case of these small inner circles, many other inner circles would be needed in order to fill the outer circle. This is not the case when the inner circle comes close to the outer circle. In that case, the inner circle contains nearly as many cases as the outer circle, and just some very minor other sufficient conditions are needed in order to account for the whole outcome (i.e., the outer circle). In QCA, this can be

\[ \text{This graphical representation works rather well for crisp sets, whereas the situation becomes more complex as soon as there are fuzzy sets involved. As mentioned, fuzzy sets are characterized by the fact that there can also be a not clear membership or a not clear non-membership of cases in a set. Therefore, set boundaries become fluid. For more information on subset relations in fuzzy sets, see the explanations in Ragin (2008: 29ff.) and Schneider and Wagemann (2007: 197ff.).} \]

\[ \text{A note has to be made on this: it can also be that the inner circle and the outer circle both contain very few cases. In this situation, the inner circle would also be very close to the outer circle, but they would be far from the rectangle. In such a situation, neither the condition nor the outcome would refer to many cases. We} \]
measured with the coverage parameter which indicates how much of the outcome is ‘covered’ by the condition (Ragin 2008: 44ff.; Schneider and Wagemann 2007: 90ff., 208ff.).

In another scenario, we could now imagine a situation in which the subset-superset relationship is prevalent, but with small deviances. In such a situation, the circle for X (i.e., the sufficient condition) would largely be contained in the circle for Y (i.e., the outcome), but there would be a small portion of the X circle which exceeds the borders of Y. Strictly speaking, X is not a perfect subset if Y any more, and X is not a perfectly sufficient condition for Y: there are a very small number of cases which show the potentially sufficient condition X, but not the outcome Y. If X were perfectly sufficient, then all instances with X should also imply the presence of Y. If the number of cases which fail this requirement is small, we can still speak about a quasi-sufficient condition or about a non perfectly consistent sufficient condition. The parameter of consistency indicates the dimension of the violation of the sufficiency rule (Ragin 2008: 44ff.; Schneider and Wagemann 2007: 86ff., 203ff.).

Such a reasoning with set theory is also possible when examining potential necessary conditions. In this case, the necessary condition X is a superset of Y and Y a subset of X: all elements which are contained in the set Y are also contained in the larger set X, but not all elements which are contained in the set X are also contained in the set Y.

would try to explain a very rare outcome (i.e., our outcome hardly exists) and would account for it with exactly the characteristics of the rare cases in which the outcome appears.
As can be seen from the figure, Y is a subset of X. Following a similar reasoning as in the case of sufficient conditions, we cannot say anything about eventual necessary conditions for \( \sim Y \) (i.e., the absence of the outcome), since the area describing \( \sim Y \) (i.e., the area outside the circle for Y) overlaps with both X and \( \sim X \) (i.e., the area outside the circle for X). Again, this is an indication of an asymmetric causal relationship.

Also the figure with the circles representing subset-superset relations for necessary conditions illustrates the consistency and coverage measures well. Whereas the interpretation of consistency measures is parallel to sufficient conditions and therefore straightforward – an inner circle for Y exceeding the border of X at a certain point indicates a non perfectly necessary condition X, since there are also cases where Y is present, but X not – the interpretation of coverage needs some more reflection: when discussing sufficient conditions, we have looked at coverage for the situation that the inner circle was much smaller than the outer circle. Applying this to necessary conditions this means that the outcome refers to much fewer cases than the condition or, in the reverse formulation, that the condition goes much beyond the outcome. Imagine the following example: air to breath is a necessary
condition for a war.\footnote{16} This is definitely a true statement. We are not aware of any war which was fought without air to breath. However, if we try to represent this with circles, we will see that the set of cases in which air to breath is present (i.e., the X circle for the necessary condition) is extremely large; the circle of wars instead (the inner circle for the outcome Y) will be just a tiny spot within the large circle of situations with air to breath. Indeed, air to breath is a necessary condition for many other actions (which would be represented by other circles within the large circle X), such as organizing peace talks, cooking, attending a talk on QCA, or watching rare birds in Northern China. In other words, the coverage measure (which tells us about the relation between the outer and the inner circles) indicates in how far the necessary condition is trivial – despite being perfectly necessary.\footnote{17}

In this way, set relations (best illustrated with circles) can be used in order to work out necessity and sufficiency relations, no matter if in form of (rare) pure necessity and sufficiency or in form of INUS and SUIN conditions.\footnote{18} Furthermore, set theory and the reasoning in terms of set relations allow us to work with coverage and consistency parameters which help us assess the quality of our results and in how far abstract hypotheses about necessary, sufficient, INUS and SUIN conditions correspond to the reality. In this way, a mathematical sub-discipline such as set theory contributes to the analysis of complex causal relationships based on necessity and sufficiency in which we might be interested.

\textbf{3.2 Causality in QCA}

In the following, three central aspects of causality in QCA shall be discussed which directly derive from the set-theoretic characteristics of the analysis and which differentiate QCA from some other causal methods: equifinality, conjunctural causation, and asymmetric causality.

\footnote{16} The example is taken from Schneider and Wagemann (2007: 98).
\footnote{17} However, as Schneider and Wagemann (2012) demonstrate, this measure covers just one aspect of trivialness. It fails assessing trivialness where both the condition and the outcome are trivial; in that case, the inner and the outer circle greatly overlap and the coverage measure is artificially high. This is why Goertz (2006) has proposed an alternative formula, then further revised by Schneider and Wagemann (2012).
\footnote{18} Of course, the presentation has to remain necessarily short in this paper. The analysis of INUS and SUIN conditions goes much beyond what can be easily demonstrated with two circles, even more so, if fuzzy sets are analyzed and therefore set memberships can also be partial.
*Equifinality* refers to the fact that, in the analysis of sufficient conditions, it is possible to find more than one sufficient condition (or combinations thereof) which implies the outcome. If we look at the hypothetical solution formula

\[ A \sim B + C \rightarrow Y, \]

then the equifinal solution is that both the combination of the conditions A and \( \sim B \) and the condition C are sufficient conditions for Y. In other words: even if the combination ‘\( A \sim B \)’ were not present, the outcome could still be present, namely, if the alternative condition, ‘C’, is present, and *vice versa*. A\( \sim B \) and C constitute two circles which, taken together, completely fill up the Y circle. Obviously, it is also possible that both conditions are present at the same time. Graphically speaking, the circles for A\( \sim B \) and C overlap in this case.\(^{19}\) The difference with equations as they are used in statistics, such as regression equations, is evident: in regression, all the variables contribute individually to the explanation of the variation of the dependent variable, without being alternatives for one another. Their effect is usually additive, and the variables are in competition. Such competition does not exist in QCA’s equifinality. Quite to the contrary, all the causal conditions can potentially imply the outcome, without being rival. Sometimes, some conditions can even be ‘superfluous’, but they are kept in the equation, since they refer to very specific and interesting theoretical or empirical phenomena (or are even directly linked to the hypothesis which the researcher wants to test). E.g., the component ‘\( \sim AB \)’ is superfluous in the solution formula \( A + B + \sim AB \rightarrow Y \), because it is already a part (= a subset) of B. However, a researcher might want to keep this part of the result, because perhaps exactly the sufficiency of this combination had been denied in the literature before: the literature could have claimed that A is a necessary condition for Y, and that, thus, Y can only be observed if A is present. However, defining \( \sim AB \) a sufficient condition would falsify this claim.

In general, equifinality is also very often present in typical hypotheses of comparative social sciences, since many outcomes (such as, e.g., democratization processes, or the success and failure of a policy) cannot be explained in a unifinal mode, where various factors are combined in an additive way in

\(^{19}\) In terms of set theory, this is the case for the segment A\( \sim BC \).
order to result in one single explanation. Often, social reality rather proposes more than one path to reach the outcome under research. Equifinality even allows for situations which – at a first glance – seem causally contradictory. In the solution

\[ AB + \sim AC \rightarrow Y, \]

the condition A takes on two different causal roles, depending on with which condition it is combined. If combined with ‘B’, then ‘A’ has to be present in order to imply the outcome. However, if it is combined with ‘C’, then ‘A’ has to be absent. In statistics, instead, it might happen that an independent variable ‘A’ with these ‘ambiguous’ properties would risk to be excluded from the final equation.

As seen, equifinality is manifest in the addition part (‘+’) of a Boolean expression. *Conjunctural causation*, by contrast, is more linked to Boolean multiplication. In our previous example,

\[ A\sim B + C \rightarrow Y, \]

only the combination of A and \( \sim B \) can be deemed a sufficient conditions, but not their isolated presence. Applying set theory to this reasoning, this does not have anything to do with subset-superset relations (as before), but with intersections of sets. The combination \( A\sim B \) corresponds to those cases in which the sets of A and \( \sim B \) overlap, i.e., where both A and \( \sim B \) are present.

In statistical techniques, situations in which two independent variables are highly correlated, are seen as enormously problematic (Ragin 2008: 9). Often, the term ‘multicollinearity’ is used for this, although ‘multicollinearity’ and ‘conjunctural causation’ are not exactly the same thing (for more details on this, see Wagemann 2007: 395). However, multicollinearity often reflects well the reality which we want to study: social processes often do evolve simultaneously, and, thus, the possibility to combine two conditions, as we can do in QCA, corresponds to this reality.
Finally, *asymmetric causality* is a typical pattern of causality in QCA. This refers to the fact that the explanation of the outcome does not automatically also include the explanation of the non-occurrence of the outcome (i.e., of the negative case). We have seen (section 3.1) that the assessment of sufficiency and necessity relations between X and Y is not reciprocal: assessing a cause X does not mean to assess, at the same time, cause ~X; and analyzing the outcome Y does not have the analysis of the outcome ~Y as a side-effect. In an extreme setting, this can mean that we even need two completely different sets of conditions in order to explain the outcome and the non-occurrence of the outcome.

In this way, QCA gives the opportunity to model causal relations as conformingly to the social reality as possible and/or necessary. This does *not* mean that arguments and hypotheses assessed with other methods, such as statistical methods, have to be seen as ‘caricatures of hypotheses’, as has sometimes been claimed (Munck 2001). Nevertheless, the specific view on causality is a characteristic pattern of set-theoretic methods which also differentiates them from other approaches.

### 3.3 Calibration of Fuzzy Values

Whereas the previous sections (3.1 and 3.2) dealt with patterns related to the actual analysis in QCA, this section deals with a previous phase of the research process, namely the construction of the data matrix. As mentioned (section 2), two versions exist for with which kind of information these data matrices can be filled, namely, a crisp set version in which conditions and the outcome are defined to be present or absent (1 or 0), and a fuzzy set version in which cases can also have partial set membership. Since a crisp set is nothing else than a fuzzy set with just two values, the fundamental problem of filling in the data matrix is the same for both versions. As becomes clear from using the term ‘set membership’, this issue of coding (or ‘calibrating’, as is preferred in QCA, see Ragin 2008: 71ff.) is again closely linked to set theoretic reasoning.

Right from the beginning, both crisp set QCA and even more fuzzy set QCA have been criticized for the issue of calibration. Critiques (usually scholars applying quantitative methods) claimed that the
decision on ‘in’ or ‘out’ (in case of a dichotomous analysis) and, even more so, the definition of fuzzy values would risk to be completely arbitrary, above all from the perspective of measurement theory.\textsuperscript{20} Without any doubt, this critique is partially justified, since some users of QCA could be tempted not to be sufficiently rigid or transparent in the attribution of fuzzy values.\textsuperscript{21} This is obviously crucially important. The point is not that the result of the analysis depends extremely much on the values (as claimed by Hall 2003: 389; Mahoney 2003: 347)\textsuperscript{22} but this is also a question of credibility, reliability and seriousness of the results.

Conceptually speaking, fuzzy values are defined through the membership of the case under research in the set which describes the concepts on which the conditions and the outcome of interest are based (Ragin 2000: 7). The membership value of the case in the set accounts for the fit of the case with the concept. It goes without saying that this process requires a profound knowledge about the object of research, not only in the sense of case knowledge, but also of a precise definition of the underlying concepts. Only if the concepts are theoretically clear, statements about their degree of presence and absence can be made; also, the attribution of single values to cases can only work out if the researcher also knows the cases under analysis very well. Obviously, there are some ‘dos’ and ‘don’ts’ in the calibration process (Ragin 2000: 164f.; Schneider and Wagemann 2010: 403).\textsuperscript{23} Nevertheless, despite these guidelines, clear rules on how to calibrate do not exist and cannot exist.\textsuperscript{24} As a consequence,

\textsuperscript{20} Scholars of the qualitative camp often have the opposite doubt, namely if it is possible at all to attribute any numerical values to societal and political patterns.

\textsuperscript{21} Since, as mentioned, crisp sets are fuzzy sets with just two values, I will not make any difference between the two and call them simply ‘fuzzy sets’. Of course, all arguments presented for fuzzy values also hold for crisp values.

\textsuperscript{22} Research practice gives us the insight that this statement has to be qualified better: e.g., it does not make a notable difference if a value of 0.2 is attributed to a case instead of 0.4. However, it does make a difference if a value of 0.6 is attributed instead of 0.4. It is not so much the difference which counts, but whether or not the 0.5 threshold is passed (for more on how QCA results depend on different calibrations and, in general, on the topic of robustness in QCA, see Schneider and Wagemann 2012, but also Hug 2009, Marx 2006, Seawright 2005 and Skaaning 2011). This underlines that also a fuzzy set analysis is inherently dichotomous.

\textsuperscript{23} For example, statistical parameters, such as the median or the arithmetic mean, should be avoided as criteria for the establishment of the cutoff-point of 0.5. Also, those strategies which simply standardize existing quantitative scales into fuzzy values between 0 and 1 are also strongly discouraged. Such a procedure would risk that there were no recurrence to the underlying concepts, and that the calibration would entirely depend on externally defined quantities, often themselves only being proxies of the underlying concept.

\textsuperscript{24} For reasons of completeness, a less theory-guided alternative has to be mentioned, also in response to the continuing critique on the aspect of theory-guided calibration. The diffusion of QCA (and, therefore, the increasing number of regrettably badly trained applicants) might have also played a role in this. More
QCA applicants are forced to invest much time in the definition of their concepts and to acquire the necessary case knowledge – and this is not necessarily a disadvantage of QCA!

Note that there is also a literature which makes a more elaborate use of set theory in concept formation (which is not necessarily linked to QCA). These contributions make much use of set-theory as a way to combine the various properties of a concept. If, e.g., the concept of democracy is defined through several aspects which are all indispensable for a minimum definition of democracy, then we make essentially a statement about necessary components of a concept. Set theory tells us that – in order to say that the composed concept is present – also all its components have to be present (Klir et al. 1997: 55ff.; in case of fuzzy sets, the composed concept is as present as its least present component (Klir et al. 1997: 93ff.; Schneider and Wagemann 2007: 186). This corresponds to an AND combination in formal logic, a so-called ‘conjunction’. On the other hand, we could also think of concepts with mutually substitutable components. Functioning political participation could be defined through the presence of one (or several) of various alternative modes of participation. This corresponds to the logical OR in formal logic (a so-called ‘disjunction’); in fuzzy algebra, the maximum value of the fuzzy values of all potential components of a concept represents at the same time the fuzzy value of the overall concept (Klir et al. 1997: 92ff.; Schneider and Wagemann 2007: 186ff.) and, therefore, the membership value of a given case in the concept. Of course, these ideas of indispensable (‘necessary’) and mutually substitutable (‘sufficient’) factors of concepts can also be

concretely, two rather standardized ways of calibration have been recently introduced (Ragin 2008: 85ff.) which, however, only work if quantitative proxies for the concept to be calibrated already exist (such as in the case of the richness of a country for whose calibration we can recur to the quantitative scale of the GDP). These so-called ‘direct’ and ‘indirect’ methods of calibration foresee the theoretical formulation of just some fuzzy values, while the rest of the fuzzy scale is determined through a logistic or an inverse logistic function which is applied to the quantitative raw values (for technical details see Ragin 2008: 85ff.). The ‘direct’ method is implemented in the fsQCA software, while the ‘indirect’ method is based on a STATA command (Ragin 2008: 96, fn. 6). Obviously, these procedures are, by and large, quantitatively inspired. Apart from the fact that, in this way, the researcher risks to lose the contact with his/her cases, the problem is not resolved, how to arrive at the few values which have to be defined theoretically is not resolved. Furthermore, many scales are only seemingly quantitative, while others are even contested (just think about the various proposals how to measure democracy).

25 Schneider and Wagemann (2012) define QCA as one set-theoretic method, but not as the only one. Following them, first, QCA aims at a causal interpretation (something what the application of set theory to concept formation does not). Second, QCA makes use of so-called truth tables. Third, QCA approaches make use of the principles of logical minimization, a process by which the empirical information is expressed in a more parsimonious yet logically equivalent manner by looking for commonalities and differences among cases that share the same outcome.
combined with one another. In the end, rather complicated recipes how to build different concepts can result. It is also obvious that the literature on concept formation is just at the beginning of such a set theoretic formalization of concepts (for important steps towards this, see Goertz 2005b and Quaranta 2010; for an application of this to the study of welfare states, see Kvist 2006 and 2007).

This idea of properties of concepts is also linked to the remaining points on the aspects of the definition of the reference population and on limited diversity.

3.4 The Construction of the Population and ‘Limited Diversity’

As is well known, sampling techniques are very important for statistical analysis. Inferential statistics is a way to infer from a sample to a population: significance tests indicate in how far the parameters obtained for a sample are reliable as estimates of the respective parameters in the population. Similarly, confidence intervals (calculated on the basis of sample results) are estimates for the parameters of the population, specifying a range within which the population value of the parameter will be contained. In the consequence, statistical techniques necessarily need to rely on carefully elaborated and un-biased sampling strategies. Experiments, where the researcher uses a random model in order to attribute the values of the independent variable to the single cases, or other forms of random selection which can count as approximations to experiments, are fundamental for the success of statistical analysis. Since experiments or random selection are strategies which are sometimes impossible (or not desirable) for many social science questions, above all, if the number of cases is low, comparative scholars have developed a parallel literature on case selection in non-random settings (see King et al. 1994: 115ff. and the respective sections in Brady and Collier 2004; George and Bennett 2005; and Gerring 2012). Often, however, the reflections on sampling and case selection have excluded the aspect of the definition of the population. Ragin formulates a clear critique on statistical approaches with regard to this. In his view, “[t]he concept of population is rarely problematized in variable-oriented research. In this approach, most populations are seen simply as

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26 Kvist’s contributions are admittedly less on concept formation, but on the definition of typologies. However, if we consider typologies to be extensions of concepts, then it becomes reasonable to also refer to his work.
I would like to modify this somewhat rigid statement, agreeing with Ragin that the important step of the accurate definition of the reference population is often overlooked in the research design process, but, in my view, this does not only happen in quantitative analysis, but also in case-oriented approaches. The population tells us if the conclusions are generalizable to, e.g., all countries in the world, or only to consolidated democracies, or only to EU member countries, or only to the EU members before Eastward Enlargement, etc. The fundamental question becomes ‘what is my case a case of’ (Ragin 2000: 53f.), and for which larger phenomenon the cases under examination are examples.

The principles of the so-called ‘truth tables’ which are typical for QCA and their ‘diversity-oriented’ character (Ragin 2000: 19) show both the range and the limits of the underlying population. This is expressed in the various combinations of case properties which can be achieved with the help of a truth table: it is indispensable to be able to recur to some ‘variance’ (in order to (ab)use a term from quantitative methodology) in all the columns which denote the properties of the single cases.27

This point is closely linked to a model, which had originally been developed by Lazarsfeld (1937), namely, the property space. In a property space, all properties of the cases can be combined with one another, and every case can be located with regard to its position on all properties.28 In the language of set theory, all sets representing properties or concepts have to be intersected with one another in order to constitute configurations or ideal types of a property space, as defined by Lazarsfeld. This idea is also most central for QCA, since the rows of a truth table can also be interpreted as indicators of the cases’ positions in a property space. All cases can be numerically and geometrically be attributed to a

27 This is a point where QCA goes beyond the comparative principles specified in Mill’s methods. Whereas in Mill’s methods either the effect or (most of) the causal conditions have to show a similarity among the cases, QCA only works well if both the explanandum and the explanans are as diversified as possible. If the explanans did not vary, then the so-called ‘limited diversity problem’ could become crucial (see below). If the explanandum did not vary, all the problems connected to the literature on ‘selection on the dependent variable’ would emerge (for a clear rejection of such a design, see King et al. 1994: 129ff.; for more moderate opinions, see Dion 1998; Munck 1998; Ragin 2004; 2008, 4). Both in case of non-variation of the explanans and of the explanandum, the more general problem of skewed set membership scores comes in (for a systematic approach to this, see Schneider and Wagemann 2012, but also the more example-based observations in Cooper and Glaesser 2011).

28 The metaphor of a space can obviously only be used for three properties and a resulting three-dimensional space. But the general idea can be generalized to all other numbers of properties.
precise location within the property space.\textsuperscript{29} The borders of the property space represent the borders for the generalizability of the results. Since it is technically obvious that no case can be located outside the property space,\textsuperscript{30} the results which are obtained through an analysis of the cases within the property space cannot be generalized to eventual cases outside the property space, or beyond the properties. It is here where the debate sets in, whether generalization beyond the cases under research is a central interest in QCA (for some contributions, see Goertz 2005a and Waldner 2005). Indeed, there is no theory which precisely defines if and how QCA can be used in an inferential way, generalizing the results beyond the cases which are examined. In an extreme opinion, it could be even claimed that QCA just structures the data without going anywhere beyond. But also without accepting such a provocation, the fact remains that QCA, through the idea of a property space, is helpful in the examination of so-called ‘middle-range’ theories (Merton 1957; see also Rueschemeyer 2003: 328).

Taking up again the issue of how a truth table is constructed (Ragin 1987: 87f.; Schneider and Wagemann 2007: 44), every case can be described through a combination of the properties of a truth table. If two or more cases are equal with regard to their properties (or similar, if fuzzy sets are used), then they belong to the same truth table row. Following this logic, the truth table rows do not really represent the cases, but the configurations thereof. Therefore, the number of cases is usually (even much) higher than the number of configurations and truth table rows. The logic of how the configurations are composed also gives an answer to the question which minimum number of cases is needed in order to carry out a QCA. Indeed, QCA is often called a ‘small-n method’. This is not completely wrong, although such a statement should not be made without any reference to the relation between the number of cases and the number of variables (Wagemann 2008: 252). Even more, diversity as a fundamental principle of QCA requires a minimum number of cases in order to be able to apply the analysis without any problems. Mathematically speaking, the number of truth table rows is an exponential function of the number of included conditions:

\[2^{9}\]

This is an important point why to have doubts whether the variant of mvQCA is linked to set theory, since the idea of the property space cannot be applied to multinomial categories.

\[30\]

In set theoretic terms, this means that every case must belong to exactly one intersection of the sets which describe all possible combinations of properties.
\[ r = 2^k, \]

with \( r \) being the number of different truth table rows (i.e., configurations) and \( k \) the number of conditions. Four truth table rows (or configurations) exist in the analysis of two conditions; 16 in case of four conditions; and no less than 1,024 in case of ten conditions. Since, as mentioned, a configuration can refer to more than one empirical case, the number of configurations is equal to the number of cases only if all cases are different (with regard to their properties) from one another. In order to have information on all truth table rows, as many different cases as truth table rows are needed. This means that not any 1,024 cases, but 1,024 different cases are needed for the analysis of ten conditions. As research experience shows, this means to include many more than just 1,024 cases in practice. If we invert the prospective, then the approximately 200 countries in the world could just be analyzed with regard to seven conditions, if the additional requirement were fulfilled that 128 countries were different from one another with regard to the seven conditions/properties. Consequently, it would be more correct to call QCA a ‘medium-n method’, even if only a modest number of conditions (three, four or five) is analyzed.

However, it is almost impossible to work with ‘complete’ truth tables in this sense. It will hardly be possible to find all the variance in the real and observable world which is theoretically contained in the list of all theoretically possible configurations. QCA uses the label of ‘limited diversity’ for the phenomenon that theoretically existing configurations do not exist in the empirical reality (Ragin 1987: 104ff.; 2000: 198ff.; Schneider and Wagemann 2007: 101ff., 195f.). There are three sources for limited diversity: first, the configuration is based on a paradox (such as the famous ‘pregnant man’); second, the configuration can theoretically exist, but does not exist empirically, because of the social world being complex and shaped by various historical, cultural and other processes (an example for this would be the combination of the properties ‘being a woman’ and ‘being the President of the United States of America’); and third, we simply analyze too small numbers of cases in order to cover all the configurations (an example would be an analysis of the 27 EU countries with five conditions which results in 32 truth table rows; even if the EU countries are maximally different, five of these
truth table rows will be void of cases). Limited diversity is a very important problem for comparative research in general, and the various technical ‘solutions’ (which – in order to be honest – do not resolve much) can lead to different results. A first proposal is to use computer simulations and to opt for the most parsimonious solution (Schneider and Wagemann 2007: 106f.). This is, of course, an option which we can easily discard, since the application of a computer algorithm is perhaps the least useful strategy, although it does not contradict the empirical information contained in the truth table, since the empirically existing configurations are not modified. Nevertheless, exactly this strategy is applied in (too) many applications. The second proposal is to base the solution only on those configurations for which empirical information is available and to esclude all remainders from the reasoning (Schneider and Wagemann 2007: 107). This solution, wrongly called ‘complex solution’, has the interesting characteristic that it is a subset of all other possible solutions which can be obtained respecting the truth value of the empirical information (Schneider and Wagemann 2012). The third strategy is based on the use of easy counterfactuals and so-called directional expectations and formulates the so-called intermediate solution (Ragin 2008: 147ff.). For this, only those possible minimizations of the truth tables are permitted which are not only supersets of the ‘complex solution’ (something which applies to all solutions by definition), but also subsets of the most parsimonious solution. This means that set theory again plays an important role – this time in establishing in how far a researcher can use theoretical and counterfactual reasoning going beyond the empirically available information in order to choose a theoretically and empirically valid solution term which at the same time fulfils subset-superset criteria.

31 The expression ‘complex’ is misleading, since this is not automatically the most complex solution which can be gained from one and the same truth table showing limited diversity. Schneider and Wagemann (2012) propose to call this the ‘conservative solution’.

32 The double subset-superset relationship can be criticized, since it excludes the technically possible solutions which are supersets of the most parsimonious solution (Schneider and Wagemann 2012).
4 QCA as a Set Theoretic Method: Applications, Agendas, Outlook

4.1 Examples for Publications with QCA

It can certainly not be the task of a short paper to present a long review of QCA applications (for more on publications with QCA, see Yamasaki and Rihoux 2008 and Rihoux et al. 2012). But it is certainly possible to list at least a few recently published or path-breaking applications of it. It is obvious that it is above all the field of (macro) Comparative Politics which invites for the application of QCA since both the described form of causal complexity and the mid-sized n are very frequent in Comparative Politics. Most importantly, we have to quote Berg-Schlosser’s (2008) contribution on success and failure of democratization in Africa; Schneider’s (2008) excellent work on the consolidation of democracy; or Avdagic’s contribution on social pacts (2010). More recently, Ackrén (2009) has presented an analysis on different autonomy regimes where not nation-states, but (more or less) autonomous regions are the units of analysis. Blatter et al. (2010) also use regions as units of analysis and discuss their foreign activities. Vis (2009) works on different governments and analyses the conditions for when they propose unpopular social reforms. Going beyond Comparative Politics, it has only been recently that first important contributions are also published in Comparative Public Policy Analysis. Examples for this are Emmenegger’s (2011) work on job security regulations in Western democracies and Mayer’s et al. (2011) analysis of the conditions for the policy impact of public regulations on private energy saving. As far as the sub-discipline of International Relations is concerned, the applications are still rare. However, Koenig-Archibugi’s (2004) analysis in which he explains government preferences for institutional change in EU foreign and security policy has been path-breaking.

Outside of political science, we find some often-quoted applications in sociology, such as Ragin’s et al. (2003) study on cooperation in Indian villages; Cress’ and Snow’s (2000) study on the mobilization of the homeless; or Hollstein’s and Wagemann’s (2012) work on the transition from school to work, only focussing on young people who have already undergone a failure when they tried
to enter the labor market in a first attempt. There are, of course, also applications in psychology, education science, business strategies, and even comparative linguistics.

4.2 QCA 2022 – What has to be achieved?

In this paper, it was demonstrated how a set theoretic method such as QCA enriches our methodological repertoire. First, it was shown how set relations contribute to the analysis of sufficiency and necessity in general and of INUS and SUIN conditions in particular. Second, this made it possible to model complex causal relations, signified by equifinal, conjunctural and asymmetric patterns. Set theory was also identified as a useful tool for the definition of concepts as it is needed in the calibration of fuzzy sets; however, set theory can also be applied in the definition of concepts in general. Finally, set theory becomes important when cases are seen as configuration of their properties. Not only does set theory help to assign cases to configurations, but subset-superset relations also allow for the use of counterfactual thinking in the most recent formulation of ‘intermediate solutions’ (Ragin 2008: 147ff.). This means that linear algebra is not the only mathematical subdisciplin which is useful for social science analysis – set theory might even correspond better in many cases to our ideas of how the social world is made up. Set theory might correspond to many of our ideas of subset-superset relations in the social world, or to the need of a complex approach towards explanation. ‘Qualitative Comparative Analysis’ (QCA) is just one set-theoretic method, but certainly the best formalized and most widely known one. QCA respects the set theoretic character of many of our hypotheses and helps us to work with causal complexity. Certainly, this also has a negative side: QCA results (and set theoretic insights in general) are often not very parsimonious and easily interpretable. They can have an impressing and often also discouraging complexity and length. Obviously, the doubt remains if the complexity of the result could actually be a positive feature of the analysis, since it seems to correspond better to the world which we analyze (Hall 2003).

QCA is certainly still a young method. This means that it can and has to be developed further. A possible agenda for the near future of QCA could be the following:
1. The discussion on the calibration of fuzzy values (see 3.3 and Ragin 2008: 71ff.) should be continued and further specified. Credibility and reliability of QCA depend very much on this. Critiques cannot only be answered with the repeated comment that transparency would solve the problem of calibration.

2. The problem of limited diversity and its various effects should be taken more seriously. QCA scholars should render the strategies how they deal with limited diversity more explicit. From the perspective of methodological development, the existing proposals on how to deal with limited diversity have to be reassessed and eventually complemented. Certainly, the most recent proposal (Ragin 2008: 147ff.) on the use of easy counterfactuals is the most important step forward. But although set theory has been praised in this paper as an important epistemological underpinning of case comparisons, this must not mean that set relations can be the only criteria for assessing different solutions for limited diversity. Going beyond what has been called the ‘Standard Analysis’, Schneider and Wagemann (2012) make a first step in elaborating these proposals further: they propose an ‘Enhanced Standard Analysis’ (ESA) which excludes all incoherent and impossible assumptions from the analysis and a ‘Theory-Guided Enhanced Standard Analysis’ (TESA) where even the idea of parsimony is given up, in the favor of a greater emphasis on theoretical assumptions. More on this should be added.

3. A code of standards for a good-quality QCA (for a proposal, see Schneider and Wagemann 2010) has to be observed. It is true that QCA technically works even if no rules of good behavior are respected, but this is also true for more diffused methods, such as regression, which also sometimes suffer from a superficial application. As clear rules exist for regression analysis and regression diagnostics, the same rigor should be used for QCA. This seems to be the most pressing point, because otherwise a broad diffusion of a ‘low-quality’ QCA could even cause serious problems in the recognition of the technique.
4. More has to be done on combining QCA with other methods and techniques. QCA offers two notable possibilities for this methodological integration: first, when formulating fuzzy values, it is clear that they can come from different sources and methods, ranging from questionnaires to texts, interviews, observations, secondary data, etc. There is also no reason why statistical techniques such as factor analysis, cluster analysis, or index-building strategies in general should not be given more weight in determining fuzzy values. Second, QCA can also be connected with other methods in terms of research design. A good possibility could be to use a QCA to map the available cases from which then a limited number (or even one) are selected for an in-depth case study (see Schneider and Rohlfing 2011 for a very good and elaborate proposal on this). Any other combinations of methods are imaginable. This can also result in a sequencing of hypothesis assessment: a covariational hypothesis could be de-composed in various hypotheses on the sufficiency and necessity of conditions, and the causal mechanisms could be tested in subsequent case studies.

In sum, this means that QCA should also be discussed in terms of triangulation (Della Porta and Keating 2008: 34; Seawright and Collier 2004: 310; Tarrow 2004: 174). It cannot stand alone as a method, as neither of the other methods can. No method should be declared the ‘winner’ of a methodological competition, as unfortunately often happens in comparisons between QCA and other techniques. On the contrary, triangulation and mixed-method designs serve at an integration of various methodologies in order to create spaces for the specific advantages of each technique. Obviously, triangulation must not be artificial.

In any case, if we do not try to promote QCA as the new method which solves all methodological problems or which would even rival the well-established statistical techniques, then we might come

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33 Hollstein and Wagemann (2012), e.g., combine QCA with network analysis.
34 Apart from this, a comparison of QCA and statistical methods does not make sense, even if the same data is used. Both methodological traditions start off from two different epistemological foundations. It is therefore not so surprising that they often lead to different results, and that one methodological tradition turns out to be inferior to the other one: if the questions (or the research interests) differ, then it is not surprising that the results also differ.
easily to the conviction that QCA is a good achievement in comparative methodology, which renders
the set-theoretic foundation of comparative research explicit, standardizes it, and makes it
manageable.
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